A TREATISE

ON

MARINE SURVEYING.

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A TREATISE ON MARINE SURVEYING.

BY

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OF THE HON. EAST INDIA COMPANY'S SERVICE.

——— When the main,
Worked into fury, many a mighty ship
Wrecks ruthless, and tow'rds every coast impels
Masts, yards, and streamers, cordage, sails, and helms,
And parted planks; they teach us as they float
What dangers lurk unseen; what snares to lure
Unthinking mortals!

GOOD'S LUCRETIUS.

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PREFACE.

It is the object of this Treatise on Marine Surveying to supply a desideratum, which has long existed; inasmuch as there is not at present any elementary treatise which nautical men can consult on this important branch of science. During the residence of the Author in India, he had often reason to regret the want of such a text-book; and taking advantage of the experience he enjoyed under Captain D. Ross, Marine Surveyor-General of India, whose splendid survey of the China Seas and Straits of Malacca has justly excited the highest admiration, he has endeavoured to compile a volume, which he ventures to hope will be found of practical utility. It is, he apprehends, unnecessary for him to dwell on the importance of a science, by the aid of which alone clear and correct geographical knowledge can be obtained of those distant countries with which it is our interest to maintain social and commercial intercourse. The loss of life, the wreck of property, the disastrous accidents which so often occur, from vessels being run upon sand-banks, or striking against rocks not accurately delineated on

charts, must be familiar to every person. Almost every newspaper we take up teems with melancholy details of shipwrecks; for although the coast of Europe may have been well surveyed, little is known of the physical character of coasts in other parts of the world. It is, therefore, evidently desirable that every person engaged in sea service, whether he have the charge of a merchant vessel or man-of-war, should understand and be able to apply practically the principles of Marine Surveying; and in this case the Author feels convinced that, being more generally attended to, this science would enjoy that progressive advancement which has recently signalized the career of every other department of human knowledge.

The arrangement which the Author has adopted in the present Treatise is the following:—He has commenced with an explanation of the principles of Plane Trigonometry, because this forms the foundation of Marine Surveying, Navigation, and almost every branch of Practical Mathematics. He has in this part of the work detailed all the most simple methods of calculating the sides and angles of triangles; illustrated each rule by appropriate examples, with the necessary calculations set down at full length; and at the end of each rule has subjoined examples for practice, with their respective answers.

He has next explained the application of Plane Trigonometry to the mensuration of heights and distances, in which, by introducing a great variety of examples,

he has attempted to bring into practice all the rules previously laid down; and has concluded this part with a short account of Terrestrial Refraction, and the method of correcting an observed angle for the curvature of the earth.

As the accuracy of a survey so much depends upon the instruments employed, he has thought it expedient to give a description of those best adapted for this purpose. These are the Theodolite, Sextant, Reflecting Circle, Prismatic Compass, and Artificial Horizon; a very minute description of which instruments has been given, with the methods of adjusting and using them. For this part of the subject the Author is much indebted to the valuable work lately published by Mr Simms on Mathematical Instruments. The description of instruments is followed by some Problems in Nautical Astronomy, for determining the time, the latitude, and the longitude from observation. On this part of the subject the Author has dwelt longer than he originally intended, and, perhaps, than may by some persons be thought necessary, seeing that we have already so many valuable treatises devoted to these subjects alone; but when it is considered that, however correct a survey may be in other respects, it would not, in a geographical point of view, be complete unless the latitude and longitude of some remarkable object or headland were given, and as he wished this Treatise should contain every thing which is requisite to be known by the surveyor, independently of other

treatises, he trusts these problems will neither be thought unnecessary nor misplaced.

In proceeding to survey a coast, the first object is to measure a base line, to be used as an element in the succeeding calculations. He has therefore shewn the method of measuring such a line, both on land and water; and has given an account of various experiments made to ascertain the velocity of sound, with a table of the results, and the names of the experimenters; also the method of applying the velocity of sound to the measurement of a base line. He has then explained the method of finding trigonometrically the latitude and longitude of any place when the latitude and longitude of some other place not very remote is known:—after which the rule and examples for finding the variation of the compass by amplitudes and azimuths, by which the true direction of the base line may be determined: also three methods of ascertaining the meridian line—1*st*, by equal altitudes of a star; 2*d*, by equal azimuths; 3*d*, by a circumpolar star.

The subject he has next entered upon is the various methods of finding the station of the ship, by protracting the measured angles subtended by three hills, or fixed objects on land, under all circumstances; with a variety of examples under each case, calculated at length, and several examples for practice. On account of its great practical utility and importance, he has treated this subject at considerable length; and he hopes that among the various problems for finding the

station-point, containing as they do such a variety of modes of investigation, all demonstrated geometrically, the young officer will find much useful matter. On this part of the present Treatise he has bestowed much labour and attention, and has taken special care that the figures should be accurately constructed, and the protracted lines and angles made to agree exactly with the results determined by logarithmic calculation.

The references from the text to the figures are numerically arranged; and, to render them as clear and comprehensive as possible, they have been constructed on a large scale, which has caused them to occupy no less than seven copper-plates.

The last subject of which he has treated is the practice of Marine Surveying under all circumstances, whether coast, harbour, or bay; with the method of protracting soundings with the Station-Pointer, and a description of that instrument: also the most easy manner of registering the soundings and angles on the Log-slate, with many useful practical hints while exploring and surveying a coast. The heads of the various subjects treated of will be found in the table of Contents.

The work concludes with a Table of Logarithms of Numbers from an unit to ten thousand; a Table of Logarithmic Sines and Tangents, &c.; also a Table of Natural Sines, which it is hoped will render the work complete, and save the expense of a larger set of

tables; the present tables having been compared with the tables of Hutton, Callet, Taylor, Briggs, &c.

Besides his obligations to Captain D. Ross, the Author has also to acknowledge the kind attentions he received from Lieut. R. Lloyd, the Assistant Marine Surveyor of India, to both of which gentlemen he feels indebted for much valuable practical knowledge. The author also begs, very respectfully, to tender his thanks to Mr Wallace, the eminent Professor of Mathematics in the University of Edinburgh; and likewise to Mr Forbes, the Professor of Natural Philosophy: to both these gentlemen he is indebted for much information; nor can he sufficiently acknowledge their urbanity in private, and the extensive knowledge which they communicate to their respective classes in public.

To the Rev. Alexander Wallace, A.M., and to Dr Stone, the author feels himself likewise under considerable obligation. The scientific and literary attainments of these gentlemen are already well known; and the author considers himself particularly fortunate in having had the advantage of becoming personally acquainted with them during his late residence in Edinburgh.

Edinburgh, *September* 1834.

A TREATISE

ON

MARINE SURVEYING.

When we reflect on the dangers which are incident to those who peril their lives and properties on the bosom of the "illimitable sea," and when we remember how vast is the importance which must ever be attached to our commercial intercourse with distant and foreign nations, we shall, without hesitation, admit that the principles by which the seaman may be enabled to steer clear of the rocks and shoals that lie scattered below the surface of the ocean, demand the most earnest investigation. Without such information being under his practical command, his vessel must frequently run the most frightful hazards of shipwreck; and the art of navigation, so far as he is concerned, will prove of no more avail than it would have done in the rudest and darkest ages of the world.

Accordingly, the art of marine surveying is one not only of geographical or scientific interest, but of great national importance, inasmuch as it will enable the nautical man to steer in safety along any unknown coast, or enter, without apprehension, any harbour in which he may desire to anchor his vessel. Thus, without any loss either of property or life, may our intercourse with the most distant countries be extended.

Let us then proceed to explain the method which must be adopted in surveying a coast or harbour. To effect this, it is necessary to measure, on a convenient plain or beach, a long base line, between the extremes of which and other objects, angles must be taken, so as to determine, with mathe-

matical precision, the exact position of at least *three* of those objects. They ought, if possible, to be conspicuous enough to be seen by the observer at a great distance; and the finer the termination of their summits the better, for thereby we obtain a greater degree of nicety in the measurement of the angle subtended by them, by which means the station of the ship may at all times be determined, while sounding and exploring the coast, by a protraction of the two angles so taken. When, however, a greater degree of correctness is required, such as to fix the position of a rock or shoal that it may be laid down on the chart in its true place, recourse must be had not only to a protraction, but to an absolute trigonometrical calculation of the sides and angles, by which we can alone expect an accurate result.

If suitable hills or remarkable objects happen not to be found, then artificial means must be resorted to, by having marks or signal-posts erected, the mutual distances of which must, as in the above case, be determined by angles taken from the extremities of a correctly measured base line. This system of surveying, whether on shore or afloat, is acknowledged to possess a decided superiority over all other methods, being not only more correct but infinitely more comprehensive and expeditious; and should an error occur in determining one point or station along the coast, it will not in the slightest degree affect the rest, unless it be afterwards made a link in the chain of triangles carried forward, but will be confined strictly to that identical part, and may at any future period be rectified, without in any manner interfering with the trigonometrical arrangement of the survey. The principal consideration, therefore, is to make a judicious selection of the *three* objects at the commencement of the survey, choosing those with well-defined sharp summits, and so situated as to form as nearly as possible an equilateral and equiangular triangle; for on this arrangement depends the accurate determination of the various rocks, shoals, and points along the whole line of the coast. As all the cases in surveying are solved by trigonometry, where great accuracy is required, it becomes absolutely necessary that those who wish to acquire a knowledge of this science should be familiar with decimal and fractional arithmetic, with the first six books of Euclid's Elements, and practical geometry: he should possess a knowledge of some of the fixed stars, and understand the logarithmic calculation of plane and spherical trigonometry; he should be able to use with facility the sextant, in determining the latitude by a meridian

altitude of the sun, moon, and stars, both with and without an artificial horizon, also by double and equal altitudes; he should be able to use the theodolite, azimuth, and prismatic compasses, and should have studied well the constitution and adjustment of the various instruments made use of in surveying, as described under that head. A knowledge of drawing, and an expert habit of protracting lines and angles, with the aids of a divider and a scale of equal parts, will also be of the utmost utility.

PLANE TRIGONOMETRY.

Two geometrical magnitudes of the same kind may be compared with each other, by assuming another magnitude of the same kind, which is a part of both, and, employing this last magnitude as a measuring unit, finding how often it is contained in each of the others. The quotient thus obtained will express arithmetically the relation of the two given magnitudes. Thus, assuming one inch as the measuring unit, if, of two straight lines, the one contain the measuring unit ten times, and the other contain it fifteen times—that is, if the one line is 10 inches and the other 15 inches—the numbers 10 and 15 will express the ratio of these lines to each other, and may be taken to represent the lines themselves.

In like manner, assuming any determinate part of the circumference of a circle as a measuring unit, and finding how often it is contained in each of two given arcs, the quotients will express the ratio of the arcs to each other, and may be used as representatives of the arcs themselves. And as angles at the centre of equal circles, or of the same circle, are to each other as the arcs on which they stand (Euc. vi. 33), the numbers representing the arcs may also be used to denote the angles.

In this way numbers may be employed to express the relation of the sides and angles of triangles to each other. This application of numbers to geometrical magnitudes is called Trigonometry.

Before proceeding to give the definitions to be employed in trigonometry, it will be necessary to obtain a proper representation for the measure of an angle.

Let ADBE be a circle (fig. 1), and draw the diameters AB, DE at right angles to each other, which will divide the cir-

cumference into four equal parts, AD, DB, BE, EA, each of which is called a *quadrant*.

Draw any straight line CF from the centre to the circumference, then (Euc. vi. 33) the angles ACF, ACD, are to each other as the arcs AF, AD; so that if the magnitude of the angle ACF be represented by the arc AF, the magnitude of the angle ACD will be represented by the arc AD; and so of all other angles. The magnitude of an angle is in all cases measured by the arc which subtends it in a circle described with a given radius.

For the purpose of exhibiting *arithmetically* the magnitude of angles, the circumference of the circle is supposed to be divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* each minute into 60 equal parts, called *seconds;* noted thus, ° degrees, ′ minutes, ″ seconds, &c. And since arcs are the measures of angles, every angle may be termed an angle of such number of degrees, minutes, and seconds, as the arc which subtends it contains. Thus, if the arc AF contains 36 degrees, 45 minutes, 33 seconds, the angle ACF (making use of the above notation) is said to be an angle of 36° 45′ 33″. The quadrants AD, DB, BE, EA, contain (as is evident) 90° each.

DEFINITIONS.

1. The difference between any angle ACF (fig. 1) and a right angle, or 90°, is called the *complement* of that angle. Thus, if ACF is an angle of 25° 5′ 2″, its *complement* FCD will be an angle of 64° 54′ 58″.

2. The supplement of an angle is the difference between it and 180°, or what it wants of two right angles. Thus, if the angle ACF (fig. 1) is 50° 33′ 24″, its *supplement* FCB will be 129° 26′ 36″.

3. The straight line AF (fig. 2), drawn from one extremity of the arc to the other, is called the chord of the arc AF.

4. FG (fig. 2), a line drawn from one extremity of the arc AF perpendicular to the diameter AB, passing through the *other* extremity, is called the *sine* of the arc AF, or of the angle ACF.

5. The versed sine is the segment of the diameter between the sine and the circumference. The line AG (fig. 2) is the versed sine of the arc AF, or of the angle ACF.

6. If a line be drawn touching the circle in A (fig. 2), and the radius CF be produced to meet it in the point K, then AK is called the *tangent*, and CK the *secant* of the arc AF, or of the angle ACF.

Corollary to Definitions 4 *and* 6.—The sine, tangent, and secant of any angle ACF (fig. 2), are also the sine, tangent, and secant of its supplement BCF.

It is evident from def. 4, that FG is the sine of the angle BCF. Let FC be produced to meet the circumference in the point M, and it is manifest that AK is the tangent, and CK the secant of the angle MCA. But MCA is equal to its vertical angle BCF; therefore AK, CK, are respectively the tangent and secant of the angle BCF.

7. If a line be drawn touching the circle in D (fig. 2), and CF be produced to meet it in L, and FH be drawn perpendicular to the diameter DE, then FH, DH, DL, and CL, become respectively the sine, versed-sine, tangent, and secant of the angle FCD, which is the *complement* of the angle ACF, and are therefore called the *cosine*, *coversed-sine*, *cotangent*, and *cosecant* of the angle ACF.

Corollary.—Since CG (fig. 2) is equal to FH (Euc. i. 34), it is equal to the cosine of the arc AF. Hence the *cosine* of any arc is equal to that part of the radius of the circle which is intercepted between the *centre of the circle* and the *extremity of the sine* of that arc.

A few necessary properties of arcs and angles demonstrated geometrically:—

Property I. (Fig. 3.)

The chord of an arc is double the sine of half that arc.

Let AF be the chord of the arc ADF. Draw CG at right angles to AF and produce it to D. Then (Euc. iii. 3) CG bisects the chord AF, and (Euc. iii. 30) it also bisects the arc AF in D. Hence the chord AF $= 2$FG, and the arc AF $= 2$FD, or FD $= \frac{1}{2}$AF. Now FG $=$ sine of arc FD $=$ sine $\frac{1}{2}$ arc AF; therefore, the chord AF $=$ (2 FG) $=$ twice the sine of $\frac{1}{2}$ arc AF.

Property II. (Fig. 3.)

The chord of 60° *is equal to the radius of the circle.*

Let AF be an arc of 60°, then the angle ACF of the triangle ACF is 60°; and since the three angles of the triangle are equal to 180°, the two remaining angles CAF, CFA, must be equal to 120°. But CA $=$ CF; hence the angle CAF $=$ CFA, and each of them is 60°. Therefore, the triangle CAF is equiangular and equilateral; wherefore the chord AF $=$ (AC or CF) $=$ radius.

Property III. (Fig. 3.)

The sine of 30° is equal to half the radius.

By Property I. the sine of an arc is half the chord of double the arc; if, therefore, AF is 60°, FD will be 30°, and its sine FG = ½AF = (by Property II.) ½ radius.

Property IV. (Fig. 4.)

The tangent of 45° is equal to the radius.

Let the arc AF = 45°, then the angle ACK = 45°; and since the angle CAK = 90°, the remaining angle AKC must be 45°: hence the angle ACK = the angle AKC, and the tangent AK = AC = radius.

RIGHT-ANGLED TRIANGLES.

A right-angled triangle consists of six parts—viz. the three sides and three angles. Of the six parts mentioned, any three being given, except the three angles, the remaining three may be found by calculation.

N.B.—The three angles of every triangle are together equal to two right angles, or 180° (Euc. i. 32); if, therefore, one of the angles of any triangle be a right angle, the two remaining angles must together be equal to 90°. When, therefore, one of the acute angles is given, the other may be found by subtracting that given angle from 90°.

The rules for computing the sides and angles of right-angled triangles are deduced from the following

PROPOSITION.

In any right-angled triangle, the hypothenuse is to either of the sides as radius is to the sine of the angle opposite to that side; and as either of the sides is to the other side, so is radius to the tangent of the angle opposite to that side.

Let ABC (fig. 5) be a right-angled triangle, having the right angle at C, with any distance AD as radius, describe an arch of a circle DE, cutting AC, AB, in D and E, draw DF and EG perpendicular to AC, then DF is the sine, and GE the tangent of angle A. Then because the angle AFD is equal to the angle ACB, being both right angles, and the angle A is common to the two triangles AFD, ACB, these triangles are similar (Euc. vi. 4.); therefore AB is to BC as AD to DF. But AD is the radius, and DF the sine of angle A; therefore AB is to BC as radius to

sine A. Again, because in the triangles ACB, AEG, the angle A is common and AEG is equal to ACB, being both right angles, these two triangles are similar (Euc. vi. 4.); therefore AC is to CB as AE to EG. But AE is the radius, and EG the tangent of angle A; therefore AC is to CB as radius to tangent A.

From the above proposition it is evident, that if the hypothenuse AB (fig. 6) of a right-angled triangle be made radius, the side BC becomes the sine of the angle BAC opposite to it, or the co-sine of the angle ABC, since these angles are the complements of each other, and the side AC becomes the co-sine of angle BAC or sine of angle ABC.

But if the side AC (fig. 7) be made radius, the other side BC becomes the tangent of the angle BAC or co-tangent of the angle ABC, and AB, the hypothenuse, becomes the secant of the angle BAC, or co-secant of ABC.

In like manner, if BC (fig. 8) be made radius, AC becomes the tangent of the angle ABC, or co-tangent of BAC, and AB becomes the secant of ABC, or the co-secant of BAC.

The solution of right-angled triangles may be divided into the four following cases:—

1. When the angles and hypothenuse are given.
2. When the angles and one side are given.
3. When the hypothenuse and one side are given.
4. When the sides are given.

In each of these cases, the three remaining parts of the triangle may be found in the following manner:—

If it be required to find a side, write the word radius on one side of the triangle, and then consider what the other sides become, and write upon them their respective names, agreeably to what is shown in figures 6, 7, 8: then say—

As the name upon the given side
Is to the name upon the required side,
So is the given side
To the required side.

If an angle be sought, write the word radius upon a given side, and, as before, mark what the other sides become: then say—

As the side made radius
Is to the other given side,
So is radius
To the name upon that other side.

Find the logarithms of the three given terms in the tables of logarithms.

Add the logarithms of the second and third terms together, and from their sum subtract the logarithm of the first term, and the remainder will be the logarithm of the fourth term; which being found in the table bearing its name, either *numbers*, or *degrees*, *minutes*, *&c.* will give the required angle or side.

PROBLEM I.

Given the angles and hypothenuse to find the remaining sides.

In the right-angled triangle ABC (fig. 9), having the right angle at C, there are given the hypothenuse AB, equal to 135 feet, the angle BAC equal to 25° 30′. Required the sides AC and BC? Since the angle BAC is 25° 30′, the angle ABC, which is found by subtracting BAC from 90°, will be 64° 30′.

GEOMETRICAL CONSTRUCTION.

At the point A, in an indefinite straight line AC, make, by means of a protractor or scale of chords, an angle BAC equal to 25° 30′; from a scale of equal parts take off 135, and apply it to the line AB, from the point B draw BC perpendicular to AC, then shall ABC be the triangle required, and BC, AC, applied to the same scale from which AB was taken, will give the length of the sides as near as protraction will admit; BC will be equal to 58·12, and AC equal to 121·85, nearly.

CALCULATION BY LOGARITHMS.

Making AB, the hypothenuse, radius.

To find BC.		To find AC.	
As radius . .	10·0000000	As radius . . .	10·0000000
Is to sine A 25° 30′	9·6339844	Is to sine B 64° 30′	9·9554882
So is AB 135 . .	2·1303338	So is AB 135 .	2·1303338
	11·7643182		12·0858220
To BC 58·12 .	1·7643182	To AC 121·85	2·0858220

Making AC, the base, radius.

To find BC.		To find AC.	
As sec. A 25° 30′	10·0445118	As sec. A 25° 30′	10·0445118
Is to tan. A 25° 30′	9·6784961	Is to radius . .	10·0000000
So is AB 135 . .	2·1303338	So is AB 135 . .	2·1303338
	11·8088299		12·1303338
To BC 58.12 . .	1·7643181	To AC 121·85 .	2·0858220

Making BC, the perpendicular, radius.

To find BC.		To find AC.	
As sec. B 64° 30′	10.3660156	As sec B 64° 30′ .	10.3660156
Is to radius . .	10.0000000	Is to tan. B 64° 30′	10.3215039
So is AB 135 . .	2.1303338	So is AB 135 . .	2.1303338
	12.1303338		12.4518377
To BC 58.12 . .	1.7643182	To AC 121.85 . .	2.0858221

By the above work it will be seen, that whichever side is made radius, the result is exactly the same.

EXAMPLES FOR PRACTICE.

1. The hypothenuse of a right-angled triangle is 374.9 yards, and one of the acute angles 38° 40′. What are the other two sides? *Ans.* 292·72, and 234·23.

2. A ladder, 85 feet long, was placed against the wall of a house, so that one end of it touched the top of the wall, and the other end made an angle with the ground of 36° 15′. Required the height of the wall, and the distance of the lower extremity of the ladder from it? *Ans.* 68·548 and 50·262.

3. A ship, from latitude 40° 36′ N., sails N.W. ½ W. 204 miles. What is her departure from the meridian, and the latitude come to? *Ans.* Dep. 157·70.
Lat. in 42° 45′ 25″.

PROBLEM II.

Given the angles and one side to find the hypothenuse and the other side.

In a right-angled triangle ABC (fig. 10), there are given the side AC 105·3 feet, and the angle A 44° 25′. Required the hypothenuse AB, and the side BC ?

GEOMETRICAL CONSTRUCTION.

Draw the straight line AC, and from a scale of equal parts, make it equal to 105·3; at the extremity A, by means of a protractor or scale of chords, make an angle CAB 44° 25′: from C draw CB perpendicular to AC, and let it meet AB in B. ABC is the triangle required; and AB, BC, applied to the same scale from which AC was measured, will give the length of AB, 147·42, and BC 103·18, nearly.

By subtracting 44° 25′ from 90°, the angle B will be found to be 45° 35′.

CALCULATION BY LOGARITHMS.

Making AB radius.

To find AB.		To find BC.	
As sine B 45° 35′	9.8538619	As sine B 45° 35′	9.8538619
Is to radius . .	10.0000000	Is to sine A 44° 25′	9.8450181
So is AC 105.3 .	2.0224284	So is AC 105.3 .	2.0224284
	12.0224284		11.8674465
To AB 147.42 .	2.1685665	To BC 103.18 .	2.0135846

Making AC radius.

To find AB.		To find BC.	
As radius . . .	10.0000000	As radius . . .	10.0000000
Is to sec. A 44° 25′	10.1461381	Is to tan. A 44° 25′	9.9911562
So is AC 105.3 .	2.0224284	So is AC 105.3 .	2.0224284
	12.1685665		12.0135846
To AB 147.42 .	2.1685665	To BC 103.18 .	2.0135846

Making BC radius.

To find AB.		To find BC.	
As tan. B 45° 35′	10.0088438	As tan. B 45° 35′	10.0088438
Is to sec. B 45° 35′	10.1549819	Is to radius . .	10.0000000
So is AC 105.3 .	2.0224284	So is AC 105.3 .	2.0224284
	12.1774103		12.0224284
To AB 147.42 .	2.1685665	To BC 103.18 .	2.0135846

EXAMPLES FOR PRACTICE.

1. Given a side of a right-angled triangle 359·6, and the adjacent acute angle 26° 13′. What are the hypothenuse and remaining side? *Ans.* Hyp. 400·83. Side 177·07.

2. What is the height of a steeple, whose angle of elevation, at the distance of 400 yards, is 34° 26′?
Ans. Height of steeple 274·23.

3. A ship sails between S. and W., on a course of 56° 15′, till her departure is 80 miles. What distance has she sailed, and what is her difference of latitude?
Ans. Dist. 96·215. Lat. 53·45.

PROBLEM III.

Given the hypothenuse and one side, to find the angles and the other side.

In a right-angled triangle ABC (fig. 11), given the base AC 4378, and the hypothenuse AB 5218, to find the angles A and B, and the other side BC.

GEOMETRICAL CONSTRUCTION.

Draw AC, and make it 4378 equal parts; erect the perpendicular BC of indefinite length; then, with a radius of 5218 equal parts, describe a circle, cutting BC in the point B, draw AB, and find the measure of the angles A = 32° 57′ 50″, and B = 57° 2′ 10″ by the line of chords, and the side BC equal to 2839·13 from the scale of equal parts from which AB and AC were taken.

CALCULATION BY LOGARITHMS.

Making AB radius.

To find the Angles B and A.		To find side BC	
As AB 5218 . . .	3·7175041	As rad.	10.0000000
Is to AC 4378 . . .	3·6412758	Is to sine A 32° 57′ 50″	9.7356870
So is radius . . .	10·0000000	So is AB 5218 . . .	3.7175041
	18·6412758		13·4531911
To sin B 57° 2′ 10″	9·9237717	To BC 2839.13 . .	3·4531911
90°			
57° 2′ 10″			
A = 32° 57′ 50″			

Making AC radius.

To find Angles B and A.		To find BC.	
As AC 4378 . .	3·6412758	As radius, . . .	10·0000000
Is to AB 5218 . .	3·7175041	Is to tan. A 32° 57′ 50″	9·8119180
So is radius . . .	10·0000000	So is AC 4378 . . .	3·6412758
	13·7175041		13·4531938
To sec A 32° 57′ 50″	10·0762283	To BC 2839·13 . .	3·4531938
90°			
32° 57′ 50″			
B = 57° 2′ 10″			

NOTE. The side BC may be found by (Euc. i. 47.) For $AB^2 = AC^2 + CB^2$; therefore $AB^2 - AC^2 = BC^2$, and $BC = \sqrt{AB^2 - AC^2}$.

EXAMPLES FOR PRACTICE.

1. A ship sails between N. and E. 120 miles, till her difference of latitude is one degree or 60 miles. Required her course and departure? *Ans.* Course 60°. Dep. 103·95.

2. At what distance from the bottom of a wall which is 34·57 feet high, must a ladder 70¾ feet in length be placed, so that its extremity may reach the top of the wall?

Ans. 61·738 feet.

3. A ship's main-stay is 14 fathoms, and the distance from the main-mast, along the deck to the night-heads, where it is set up is 50 feet. Required the whole height of the mast from the upper deck, the mast head being 14 feet?

Ans. 81·498 feet.

PROBLEM IV.

Given two sides, namely the base and perpendicular, to find the hypothenuse and the angles.

In the triangle ABC (fig. 12.), right-angled at C, there are given the side AC 335 feet, and the side BC 474 feet. Required the hypothenuse AB, and the angles A and B?

GEOMETRICAL CONSTRUCTION.

Draw AC and make it equal to 335, from a scale of equal parts; erect CB at right angles to AC, and make it equal to 474 from the same scale; join AB, and ABC is the triangle required. Measure AB upon the scale from which AC and CB were taken, and the angles A and B from a scale of chords, AB will be equal to 580·431, angle B = 35°. 15′. 3″, and the angle A = 54°. 44′. 57″.

CALCULATION BY LOGARITHMS.

Making AC radius.

To find the Angle A.		To find AB.	
As AC 335 . .	2.5250448	As radius	10.0000000
Is to BC 474 . .	2.6757783	Is to sec. A 54° 44′ 57″	10.2387059
So is rad. . . .	10.0000000	So is AC 335 . . .	2.5250448
	12.6757783		12.7637507
To tan. A 54° 44′ 57″	10.1507335	To AB 580.431 . .	2.7637507
90°			
54° 44′ 57″			
B = 35° 15′ 3″			

Making BC radius.

To find Angle A.		To find AB.	
As BC 474 . . .	2.6757783	As radius, . . .	10.0000000
Is to AC 335 . .	2.5250448	Is to sec. B 35° 15′ 8″	10.0879729
So is rad. . . .	10.0000000	So is BC 474 . . .	2.6757783
	12.5250448		12.7637512
To tan. B 35° 15′ 3″	9.8492665	To AB 580·431. . .	2.7637512
90°			
35° 15′ 3″			
A = 54° 44′ 57″			

EXAMPLES FOR PRACTICE.

1. A ship from latitude 56°. 51′. N. sails between S. and W. till she is in latitude 54° 25′ N., her departure being 32 miles. Required her course and distance?

Ans. Course S. 12° 21′ 45″, W. Dist. 149.47.

2. The height of a pole is 35 feet—what is the length of a rope stretched from the top of the pole to a point on the ground 17 feet from its base, and what angle does it make with the ground? *Ans.* 64° 5′ 37″. Length 38·91.

3. Three towns, A, B, and C, are so situated, that A bears from B S. 77° 45′ W., and C from B N. 12° 15′ W.; the distance of A from B is 42 miles, and that of C from B 65 miles. What is the distance between A and C, and how do they bear from each other?

Ans. C bears from A N. 20° 37′ 7″ E., or N. by E. ¾ E. nearly. A from C distant 77·389.

OBLIQUE-ANGLED TRIANGLES.

Of the three sides and three angles of an oblique-angled triangle, any three being given, one of which, however, must be a side, the other three may be found.

It is necessary that one of the given parts be a side, because the angles do not determine the magnitude of the triangle, and any number of equiangular and similar triangles may be constructed, the sides of which shall all be unequal. When two of the angles of a triangle are given, the third in effect is also given, being the difference between the sum of the other two and 180°. The solution of oblique-angled triangles may be comprehended under the four following cases:—

1. *When two angles and a side opposite to one of them are given.*
2. *When two sides and an angle opposite to one of them are given.*
3. *When two sides and the included angle are given.*
4. *When the three sides are given.*

The rule for the solution of cases 1 and 2 is derived from the following

PROPOSITION.

The sides of a plane triangle are to one another as the sines of the angles opposite to them,

From B (fig 13) any angle in the triangle ABC, let BD be drawn perpendicular to AC; and because the triangle BAD is

right-angled at D, BA: BD:: R: sin. A: and for the same reason, BC: BD:: R: sin. C, and inversely BD: BC:: sin C: R therefore, *ex æquo* inversely, BA: BC:: sin. C.·. sin. A, in the same manner it may be demonstrated that BA: AC:: sin. C: sin B. Hence the following rules

To find a side.

As the sine of the angle opposite to the given side
Is to the sine of the angle opposite to the required side,
So is the given side
To the required side.

To find an angle.

As the side opposite to the given angle
Is to the side opposite to the required angle,
So is the sine of the given angle
To the sine of the required angle.

PROBLEM I.

Given two angles and a side opposite to one of them.

In the oblique-angled triangle (fig. 14.) ABC, there are given the side AB 230·5, the angle A 24° 26′, and the angle C 56° 30. Required the angle B and the sides AC and BC?

Add the angle A 24° 26′ to angle C 56° 30′, the sum is 80° 56′, this subtracted from 180°, leaves the angle B 99° 4′, which being greater than a right angle, or 90°, the sine of its supplement, 80° 56′, must be used in the calculation.

GEOMETRICAL CONSTRUCTION.

Draw an indefinite line AC, and at the extremity A, by means of a protractor or scale of chords, make an angle CAB equal to 24° 26′; from a scale of equal parts lay off from A to B 230·5, then at the point B, make an angle ABC equal 99° 4′, and let BC meet AC in the point C: and the triangle is completed. By the scale of equal parts, it will be found that BC is equal to 114·335 and AC 272·963.

CALCULATION BY LOGARITHMS.

To find BC.		To find AC.	
As sin. C 56° 30′ .	9.9211066	As sin. C 56° 30′ .	9.9211066
Is to sin. A 24° 26′	9.6166164	Is to sin. B 80° 56′ .	9.9945396
So is AB 230.5 .	2.3626709	So is AB 230.5 .	2.3626709
	11.9792873		12.3572105
To BC 114.335 .	2.0581807	To AC 272.963 .	2.4361039

EXAMPLES FOR PRACTICE.

1. Wishing to know my distance from an inaccessible object, I measured a base line 455 fathoms, and found that the angles, subtended by lines drawn from the extremities of the base to the object, were 85° 25′ and 23° 19′. What was the distance of the object from each end of the base line?

Ans. One side 478·915, other side 190·17.

2. In an oblique-angled triangle are given the side AC 624·4, angle B 64° 28′, and angle C 27° 30′. Required the sides AB, CB?

Ans. AB. 319·52, BC 691·575.

3. Sailing along a coast, I observed a cape to bear from me N.W. by N., and having run 25 miles N.N.E., I again took its bearings, and found it to be W. by S½S. What was my distance from the cape at each station?

Ans. Dist. when first bearing was taken, 20·195.
Dist. when second bearing was taken, 21·23 miles.

PROBLEM II.

Given two sides and an angle opposite to one of them.

In a plane triangle ABC (fig. 15), there are given the side AB 475 yards, the side BC 380 yards, and the angle A 38° 12′. Required the other angles B and C, and the side AC?

GEOMETRICAL CONSTRUCTION.

From a scale of equal parts, make the line AB 475. At the point A, by means of a scale of chords, or a protractor, make the angle BAC 38° 12′. From B is a centre, with a radius equal to 380, taken from the same scale from which AB was measured, describe an arc of a circle, cutting the line AC in the point C, or, as in this figure, in the points C and C′; join BC and BC′: then ACB, or AC′B, is the triangle required. By measurement, it will be found that angle ABC is 91° 10′ 30″, ACB 50° 37′ 30″, AC 614·35; angle ABC′ 12° 25′ 30″, AC′B 129° 22′ 30″, AC′ 132·213.

When the radius BC cuts the line AC in two points, as in this figure, the angle ACB is rendered ambiguous, or it may be either acute or obtuse. Trigonometry gives us only the *sine of an angle, not the angle itself;* and, as the sine of an angle is also the sine of its supplement, unless there be some limitation included in the question, it is impossible to determine which of the angles should be taken. In the above

example, it is manifest that there are two triangles ACB, AC′B, each having all the parts given, namely, the angle A, the sides AB, and BC, or BC′, the same in each; but the remaining angles, and the side AC or AC,′ different.

It is obvious from the figure, that BC and BC′ are equal, being radii of the same circle: hence the angles BCC′, BC′C are equal (Euc. i. 5.) But (Euc. i. 13), the angle BC′A is the supplement of BCA; therefore, when BCA is found, BC′A becomes known, from which the angles ABC, ABC′ may easily be found.

When BC is greater than BA, the quantities sought cannot have more than one value; but the ambiguity will always occur when the side opposite the given angle is less than the other given side.

CALCULATION BY LOGARITHMS.

To find angle ACB.

As BC 380 . .	2.5797836
Is to AB 475 . .	2.6766936
So is sine A 38° 12′	9.7912754
	12.4679690
To sine C 50° 37′ 30″	9.8881854

From . . .		180°
Take A + C	=	88° 49′ 30″
ABC	=	91° 10′ 30″
From . . .		180°
Take A + C′	=	167° 34′ 30″
ABC′	=	12° 25′ 30″
From . . .		180°
Take BCA	=	50° 37′ 30″
BC′A	=	129° 22′ 30″

To find AC.

As sine A 38° 12′ . .	9.7912754
Is to sine ABC 91° 10′ 30″	9.9999087
So is BC 380 . . .	2.5797836
	12.5796923
To AC 614.35 . .	2.7884169

To find AC′.

As sine AC′B 129° 22′ 30″	9.8881834
Is to sine ABC′ 12° 25′ 30″	9.3327644
So is AB 475 . . .	2.6766936
	12.0094580
To AC′ 132.213 . .	2.1212746

EXAMPLES FOR PRACTICE.

1. In the triangle ABC, let the angle A be 56° 25′, the side AB 32.56, and the side BC 73.54, what are the angles and the side AC?

Ans. Angle C 21° 36′. Angle B 102° 9′. Side AC 86.46.

NOTE.—In this example, the side BC being greater than AB, the angle C must be acute, consequently there is no ambiguity.

2. In the plane triangle ABC, of which the side AB is 355·25, the side BC 180·6, and the angle A 23° 15′, what are the other angles, and the side AC?

This question admits of two answers:—

When angle C is acute, C is 50° 56′ 23″, side AC 440·19.

When angle C is obtuse, C is 129° 3′ 37″, side AC 212·598.

3. In the triangle ABC, let the angle ABC be 97° 12′, the side AB 369·5, and the side AC 400·2. Required the other parts of the triangle?

Ans. Angle A 16° 27′ 3″. Angle C 66° 20′ 57″.
Side BC 114·24.

PROBLEM III.

Given two sides and the included angle to find the two remaining angles and the third side.

This case is resolved by the following

PROPOSITION. (Fig. 16.)

The sum of any two sides of a triangle is to their difference as the tangent of half the sum of the angles opposite to those sides to the tangent of half their difference.

Let ABC be a triangle, the sum of AB and AC, any two sides, is to the difference of AB and AC as the tangent of half the sum of the angles ACB and ABC is to the tangent of half the difference of ACB and ABC.

About the centre A, with the radius AB, the greater of the two sides describes a circle meeting BC produced in D, and AC produced both ways in E and F. Join DA, EB, FB, and draw FG parallel to BC meeting EB produced in G.

Because the exterior angle EAB is equal to the two interior ABC, ACB (Euc. i. 32) and the angle EFB, at the circumference, is equal to half the angle EAB at the centre (Euc. iii. 20); therefore EFB is half the sum of the angles ABC, ACB, opposite to the sides AB, AC.

Again, the exterior angle ACB is equal to the two interior CAD, ADC, and therefore CAD is the difference of the angles ACB, ADC, that is, of ACB, ABC, for ABC is equal to ADC (Euc. i. 5). Wherefore, also, DBF, which is the half of CAD (Euc. iii. 20), or BFG, which is equal to DBF (Euc. i. 29), is half the difference of the angles ACB, ABC, opposite to the sides AB, AC.

Now, because the angle FBE in a semicircle is a right angle

(Euc. iii. 31), BE is the tangent of the angle EFB, and BG the tangent of the angle BFG, to the radius FB: and BE is, therefore, to BG as the tangent of EFB is to the tangent of BFG, or, as the tangent of half the sum of the angles ACB, ABC, to the tangent of half their difference. Also, CE is the sum of CA and AB, the sides of the triangle ABC, and CF their difference: and because BC is parallel to FG, CE : CF :: BE : BG (Euc. vi. 2), that is, the sum of the two sides of the triangle ABC is to their difference, as the tangent of half the sum of the angles opposite to those sides is to the tangent of half their difference.

When one of the angles of a triangle is given, the sum of the two remaining angles may be found by subtracting the given angle from 180°, and the remainder, divided by 2, will give half the sum of these angles. By the above proposition, if two sides and an included angle of a triangle be given, half the difference of the remaining angles may be found. Then half the sum and half the difference of these angles being known, their values may be obtained separately by the following

PROPOSITION.

Half the sum of two magnitudes, added to half their difference, gives the greater; and half the difference, subtracted from half the sum, gives the less.

For, let AC and CB be two magnitudes, of which AC is greater than BC, and let their sum be AB. Also let AD be half the given sum, then DB will also be half the given sum. To each of these add DC, and AC will be equal to BD and DC together, that is, to BC and twice DC: therefore, twice DC is the difference, and DC half the difference, of the two given magnitudes. But AC, the greater, is equal to AD and DC together, that is, to half the sum added to half the difference; and BC, the less, is equal to the excess of BD, half the sum, above DC, half the difference.

By this proposition the angles may be found. The third side of the triangle may then be calculated as in the last problem.

In the triangle (fig. 17), ABC, are given the side AB 721, the side BC 834, and the included angle B 63° 10′. Required the angles A and C and side AC?

GEOMETRICAL CONSTRUCTION.

Draw two straight lines AB, BC, containing an angle of 63° 10′; take from a scale of equal parts AB = 721, and BC = 834;

join AC, and ABC will be the triangle required. By the same scale or protractor it will be found that A = 65° 9′ 28″, C = 51° 40′ 32″, and the side AC 820·081, nearly.

CALCULATION BY LOGARITHMS.

To find the angles A and C.			
As BC + BA 1555 .	3·1917304	As sin. A 65° 9′ 28″	9·9578314
Is to BC — BA 113 .	2·0530784	Is to sin. B 63° 10′	9·9505223
So is tang. ½(A+C) 58° 25′	10·2112639	So is BC 834 .	2·9211661
	12·2643423		12·8716884
To tan. ½(A−C) 6° 44′ 28″	9·0726119	To AC 820·081 .	2·9138570
A = 65° 9′ 28″			
C = 51° 40′ 32″			

EXAMPLES FOR PRACTICE.

1. In a triangle ABC are given the sides AB 1327, AC 1112, and the included angle at A 55° 10′. Required the angles C and B, and the side BC?

Ans. Angle C, 71° 59′ 40″. Angle B, 52° 50′ 20″. Side BC, 1145·32.

2. In a triangle ABC are given the side AC 1313, side BC 1523, and the angle at C 70°. Required the angles A and B, and the side AB?

Ans. Angle A, 61° 2′ 12″. Angle B, 48° 57′ 48″. Side AB, 1635·73.

3. From a station B (in a triangle ABC as in the last cases), I took the bearings of the objects A and C as follows:—A from B S.W. ½ W. distant 10¾ miles, C from B S.S.E. distant 12¾ miles. Required the distance of A from C?

Ans. A from C, 14·0164 miles.

PROBLEM IV.

Given the three sides of a triangle to find the angles.

PROPOSITION (Fig. 18.)

If a perpendicular be drawn from any angle of a triangle to the opposite side or base, the sum of the segments of the base is to the sum of the other two sides of the triangle as the difference of those sides to the difference of the segments of the base.

Let ABC be a triangle, AD a perpendicular drawn from the angle A on the base BC, so that BD, DC, are the segments of the base, CD + DB : AC + AB :: AC—AB : CD—DB.

From A as a centre, with the radius AC, the greater of the two sides, describe the circle CFG; produce AB both ways to meet the circumference in E and F, and CB to meet it in G; then because AF = AC, BF = AB + AC the sum of the sides; and since AE = AC, BE = AC — AB the difference of the sides; also, because AD, drawn from the centre, cuts GC at right angles, it bisects it (Euc. iii. 3); therefore, when the perpendicular falls within the triangle, BG = DG — DB = DC — DB = the difference of the segments of the base, and BC = BD + DC = the sum of the segments. But when AD falls without the triangle (fig. 19), BG = DG + DB = CD + DB = the sum of the segments of the base, and BC = CD — DB = the difference of the segments of the base. Now, in both cases, because B is the intersection of the two lines FE, GC, drawn in the circle, (Euc. iii. 35) FB · BE = CB · BG; that is, as has been shewn, (AC + AB) (AC — AB) = (CD + DB) (CD — DB). Therefore (Euc. vi. 16) (CD + DB) : (AC + AB) : : (AC — AB) : (CD — DB).

Hence, if in a triangle the three sides be given, and a perpendicular be drawn from one of the angles to the opposite side, the sum and difference of the segments of the base become known. Then, half the sum of the segments added to half their difference, will give the greater; and half the difference subtracted from half the sum, will give the less segment. The angles of the triangle may be found by the rules for right-angled triangles.

In a triangle (fig 20) ABC are given, the sides AC = 330 BC = 530 and AB = 700. Required the angles A, B, and C?

GEOMETRICAL CONSTRUCTION.

Take a straight line AB = 700 from a scale of equal parts; from A as a centre, with 330 as radius, describe an arc, and from B as a centre, with 530 as radius, describe another arc, cutting the former in C; join AC and CB, and ACB will be the triangle: by the protractor or scale of chords, it will be found that A = 46° 29′ 23″ C = 106° 38′ 44″, and B = 26° 51′ 53″, nearly.

CALCULATION BY LOGARITHMS.

To find the Segments.

As BD + DA = 700	. .	2·8450980
Is to BC + CA = 860	. .	2·9344985
So is BC — CA = 200	. .	2·3010300
		5·2355285
To BD — DA = 245·7	. .	2·3904305
½ (BD — DA) = 122·8		
½ (BD + DA) 350		
Greater BD 472·8		
Lesser AD 227·2		

To find the Angles.

As AC 330 . . .	2·5185139
Is to AD 227·2 . .	2·3564083
So is rad. . .	10·0000000
	12·3564083
To cos. A 46° 29′ 23″	9·8378944

As BC 530 . . .	2·7242759
Is to BD 4728 . .	2·6746775
So is rad. . . .	10·0000000
	12·6746775
To cos. B 26° 51′ 53″	9·9504016

Hence angle C = 106° 38′ 44″.

EXAMPLES FOR PRACTICE.

1. In a triangle ACB are given the side AC = 477·5, BC = 677·8, and AB = 870·7. Required the angles A, B, and C?

Ans. Angle A 50° 41′ 46″, B 33° 2′, C 96° 16′ 14″.

2. In a triangle ACB, we have given the side AC = 18·66, BC = 32·8, and AB = 23·05. Required the three angles A, C, and B?

Ans. Angle A 104° 51′ 50″, B 32° 21′ 10″, C 42° 47′.

3. There are three towns, A, B, and C; A is 125 miles from B, and 86 miles from C, and the distance between B and C is 66 miles. Required the bearing of C from B, and B from A, the bearing of C from A being due north?

Ans. B from C N. 70° 3′ 6″ E. or E.N.E. ¼ E;
B from A N. 29° 45′ 25″ W. or N.W. ¾ W?

MENSURATION OF HEIGHTS AND DISTANCES.

The heights and distances of objects are determined by the mensuration and protraction of lines and angles.

Lines which are accessible throughout their whole length, may be measured by applying to them some line whose length is known. For this purpose, a tape or chain, or, when great accuracy is required, a 20 feet deal or glass rod, may be employed. The length of those lines which are inaccessible, or to which a measure cannot be applied, may be found by the rules for trigonometrical calculation. They are then to be considered as the sides of triangles, of which as many parts can be measured as are sufficient to determine all the others. Various instruments have been invented for measuring angles of such triangles, of which the most convenient and useful are the theodolite and sextant. For a description of these instruments and the method of using them, the reader is referred to the "Description of Instruments," in another part of this work.

Example I. (Fig. 21.)

To determine the height of a building, CA, with a chain or deal rod, measure a base line, AB, taking care to observe that the angle be not too acute or too obtuse; say, for example, that you make AB = 180 feet, then place your theodolite at B, and measure the angle CBA = 37° 20′; then to find the side AC, say,

As radius . . .		10·0000000
Is to tang. B 37° 20′ .		9.8823627
So is AB 180 feet . .		2·2552725
		12.1376352
To AC . .	137·289	2·1376352
Add height of the eye,	4·	
	141·289	

Or thus, to find AC.

As sin. of C 52° 40′ . .		9·9004331
Is to sin. of B 37° 20′ .		9·7827958
So is BA 180 . . .		2.2552725
		12·0380683
To AC . .	137.289	2·1376352
	4·	
	141·289	

NOTE.—A traveller, desirous of obtaining information, &c. might at all times, and at no great labour, determine the heights of curious and remarkable objects, within a few feet, with the assistance of a small pocket sextant; the base line he might measure near enough for such purposes by pacing the ground; he might also be provided with a small artificial horizon, which would enable him to determine the latitude of the place at the same time. Such a course, independent of its being a pleasing and rational amusement, would no doubt prove, if adopted by those visiting and exploring different countries, a source of much general good.

EXAMPLE II. (Fig. 22.)

Wishing to know the height of one of the minarets at the city of Benares, I measured a base line AB = 300 feet, and with a sextant or theodolite, I found the altitude of the summit of the dome to be 25° 5′. What was the height of the building? *Ans.* 140 feet.

As rad. . . .	10.0000000	As sin. C . . .	9.9569806
Is to tan. A 25° 5′	9.6703197	Is to sin. A 25° 5′	9.6273003
So is AB = 300 .	2.4771213	So is AB 300 . .	2.4771213
	12.1474410		12.1044216
To BC 140 . .	2.1474410	To BC 140 . .	2.1474410

EXAMPLE III. (Fig. 23.)

To determine the height of a monument, I measured from its base a line AB = 420 feet, and found the altitude of its summit with a sextant to be 19° 39′ 14″, the eye being raised 5 feet above the horizontal line AB. What is its height? *Ans.* 155 feet.

As rad.		10.0000000
Is to tang. A 19° 39′ 14″		9.5528101
So is AB 420 . . .		2.6232493
		12.1760594
To BC	150	2.1760594
	5	
	155	

Remark.—The altitude in measuring heights ought not to be measured between the top of the object and the inner extreme of the base line, because the eye must necessarily be raised above that line; a corresponding height to that of the eye or theodolite must be marked on the side of the object, between which and the top the angle must be measured; and the height of the eye being added to the height found, the calculation will give its true elevation.

Example IV. (Fig. 24.)

To determine the height of a building, surrounded in such a manner, by water or otherwise, that a base line cannot be measured in any direction, at a point G (which suppose in the same horizontal plane with the point H at the base of the object) fix a theodolite, and take the angles ABH $= 2° 45'$, CBH $= 28° 17'$, and the height BG from the centre of the theodolite to the ground call 5 feet, then, in the parallelogram AHBG, AH $=$ BG; hence, in the right-angled triangle, AH $= 5$ feet, and the angle ABH $= 2° 45'$. *Ans.* 61·01 feet.

To find AB.

As tan. B 2° 45′ .	8·6815437
Is to rad. . . .	10·0000000
So is AH 5 . . .	0·6989700
	10·6889700
To AB 104·094 . .	2·0174263

To find AC.

As sin. C 61° 43′ .	9·9447862
Is to sin. B 28° 17′	9·6756245
So is AB 104·094 .	2·0174263
	11·6930508
To AC 56·01 . .	1·7482646
5	
61·01	

Note.—This method may be used, but the result can scarcely be relied upon, on account of the angle which is to determine the base of the larger triangle being so very small.

Example V. (Fig. 25.)

Let it be required to find the height of a building AB, where a straight line cannot be measured from its base; at C fix the theodolite, and observe the angle BCD $= 33° 30'$, then measure a line CG as a base $= 200·7$ feet, and again take the angle DGB 26° 40′; the height of the eye being 5 feet.

Ans. 422·839 feet.

To find BC.

As sin. GBC 6° 50′	9·0754799
Is to sin. G 26° 40′	9·6520521
So is GC 200·7 .	2·3025474
	11·9545995
To BC 757·04 .	2·8791196

To find BD.

As rad. . . .	10·0000000
To sin. C 33° 30′	9·7418895
So is BC 757·04 .	2·8791196
	12·6210091
To BD 417·839 .	2·6210091
5	
422·839	

In the triangle GBC, BCG is the supplement of angle C in the triangle BCD (Euc. ii. 13), therefore

		180°
BCD	.	33° 30′
BCG	.	146° 30′
BGC	+	26° 40′
		173° 10′
		180° 00′
GBC	=	6° 50′

Example VI. (Fig. 26.)

Wanting to know the height of a church steeple, AB, inaccessible on account of uneven ground on one side and a ditch on the other side, at C I took an angle, BCD, = 50° 20′, and measured a line, CG, = 100 feet, at which station I took another angle, BGC, = 25° 30′. What was the height of the steeple and the distance from C to D?

Ans. Height of steeple, 83·9073.
Distance from C to D, 65·4326.

To find BC.

As sin. GBC 24° 50′	9.6232287
Is to sin. G 25°.30′	9.6339844
So is CG 100 . .	2.0000000
	11.6339844
To BC 102·5	2.0107557

To find CD.

As sin. C 50°.20′ .	9.8863616
Is to sin. CBD 39°.40′	9.8050385
So is BD 78·9073 .	1.8971173
	11·7021558
To CD 65.4326 .	1·8157942

To find AB.

As rad. . . .	10.0000000
Is to sin. C 50° 20′	9.8863616
So is BC 102·5 .	2.0107557
	11.8971173
To BD 78.9073 .	1.8971173
5.	
BA 83·9073	

Example VII. (Fig. 27.)

Wanting to obtain the height of an inaccessible object, BA, at a distance from it, and on some even ground, I took an angle, BGD, = 27° 16′, then, walking towards it in a right line, I measured GC, = 70 yards, and took the angle BCD, 49° 39′, the height of my eye being five feet. What was the height of the object? *Ans.* 69·187 yds.

To find BC.

As sin. GBC 22° 23′	9.5806986
Is to sin. G 27° 16′	9.6609911
So is CG 70 . .	1.8450980
	11.5060891
To BC 84·2152 .	1.9253905

To find AB.

As rad.	10.000000
Is to sin. C 49°.39′	9.8820140
So is BC 84·2152	1.9253905
	11.8074045
To BD 64.187 . .	1.8074045
5.	
AB 69.187	

Example VIII. (Fig. 28.)

Two observers wishing to ascertain the height to which a balloon would ascend, took stations in a direct line with it, and 2000 feet from each other; the angle at the nearest station, C, was 38° 10′, and at G 32° 28′. Required the height of the

balloon when the altitudes were taken, supposing the instruments used to be each five feet above the level of the ground? *Ans.* 6684·87.

To find BC.		To find AB.	
As sin. GBC 5° 42′	8·9970356	As rad. . . .	10·0000000
Is to sin. G 32° 28′	9·7288197	Is to sin. C 38° 10′	9.7909541
So is CG 2000 . .	3.3010300	So is BC 1080·97	4·0338141
	13.0308497		13.8247682
To BC 1080.97 .	4·0338141	To BD 6679.87 .	3·8247682
		5.	
		BA 6684.87	

Example IX. (Fig. 29.)

Standing in for the land, I observed a peak bearing from me due west; I took the angle of elevation, and found it to be 12° 25′; I steered direct for it, and, after having run 3½ miles, I again took an angle of elevation, which was 30° 13′ 25″. Required the height of the peak, and the distance from the second station? *Ans.* Height, 1·2388. Distance, 2·1268.

To find BC.		To find AB	
As sin. GBC 17° 48′ 25″	9·4854526	A rad.	10·0000000
Is to sin. G 12° 25′	9·3324777	Is to sin. C 30° 13′ 25″	9·7018925
So is CG 3·5 . .	0·5440680	So is BC	0·3910931
	9·8765457		10·0929856
To BC	0·3910931	To BA 1·2388 .	0·0929856

Note.—The side BC not being required, its logarithm has been used only. Find CA as before = 2·1263.

Example X. (Fig. 30.)

Wishing to find the height of a fortified hill, but not being able to approach its base, I adopted the following method:— I measured a base line, GK, = 475·6 yds. in front of the hill, from the extremes of which line I could distinctly see the summit of the tower at P; I then fixed the instrument, and took the following angles: GKP, = 70° 28′ 30″, KGP, = 66° 12′ 15″, and PGO, = 30° 28′ 30″; then in the triangle GPK, we have the side GK, and the adjacent angles at G and K, to find the other sides GP, KP.

To find PG.

As sin. GPK 43° 19′ 15″ . . .	9·8363771
Is to sin. GKP 70° 28′ 30″ . .	9.9742794
So is GK 475·6	2·6772418
	12·6515212
To PG 653·3472	2·8151441

To find height PO.

As rad.	10·0000000
Is to sin. PGO 30° 28′ 30″ . .	9.7050397
So is PG 653·3472	2·8151441
	12.5201838
To OP 331·271	2·5201838

EXAMPLE XI. (Fig. 31.)

Wishing to know the height of Arthur's Seat, near Edinburgh, I measured a base line, GK, = 356 feet, in front of the hill, and took the following angles at its extremities: KGP, = 48° 30′, GKP, = 114° 19′ 34″, and PKO, = 63° 15′. From these data it is required to find the height?

To find PK.

As sin. GPK 17° 10′ 26″ . . .	9.4702232
Is to sin. KGP 48° 30′	9.8744561
So is GK 356	2.5514500
	12·4259061
To PK 902·99	2·9556829

To find PO.

As rad.	10·0000000
Is to sin. PKO = 63° 15′ . . .	9·9508412
So is PK 902·99	2·9556829
	12.9065241
To OP 806·35	2.9065241

EXAMPLE XII. (Fig. 32.)

Wishing to know the height of Ben-Nevis, in Invernessshire, I took as a base the distance between two points, G and K, on the same horizontal plane, which was ascertained to be $2\frac{1}{3}$ miles, and at K I found the vertical angle PKO to be 16° 25′, and the horizontal angle OKG to be 22° 12′; also at the other end of the base G, the horizontal angle was 104° 25′. What was the height of the mountain?

To find OK.

As sin. GOK 53° 23′ .	9·9045230
Is to sin. OGK 104° 25′	9·9861045
So is GK 12320 . .	4·0906107
	14·0767152
To OK 14866 . . .	4·1721922

To find PO.

As sin. OPK 73° 35′ .	9·9819236
Is to sin. PKO 16° 25′	9·4512037
So is OK 14866 . .	4·1721922
	13.6232959
To PO 4380 . . .	3.6414723

Example XIII. (Fig. 33.)

Wishing to know the distance between two hills, C and D, I measured a base, AB, 735 feet; at one end of the base, A, the angles subtended by the other end, and each of the hills, were CAD, 63° 30′, DAB 35° 10′; and at the other end of the base B, the corresponding angles were DBC 80° 16′, CBA 28° 20′. What is the distance between C and D?

To find AC.

As sin. ACB, 53° . .	9.9023486
Is to sin. ABC, 28° 20′	9.6763281
So is AB 735 . . .	2.8662873
	12.5426154
To AC 436.784 . .	2.6402668

To find angle ADC.

As AD + AC 1615.33	3.2082613
Is to AD—AC 741.77	2.8702693
So is tan. ½(C+D) 58° 15′	10.2084365
	13.0787058
To tan. ½(C—D) 36° 34′ 41″	9.8704445

ADC = 21° 40′ 19″

To find AD.

As sin. ADB 36° 14′ .	9.7716426
Is to sin. ABD 108° 36′	9.9767022
So is AB 735 . . .	2.8662873
	12.8429895
To AD 1178.55 . .	3.0713469

To find CD.

As sin. ADC 21° 40′ 19″	9.5673696
Is to sin. CAD 63° 30′	9.9517912
So is AC 436.784 . .	2.6402668
	12.5920580
To CD 1058.49 . .	3.0246884

Example XIV. (Fig. 34.)

Wishing to know the distance between three hills, C, D and E, I measured a base line, AB, 2500 feet, and at A, one extremity of the base, found the angles subtended by the other extremity, and each of the hills to be 112° 36′, 87° 24′, and 33° 40′ respectively; at B, the other extremity of the base, the corresponding angles were 108° 20′, 54° 16′, and 32° 8′. Required the distances CD, DE, and CE?

In the triangle ABC.
To find AC.

As sin. ACB 35° 16′ .	9.7614638
Is to sin. ABC 32° 8′ .	9.7258229
So is AB 2500 . . .	3.3979400
	13.1237629
To AC 2303 . . .	3.3622991

In the triangle ACD.
To find angle ADC.

As AD+AC 5575 . .	3.7462449
Is to AD—AC 969 . .	2.9863238
So is tan. ½(C+D) 77° 24′	10.6506710
	13.6369948
To tan. ½(C—D) 37° 52′	9.8907499

ADC = 39° 32′

In the triangle ABD.
To find AD.

As sin. ADB 38° 20′ .	9.7925566
Is to sin. ABD 54° 16′	9.9094190
So is AB 2500 . . .	3.3979400
	13.3073590
To AD 3272 . . .	3.5148024

In the triangle ACD.
To find CD.

As sin. CDA 39° 32′ .	9.8038168
Is to sin. DAC 25° 12′ .	9.6291845
So is AC 2303 . . .	3.3622991
	12.9914836
To CD 1540.5 . . .	3.1876668

In the triangle ABE.
To find AE.

As sin. AEB 38° . .	9.7893420
Is to sin. ABE 108° 20′	9.9773772
So is AB 2500 . . .	3.3979400
	13.3753172
To AE 3854.6 . . .	3.5859752

In the triangle AED.
To find angle AED.

As AE+AD 7126.6 .	3.8528824
Is to AE—AD 582.6 .	2.7653705
So is tan. ½ (D+E) 68° 8′	10.2953368
	13.0607073
To tan. ½ (D—E) 9° 10′	9.2078249
AED =	53° 58′

In the triangle AED.
To find DE.

As sin. AED 53° 58′	9.9077740
Is to sin. DAE 53° 54′	9.9074059
So is AD 3272 . .	3.5148024
	13.4222083
To DE 3269.2 .	3.5144343

In the triangle AEC.
To find angle AEC.

As AE+AC 6157.6 .	3.7894115
Is to AE—AC 1551.6 .	3.1907798
So is tan. ½ (C+E) 50° 32′	10.0844104
	13.2751902
To tan. ½ (C—E) 17° 1′	9.4857787
AEC =	33° 31′

In the triangle EAC.
To find CE.

As sin. AEC 33° 31′ .	9.7420803
Is to sin. CAE 78° 56′	9.9918480
So is AC 2303 . . .	3.3622991
	13.3541471
To CE 4093.2	3.6120668

Example XV. (Fig. 35.)

In a survey of part of a coast, it was thought necessary to verify the base line CD, which had been measured. The following was the method adopted:—There were two hills, A and B, whose distance had already been ascertained to be 5½ miles; at each end of the base line the following angles were taken: ACB 18° 15′, BCD 125° 20′, CDA 22° 12′, ADB 18° 36′. Required the length of CD?

Assume the required side CD, equal to 1 or any other convenient number, and from this assumed base and the given angles calculate the length of AB. Then, as the value of AB thus found is to its true value, so is 1 to the true value of CD.

CALCULATION BY LOGARITHMS.

In the triangle ACD.
To find AC.

As sin. CAD 14° 13′ .	9.3902096
Is to sin. ADC 22° 12′	9.5773088
So is CD. 1. . . .	0.0000000
	9.5773088
To AC 1.5385 . . .	0.1870992

In the triangle BCD.
To find BC.

As sin. CBD 13° 52′ .	9.3796015
Is to sin. BDC 40° 48′	9.8151928
So is CD .1	0.0000000
	9.8151928
To CB 2.7264 . . .	0.4355913

In the triangle CAB.
To find angle CBA.

As BC + CA 4·2649 .	0·6299089
Is to BC — CA 1·1879 ·	0·0747799
So is tan. $\frac{1}{2}$(A + B) 80° 52′ 30″	10·7933894
	10·8681693
To tan. $\frac{1}{2}$(A − B) 59° 58′ 58″	10·2382604

CBA = 20° 53′ 32″

In the triangle CBA.
To find AB.

As sin. CBA 20° 53′ 32″	9·5521949
Is to sin. ACB 18° 15′	9·4957716
So is AC 1.5385 . .	0·1870992
	9·6828708
To AB 1·3511 . . .	0·1306759

1·3511 : 5·5 : : 1 : 4·07 = true value of AB.

Example XVI.

At the distance of 36 feet from the bottom of a wall, the angle of elevation of its top was 62° 40′. What was the height of the wall, and the length of a ladder that would reach from my station to the top of the wall?

Ans. Height of wall, 69·64.
Length of ladder, 78·4.

Example XVII.

A ladder 30 feet long can be so placed that it shall touch a point 16 feet from the ground, on the wall of a house on one side of the street, and, by turning it round without moving its foot out of the place, it will reach a point which is 20 feet from the ground on the wall of a house on the opposite side of the street. Required the breadth of the street?

Ans. 47·73 feet.

Example XVIII.

What is the height of a tower whose angle of elevation is 52° 30′, at the distance of 170 feet from the bottom, the height of the eye being 5 feet? *Ans.* 226·55 feet.

Example XIX.

A steeple 87 feet high casts a shadow 135 feet long upon the ground. What is the sun's altitude at the time?

Ans. 32° 48′.

Example XX.

To find the height of a tower standing upon a slope, I measured down the slope from the bottom of the tower 104 feet, and found the angle contained by the horizontal plane and a

line to the bottom of the tower to be 7° 30′, and also the angle contained by the horizontal plane and a line to the top of the tower to be 44° 10′. What was the height of the tower, supposing the height of the instrument to be 5½ feet?

Ans. 92·079 feet.

Example XXI.

Being on a horizontal plane, and wishing to know the height of an eminence whose base was inaccessible, I took two stations, distant from each other 340 feet, and in the same vertical plane with the object. At the station nearest the eminence the angle of elevation of its summit was 52° 25′, and at the other station the angle of elevation was 40°. What was the height of the eminence? *Ans.* 805·47 feet.

Example XXII.

A base of 320 feet was measured on the sloping side of a hill, in a vertical plane passing through its summit, which made with the horizontal plane an angle of 11°. At the higher end of the base the elevation of the hill was 61° 20′, and at the other end the angle of elevation was 35° 30′. What was the height of the hill above the lower end of the base, the height of the instrument being 5 feet? *Ans.* 333·264 feet.

Example XXIII.

Being on a horizontal plane, and wishing to know the height of a tower placed on the summit of an inaccessible hill, I took the angle of elevation of the top of the hill 39° 20′; then measuring in a line directly from it to the distance of 340 feet, I found the angle to the top of the hill to be 25° 10′, and to the top of the tower 30° 50′. What was the height of the tower?

Ans. 101·255 feet.

Example XXIV.

At the top of a tower which stood on the sea-shore, whose height was 89 feet, the angle of depression of a ship at anchor was 18° 52′, and at the bottom of the tower its depression was 13° 20′. Required the height of the bottom of the tower above the level of the sea, and the horizontal distance of the vessel.

Ans. 201·42, height of the bottom of the tower.
849·87, distance of the vessel.

Example XXV.

Wishing to know the height of, and my distance from, an

object on the other side of a river which seemed to be on a level with the place where I stood, close by the side of the river, and not having room to measure backward on the same plane because of the immediate rise of the bank, I placed a mark where I stood, and measured in a direction from the object up the ascending ground to the distance of 270 feet, where I was evidently above the level of the top of the object; there the angles of depression were found to be, of the mark left at the river side 45° 55′, of the bottom of the object 21° 10′, and of its top 4° 53′. What was the height of the object, and the distance of the mark from its bottom?

Ans. 151·15, height of the object.
313·05, distance of the mark.

Example XXVI.

Being required to ascertain the height of a tower which stood on the summit of a hill, and being prevented by the inequalities of the ground from taking a base in a direct line with the hill, I measured one of 450 feet in front of it. At one extremity of this base the elevation of the summit of the hill was 32° 12′, of the top of the tower 27°, and the angle contained by a line to the top of the tower and the other extremity of the base was 66° 30′; at the other end of the base the angle contained by lines to the top of the tower and to the first station was 90° 34′. Required the height of the tower, and the height of the hill above the first station?

Ans. 83·266, height of the tower.
541·004, height of the hill.

Example XXVII.

Being on the side of a river, I observed a tree close to the opposite bank, and having measured a base of 528 feet in a straight line by the side of the river, at each end of it I took the horizontal angles formed between the tree and the other end of the base; these angles were 47° 30′ and 70° 12′. Required the distance of the tree from each end of the base, and also the perpendicular breadth of the river?

Ans. 561·09, distance from one end of base.
439·67, distance from other end of base.
413·68, perpendicular breadth of river.

Example XXVIII.

Wanting to know the length of a sheet of water, I took a station inland, from which a straight line could be measured

to each extremity of the water, and there found the horizontal angle contained by these lines to be 77° 10′; the distance of the station from the extremities of the piece of water being 471 and 453 feet. Required the length of the water?

Ans. 576·43 feet.

Example XXIX.

Wishing to know the distance between two inaccessible objects, I measured a base line of 480 feet, and at each end of it took the angles contained by the other end and each of the objects; these angles were, at the one extremity of the base, 82° 30′ and 38° 5′, and at the other extremity, 99° 30′ and 41° 25′. What was the distance between the objects?

Ans. 505·34 feet.

Example XXX.

To find the distance between two headlands, there was measured a base of 485 yards, and at the one extremity of it the angles, subtended by the other extremity and each of the headlands, were 50° 20′ and 95° 20′; at the other end of the base the corresponding angles were 53° 30′ and 98° 45′. Required the distance between the headlands? *Ans.* 775·66 yards.

Example XXXI.

A ship from latitude 42° 25′ N. sails N.W. by N. 260 miles. Required the latitude in and departure from the meridian? *Ans.* 45° 31′ N. latitude in.
144·45 miles, departure from meridian.

Example XXXII.

Travelling through a level country, I observed a church which bore from me N.W. by N., and after having gone 5 miles in a N.N.E. direction, the same church bore from me W.S.W. What was my distance from the church at each observation? *Ans.* 3·6 miles from first station.
4·2 miles from second station.

Example XXXIII.

From a ship at sea, a headland was observed to bear N.E. $\frac{3}{4}$ E., and after sailing $1\frac{1}{4}$ miles N.W. by N. $\frac{1}{2}$ W. it bore E. by N. $\frac{1}{4}$ E. Required the distance of the headland from each station? *Ans.* 2·274 miles from first station.
2·648 miles from second station.

Example XXXIV.

Coasting along the shore, two headlands were observed, the first of which bore N.N.W. and the second N.N.E. $\frac{3}{4}$ E.; then steering E. by N. $\frac{1}{2}$ N. 12 miles, the first headland bore W.N.W., the second N.W. by N. $\frac{1}{4}$ W. Required the bearing and the distance of the two headlands from each other?

Ans. 10·43 miles, distance of headlands.

S. 87° 1′ W., or W. by S. $\frac{3}{4}$ W. nearly, the bearing of the first headland from the second.

Example XXXV.

Wanting to know the height of an object on the top of a hill, and also its distance from the place where I stood, I measured a base of 630 feet up sloping ground, which was inclined to the horizon at an angle of 9° 30′. At my first station, the angle of elevation was 5° 34′, and the horizontal angle contained by lines to the centre of the hill and the other extremity of the base was 64° 10′; at the second station the horizontal angle was 76° 17′. What was the height of the object and its distance from the first station?

Ans. 196·5 feet, the height of the object.
948 feet, distance from first station.

OF TERRESTRIAL REFRACTION.

It has been established by experiment, that a ray of light, passing from a rarer to a denser medium, is bent out of the rectilineal direction towards the denser medium. Rays, therefore, proceeding from any of the heavenly bodies, as they fall obliquely upon the atmosphere by which the earth is surrounded, do not reach the eye in a straight direction, but are bent downwards in lines nearer to a perpendicular from the place of the observer, or nearer his zenith, than they would be if the atmosphere were removed. The effect produced by this bending of the line in which the rays enter the eye, is to make the object appear higher than it really is; and hence it is necessary to make a deduction from the observed altitude to get the true altitude of the body. This incurvation of the ray is called *refraction*, which is greater the nearer the bodies

are to the horizon, and diminishes as the altitude increases. When the object is in the zenith there is no refraction.

To explain the nature of refraction, let AB (fig. 36) be a portion of the earth's surface, GH the upper boundary of the atmosphere, S a star, P the place of the observer, and Z his zenith. The atmosphere may be considered as composed of a great number of thin strata of air of different densities, the densities being greater the nearer they are to the surface of the earth. Let EF, CD, represent the boundaries of these strata; then a ray of light will proceed from the star S in a straight line until it arrive at the point K, where it enters the denser medium contained between GKH and ELF. It will then no longer pursue its direction SKN, but will be deflected in the direction KL. At the point L it enters a still denser medium, contained between ELF and CMD, and will be again deflected from its direction KLO, and will now proceed in the direction LM. Similar effects will be produced, whatever be the number of media through which the ray passes, and it will at length reach the point P, in the line MP. Hence the ray, instead of being a straight line, is broken into parts KL, LM, MP; and if we suppose the media through which it passes to be indefinitely increased, and their boundaries to approach each other by spaces extremely small, the parts KL, LM, MP, may be considered as curvilinear, and the course of the ray a curve line, as SBP (fig. 37), where P is the place of the observer, HPN his horizon, Z the zenith, and S a star. In this case, the ray is no longer considered as passing through different strata of variable density, but through a medium of continually varying density, such as the atmosphere of the earth, whose density is greatest at its surface, and decreases towards the higher regions. In passing through such a medium, a ray of light will be deflected into a curve line, concave towards the earth's surface, and will enter the eye of the observer in the direction of a tangent to that curve. A ray from the star S will, therefore, not proceed in the straight line SP, but in the direction of the curve line SBP; and to the observer at P, the star S will appear as if at S′ in the direction of a straight line, which touches the curve at P. The angle SPN is, therefore, the apparent altitude of the star, and the angle SPN its true altitude: their difference, the angle S′PS, will consequently be the refraction.

Similar effects take place with regard to the rays of light by which terrestrial objects are rendered visible. In their passage through the atmosphere, they are bent out of their rectilineal

direction, and enter the eye of the observer in a curve line, so that the apparent or observed altitude of an object is always greater than its true altitude. This refraction is called *terrestrial*, to distinguish it from that which affects the heavenly bodies.

In measuring the heights of objects which are at a considerable distance, or near the horizon, where great accuracy is required, it is always necessary to make an allowance for the effects of refraction; but it is difficult to determine the exact quantity of refraction at any given time, as it is not constant at the same elevation and distance, but continually changes according to the state of the atmosphere, being less in dry and warm weather than when the atmosphere is saturated with moisture, and consequently more dense. This change is so great, that at a distance of 8 or 10 miles it is found to vary from 30 seconds to 2 minutes. The tables of refraction usually given in nautical works, are calculated at a mean rate for the British channel, the barometer being at 29½ inches, and Fahrenheit's thermometer at 50 degrees.

The following method will shew how the refraction may be found in any case:—

Let A and B (fig. 38) be two stations, C the centre of the earth, and SS the arc of the earth's circumference intercepted between the stations; draw lines CA, CB; from A and B draw AG, BH, at right angles to AC, CB meeting CA, CB, produced in G and H; then AG and BH will be the horizontal lines at A and B. Let *a* and *b* represent the apparent places of the objects A and B, then the angle BA*b* is the refraction at A, and AB*a* that at B; and these will be nearly equal, if the angles be observed at the same instant, which may be done by watches previously regulated for that purpose, or by signals made at one station, so as to be seen from the other. Hence, half the sum of the angles BA*b*, AB*a* will be the refraction.

In the quadrilateral figure ACBO, since the angles at A and B are right angles, the angles AOB, ACB, are together equal to two right angles; but the angles AOB, OAB, OBA of the triangle AOB, are together equal to two right angles; therefore, the angles AOB and ACB are together equal to the angles AOB, OAB, OBA. Take away the common angle AOB, and there remains the angle ACB, or the arc SS, equal to the angles OAB, OBA. If, now, the sum of the two depressions, or the angles HB*a*, GA*b*, be taken from the sum of the angles OBA, OAB, or the angle C, the remainder is the sum of both refractions, or the angles BA*b*, AB*a*; therefore, *the refraction is equal to*

half the difference between the sum of the two depressions, and the arc of the earth's circumference intercepted between the stations.

It has been supposed, in the above demonstration, that each of the objects is depressed below the horizon of the other; but if one of them be elevated instead of depressed, it may be shewn in a similar manner, that *the refraction is half the difference between the depression and the sum of the intercepted arc and elevation.*

To apply these rules, it is necessary that the distance between the places of observation should be known sufficiently near to give the arc intercepted between the stations true, to a very few seconds of a degree. When the distance between the stations is given in feet, the corresponding arc on the earth's surface may be found by dividing the given number by $6094\frac{1}{2}$.

Example.

In the British Trigonometrical Survey, on Wisp Hill and Cross Fell are two stations in a triangle, which connects the north of England with the borders of Scotland. Their distance is computed at 235018·6 feet, which corresponds on the surface of the earth to an arc of 38′ 33″·7· From Cross Fell, Wisp Hill was seen depressed 30′ 48″ below the horizon, and, from the latter place, the former was found to have a depression of 2′ 31″. Required the refraction?

Depression of Wisp Hill,	30′ 48″	Intercepted arc,	38′ 33″.7
Depression of Cross Fell,	2′ 31″	—	33′ 19″
Sum	33′ 19″	Difference, .	5′ 14″.7
		Half difference,	2′ 37″.3 = refrac.

Besides the refraction, it is necessary to correct the observed angle on account of the earth's curvature, in estimating the difference between the heights of two stations which are at a considerable distance from each other. This may be explained in the following manner:—

Let A and B (fig. 39) be two stations, C the centre of the earth, and SS the arc of the earth's circumference intercepted between the stations; draw CA CB, and from A draw the horizontal line AH, at right angles to AC and meeting CB, produced in H; make CD equal to CA, and join AB, AD;—then, because the points A and D are at the same height above the surface of the earth, BD is the difference between the heights of the stations A and B.

In the isosceles triangle ACD, the sum of its angles CAD, ADC, ACD is equal to two right angles; but CAD and ADC are equal;

therefore the sum of 2CAD and ACD is equal to two right angles, and CAD together with $\frac{1}{2}$ACD, is equal to one right angle; but CAH is a right angle, therefore CAH is equal to CAD together with $\frac{1}{2}$ACD; and taking away the common angle CAD there remains the angle HAD, equal to $\frac{1}{2}$ACD; hence the angle BAD is equal to the difference between $\frac{1}{2}$ACD, or the arc SS and HAB—that is, when B is below the horizon, as seen from A, the vertical angle, corrected for the earth's curvature, will be *the excess of half the intercepted arc above the apparent depression.* If the station B be above the horizon, the corrected vertical angle will be *the sum of the apparent elevation and half the intercepted arc.* The angle BAD being thus found, the side BD may be computed;—for in the triangle BAD the angle BDA may be considered a right angle, which it is very nearly, and the side AD is known, being the distance between the two stations; therefore (Trig. case ii. of right-angled triangles),

As radius
Is to tan. BAD,
So is AD
To BD.

Example.

In the British Trigonometrical Survey, the distance between the stations on Wisp Hill and Cross Fell was found to be 235018·6 feet, which corresponds to 38′ 33″·7 on the earth's surface. From Wisp Hill, Cross Fell appeared depressed below its horizon 2′ 31″; but, by a corresponding observation at the other station, the error arising from refraction was estimated, as in last example, at 2′ 37″·3· Required the difference between the height of the stations?

Observed Depression of Cross Fell, . .	2′ 31″
Refraction, . .	+ 2′ 37″·3
Apparent Depression, .	5′ 8″·3

Half-intercepted arc, .	19′ 16″·8
Apparent Depression,	— 5′ 8″·3
Cor. vertical angle, .	14′ 8″·5

As radius . .	10·0000000
Is to tan. 14′ 8″·5	7·6142291
So is 235018·6	5·3711022
	12·9853313
	10·0000000
To 966·8	2·9853313

Hence the difference between the height of the station on Cross Fell above that on Wisp Hill is 966·8 feet.

DESCRIPTION OF INSTRUMENTS.

THE THEODOLITE.

As an angular instrument, the theodolite has, from time to time, received such improvements that it may now be considered as the most valuable instrument employed in surveying. Instruments of this kind, of the best construction, may, to a certain extent, be used as altitude and azimuth instruments; and several astronomical operations, such as those required for determining the time, the latitude of place, &c., may be performed by them, and to a degree of accuracy sufficient for most of the purposes that occur in the ordinary practice of a surveyor.

This instrument (as represented in fig. 40) consists of two circular plates, A and B, called the horizontal limb, the upper, or vernier plate, A, turning freely upon the lower, and both have a horizontal motion by means of the vertical axis C. This axis consists of two parts, external and internal, the former secured to the graduated limb B, and the latter to the vernier plate A. Their form is conical, nicely fitted and ground into each other, having an easy and very steady motion; the external centre also fits into a ball at D, and the parts are held together by a screw at the lower end of the internal axis.

The diameter of the lower plate is greater than that of the upper one, and its edge is chamfered off and covered with silver, to receive the graduations. On opposite parts of the edge of the upper plate, or 180 degrees apart, a short space, *a*, is also chamfered, forming with the edge of the lower plate a continued inclined plane. These spaces are likewise covered with silver, and

form the verniers. The lower limb is usually graduated to 30 minutes of a degree, and it is subdivided by the vernier to single minutes, which being read off by the microscope E, half, or even quarter minutes, can easily be estimated.

The parallel plates F and G are held together by a ball and socket at D, and are set firm and parallel to each other, by four milled-headed screws, three of which, *b*, *b*, *b*, are shown in the figure. These turn in sockets fixed to the lower plate, while their heads press against the under side of the upper plate, and being set in pairs opposite each other, they act in contrary directions. The instrument by this means is set up level for observation.

Beneath the parallel plates is a female screw adapted to the staffhead, which is connected by brass joints to three mahogany legs, so constructed, that when shut up they form one round staff, secured in that form for carriage by rings put on them; and when opened out they make a very firm stand, be the ground ever so uneven.

The lower horizontal limb can be fixed in any position by tightening the clamping screw H, which causes the collar *c* to embrace the axis C, and prevents its moving; but it being requisite that it should be fixed in some precise position more exactly than can be done by the hand alone, the whole instrument, when thus clamped, can be moved any small quantity by means of the slow motion screw I, which is attached to the upper parallel plate. In like manner, the upper or vernier plate can be fixed to the lower in any position, by a clamp (in the plate this clamp is concealed from view), which is also furnished with a slow motion, the screw of which is generally called the tangent screw. The motion of this limb and of the vertical arc, hereafter to be described, is sometimes effected by a rack and pinion; but this is greatly inferior, where delicacy is required, to the slow motion produced by the clamp and tangent screw.

Upon the plane of the vernier plate, two spirit levels, *d*, *d*, are placed at right angles to each other, with their proper adjusting screws: their use is to determine when the horizontal limb is set level. A compass also is placed at J.

The frames K and L support the pivots of the horizontal axis of the vertical arc (or semicircle) M, on which the telescope is placed. The arm which bears the microscope N, for reading the altitudes or depressions, measured by the semicircle, and denoted by the vernier *e*, has a motion of several degrees between the bars of the frame K, and can be moved before the

face of the vernier for reading it off. Another arm clamps the opposite end of the horizontal axis by turning the screw O, and has a tangent screw of slow motion at P, by which the vertical arc and telescope are moved very small quantities up or down, to perfect the contact when an observation is made.

One side of the vertical arc is inlaid with silver, and divided to single minutes by the help of its vernier; and the other side shows the difference between the hypothenuse and base of a right-angled triangle, or the number of links to be deducted from each chain's length, in measuring up or down an inclined plane, to reduce it to the horizontal measure. The level, which is shown under and parallel to the telescope, is attached to it at one end by a joint, and at the other by a capstan-headed screw, *f*, which being raised or lowered, will set the level parallel to the optical axis of the telescope, or line of collimation; the screw, *g*, at the opposite end, is to adjust it laterally for true parallelism in this respect. The telescope has two collars or rings, of bell metal, ground truly cylindrical, on which it rests in its supports, *hh*, called Y's, from their resemblance to that letter; and it is confined in its place by the clips, *i*, *i*, which may be opened by removing the pins, *j*, *j*, for the purpose of reversing the telescope, or allowing it a circular motion round its axis during the adjustment.

In the focus of the eye-glass are placed three lines, formed of spider's web, one horizontal, and two crossing it, so as to include a small angle between them—a method of fixing the wires which is better than having one perpendicular wire, because an object at a distance can be made to bisect the said small angle with more certainty than it can be bisected by a vertical wire. The screws adjusting the cross wires are shown at *m*. There are four of these screws, two of which are placed opposite each other, and at right angles to the other two, so that by easing one and tightening the opposite one of each pair, the intersection of the cross wires may be placed in adjustment.

The object glass is thrust outwards by turning the milled-head, Q, on the side of the telescope, that being the means of adjusting it to show an object distinctly.

A brass plummet and line are packed in the box with the theodolite, to suspend from a hook under its centre, by which it can be placed exactly over the station from whence the observations are to be taken; likewise, if required, two extra eye-pieces for the telescope, to be used for astronomical obser-

vations; the one inverts the object, and has a greater magnifying power, but, having fewer glasses, possesses more light; the other is a diagonal eye-piece, which will be found extremely convenient when observing an object that has a considerable altitude; the observer avoiding the unpleasant and painful position he must assume in order to look through the telescope when either of the other eye-pieces is applied. A small cap containing a dark-coloured glass is made to apply to the eye-end of the telescope, to screen the eye of the observer from the intensity of the sun's rays, when that is the object under observation. A magnifying glass, mounted on a horn-frame, a screw-driver, and a pin to turn the capstan-screws for the adjustments, are also furnished with the instrument.

THE ADJUSTMENTS.

THE first adjustment is that of the line of collimation; that is, to make the intersection of the cross wires coincide with the axis of the cylindrical rings on which the telescope turns. It is known to be correct, when an eye looking through the telescope observes their intersection continue on the same point of a distant object during an entire revolution of the telescope. The usual method of making this adjustment is as follows:—

First, make the centre of the horizontal wire coincide with some well-defined part of a distant object, then turn the telescope half round in its Y's till the level lies above it, and observe if the same point is again cut by the centre of the wire; if not, move the wire one-half the quantity of deviation, by turning the screws at *m* (releasing one before tightening the other), and correct the other half by elevating or depressing the telescope. Now, if the coincidence of the wire and object remains perfect in both positions of the telescope, the line of collimation, in altitude or depression, is correct; but if not, the operation must be repeated carefully, until the adjustment is satisfactory. A similar proceeding will also put the vertical line correct, or rather the point of intersection, when there are two oblique lines instead of a vertical one.

The second adjustment is that which puts the level attached to the telescope parallel to the rectified line of collimation. The clips, *i*, *i*, being open, and the vertical arc clamped, bring

ıe air bubble of the level to the centre of its glass tube, by ırning the tangent screw P; which done, reverse the tele-ope in its Y's, that is, turn it end for end, which must be me carefully, that it may not disturb the vertical arc, and if e bubble resume its former situation in the middle of the be, all is right; but if it retires to one end, bring it back ıe-half by the screw, *f*, which elevates or depresses that end the level, and the other half by the tangent screw, P. This ocess must be repeated until the adjustment is perfect; but make it completely so, the level should be adjusted later-y, that it may remain in the middle of the tube, when in-ned a little on either side from its usual position imme-ıtely under the telescope, which is effected by giving the rel such an inclination, and, if necessary, turning the two eral screws at *g*. If making the latter adjustment derange ɜ former, the whole operation must be carefully repeated.

The third adjustment is that which makes the azimuthal is, or axis of the horizontal limb, truly vertical.

Set the instrument as nearly level as can be done by the ɜ, fasten the centre of the lower horizontal limb by the staff-ıd clamp, H, leaving the upper limb at liberty, but move it the telescope is over two of the parallel plate screws; then ng the bubble of the level under the telescope, to the mid-of the tube, by the screw, P; now turn the upper limb f round, that is 180°, from its former position; then, if the ɔble returns to the middle, the limb is horizontal in that ɜction; but if otherwise, half the difference must be cor-ted by the parallel plate screws over which the telescope , and half by elevating or depressing the telescope, by turn-the tangent screw of the vertical arc; having done which, mly remains to turn the upper limb forward or backward , that the telescope may lie over the other two parallel plate ːws, and by their motion set it horizontal. Having now ːlled the limb plates by means of the telescope level, which ıe most sensible upon the instrument, the other air bubbles d upon the vernier plate may be brought to the middle of r tubes, by merely giving motion to the screws which fas-them in their places.

ˈhe vernier of the vertical arc may now be attended to. ɜ correct if it points to zero when all the foregoing adjust-ıts are perfect; and any deviation in it is easily rectified, ˈeleasing the screws by which it is held, and tightening n again after having made the adjustment: or, what is per-

haps better, note the quantity of deviation as an index error, and apply it, plus or minus, to each vertical angle observed. This deviation is best determined by repeating the observation of an altitude or depression in the reversed positions, both of the telescope and the vernier plate; the two readings will have equal and opposite errors, one half of their difference being the index error. Such a method of observing angles is decidedly the best, since the mean of any equal number of observations, taken with the telescope reversed in its Y's, must be free from the effects of any error that may exist in the adjustment of the vernier or zero of altitude.

The theodolite, as constructed in the manner described, is not inconveniently heavy, as the diameter of the horizontal limb seldom exceeds five inches; but when the diameter is increased, the other parts must be made proportionally large and strong, and the instrument becomes too weighty and cumbersome to be easily carried from station to station. The object of increasing the dimensions, is to enable the instrument to furnish more accurate results, by applying a telescope of greater power, and by a more minute subdivision of the graduated arcs. With the increase of size, a small variation takes place in the construction, principally consisting in the addition of a second telescope, and in the manner of attaching the supports, K and L, to the horizontal limb, to afford the means of adjusting the horizontal axis, and of course making the telescope and vertical arc move in a vertical plane. In the smaller instruments this is done by construction; but in the larger ones, the supports, K and L, are attached to a stout frame, which also carries the compass-box, instead of being fixed, as represented in the figure to the upper horizontal plate. The frame is attached to the limb by three capstan-headed screws, forming an equilateral triangle, two of them lying parallel to the horizontal axis, and the third in the direction of the telescope: the adjustment is made by means of these screws. To prove its accuracy, set up the theodolite in such a situation that some conspicuous point of an elevated building may be seen through the telescope, both directly and by reflection, from a basin of water, or, what is better, of oil or quicksilver. Let the instrument be very correctly levelled, and if, when a vertical motion is given to the telescope, the cross wires do not cut the object seen, both directly and by reflection, it is a proof that the axis is not horizontal; and its correction is effected by giving motion to the screws above spoken of, which are at

ght angles to the telescope, or in the direction of the hori-
ontal axis. The third screw, or that which is under the tele-
:ope, serves for adjusting the zero of altitude, or vernier of
ıe vertical arc.

A second telescope is sometimes attached to the instrument
eneath the horizontal limb: it admits of being moved, both
ı a vertical and horizontal plane, and has a tangent screw
tached for slow motion. Its use is to detect any accidental
erangement that may occur to the instrument whilst obser-
g, which may be done by it in the following manner:—After
velling the instrument, bisect some very remote object with
e cross wires of this second telescope, and clamp it firm. If
e instrument is steady, the bisection will remain permanent
hilst any number of angles are measured, and by examin-
g the bisection from time to time during the operation at
e place where the instrument is set up, any error arising
m this cause may be detected and rectified.

At the suggestion of Captain Everest, surveyor-general of
dia, several small theodolites, differing considerably in con-
uction from the one last described, have lately been made
Messrs Troughton and Simms, for the great Indian sur-
y. In principle, they are similar to the theodolites of much
ger dimensions, and, consequently, the whole of their essen-
l adjustments are made in the same manner. Their use
l adjustments must, therefore, be understood as equally ap-
able to the larger theodolites, which are usually employed
extensive trigonometrical operations.

The horizontal circle (or limb) of this instrument consists
one plate only, which, as usual, is graduated at its circum-
nce. The index is formed with four radiating bars, having
niers at the extremities of three of them for reading the
izontal angles, and the fourth carries a clamp to fasten the
x to the edge of the horizontal limb, and a tangent screw
slow motion. These are connected with the upper works
ch carry the telescope, and, turning upon the same centre,
v any angle through which the telescope has been moved.
instrument has also the power of repeating the measure-
t of an angle; for the horizontal limb being firmly fixed
centre, moveable within the tripod support, and governed
clamp and tangent screw, can be moved with the same
acy, and secured with as much firmness, as the index
e it. Large theodolites, when required, have the power
epeating given them, by means of a particular kind of
l, called a repeating table.

The tripod support, which forms the stand of the instrument, has a foot screw at each extremity of the *arms* which form the tripod. The heads of the foot screws are turned downwards, and have a flange (or shoulder) upon them; so that when they rest upon a triangular plate fixed upon the staff-head, another plate locks over the flange, and, being acted upon by a spring, retains the whole instrument firmly upon the top of the staff, which is similar to that of the theodolite represented in fig. 40. The great advantage of the tripod stand is, that it can easily be disengaged from the top of the staff, and placed upon a parapet or other support, in situations where the staff cannot be used.

The telescope is mounted in the manner of a transit instrument, that is, the horizontal axis and the telescope form one piece, the axis crossing the telescope about its middle, and terminating at each extremity in a cylindrical pivot. The pivots rest upon low supports, carried out from the centre on each side by a flat horizontal bar, to which a spirit level is attached, for adjusting the axis to the horizontal plane. The vertical angles are read off on two arcs of circles, which have the horizontal axis as their centre, and, being attached to the telescope, move with it in a vertical plane. An index, upon the same centre, carries two verniers, and it has a spirit level attached to it, by which the index can be set in a horizontal position, so that whatever position the telescope, and, consequently, the graduated arcs, may have when an observation is made, the mean of the two readings will denote the elevation or depression of the object observed from the horizontal plane.

The following are the adjustments of this instrument:— First, To set the instrument level. To accomplish this, bring the spirit bubble attached to the horizontal bar in a direction parallel to two of the foot screws, and, by their motion, cause the air bubble to assume a central position in the glass tube; then turn the telescope, level, &c. half round, and if the bubble is not central, correct half the deviation by raising or lowering one end of the level itself, and the other half by the foot screws, which, in this instrument, perform an office similar to that of the parallel plate-screws of the theodolite already described. Having perfected this part, turn the telescope a quarter round, and the level will be over the third foot screw, which must be moved to set the level correct, and this part of the adjustment will be complete.

The line of collimation must be next attended to. Direct

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The line of collimation must be next attended to. Direct

till the needle reads zero, at the same time reading off the izontal limb; then turning the upper plate about, bisect the ect, and read again; the difference between this reading and former will be the bearing required.

n taking angles of elevation or depression, it is scarcely essary to add, that the object must be bisected by the hontal wire, or rather by the intersection of the wires; and ;, after observing the angle with the telescope in its natural ition, it should be repeated with the telescope turned half nd in its Y's, that is, with the level uppermost. The mean he two measures will neutralize the effect of any error that exist in the line of collimation.

he proof of the accuracy of a number of horizontal angles, ey quite surround the station from whence they are taken, add them all together, and their sum, if correct, will be °. If they are taken at several stations, consider them as internal angles of a geometrical figure, and the lines coning the stations as the sides of such figure; then, if the re has three sides, their sum will = 180°, if four sides, 60°, if more than four, multiply 90° by double the numof sides, and subtract 360° from the product; the remainwill be the sum of the internal angles.

he altitude and azimuth of a celestial object may likewise bserved with the theodolite, the former being merely the ation of the object taken upon the vertical arc, and the r its horizontal angular distance from the meridian.

LEVELLING WITH THE THEODOLITE.

he use of the theodolite as a levelling instrument, consists king a series of angles of elevation and depression along ine, the section of which is required. This must be done ery point where the inclination of the line changes, and listance measured between the instrument and the station-

This distance, it will be evident, is the hypothenuse of ht-angled triangle; the perpendicular of which is the difce of level. To insure accuracy, the angles should be ved both forwards and backwards, by making the instruand staff change places; and a mean of the two measures ld be taken as the correct angle. The instrument should t up (as nearly as possible) at a constant height from the nd, and the staff used for the observations should have a vane, or conspicuous mark on it, at exactly the same

height from the ground as the centre of the telescope, which mark must be bisected by the cross-wires in observing. Great care should be taken that the adjustments of the instrument are correct, more particularly the line of collimation, and the level attached to the telescope.

With the measured distance, and the observed angle, the difference of level may be computed, by adding to the logarithm of the measured distance the log. sine of the vertical angle, and their sum, rejecting 10 from the index, will be the log. of the difference of level (in feet or links, as the distance was measured*). Having a series of elevations and depressions, the final difference of level between the extreme, or any two stations, may be found by simply taking the difference of the sums of the intervening elevations and depressions.

THE SEXTANT.

Let ABC (fig. 41) represent a sextant, having an index, AG (to which is attached a mirror at A), moveable about A as a centre, and denoting the angle it has moved through, on the arc BC; also let the half-silvered (or horizon) glass, *ab*, be fixed parallel to AC; now a ray of light, SA, from a celestial object, S, impinging against the mirror, A, is reflected off at an equal angle, and striking the half-silvered glass at D, is again reflected to E, where the eye likewise receives through the transparent part of that glass a direct ray from the horizon. Then the altitude, SAH, is equal to double the angle, CAG, measured upon the limb, BC, of the instrument.

For the reflected angle, BAG, (or DAF) = the incident angle SAI, and the reflected angle *b*DE = the incident *a*DA = DAE = DEA, because *ab* is parallel to AC. Now, HAI = DFA = (FAE + FEA), and DAE being equal to DEA, it follows that HAI = (DAE + FAE). From HAI and (DAE + FAE) take the equal angles, SAI and DAF, and there remains SAH = 2 FAE, or 2 GAC; or, in other words, the angle of elevation, SAH, is equal to double the angle of inclination of the two mirrors, DGA being equal to GAC.

* When the distance has been measured in links of Gunter's chain, and the difference of level is required in feet, add the above logarithm to the constant log. 9.8195439. Their sum, rejecting 10 from the index, will be the log. of the difference of level in feet.

telescope to some well-defined object, and make the verti-wire bisect it; then turn the axis end for end, an operation ich of course inverts the telescope, and if the object be not v bisected by the vertical wire, correct half the deviation the collimating screws at the eye-end of the telescope, and other half by giving motion in azimuth to the instrument; this must be repeated till the adjustment is satisfactorily omplished.

'inally, For the zero of altitude. Take the altitude or ression of an object with the vertical sector in reversed po-ons, half the sum will be its true altitude or depression; to this let the verniers be set. Again, carefully direct the scope to the object, making the bisection by the screws ch retain the index in a horizontal position, and finally, ect the level by the adjusting screws at one of its ends.

THE METHOD OF OBSERVING WITH THE THEODOLITE.

'he instrument being placed exactly over the station from nce the angles are to be taken, by means of the plumb-suspended from its centre, it must be set level by the pa-l plate-screws *b*, *b*, &c., bringing the telescope over each alternately; one must be unscrewed while its opposite one rewed up, until the two spirit levels on the vernier plate lily keep their position in the middle of their tubes while nstrument is turned quite round upon its staff-head, when ll be ready for commencing operations—(supposing the stments before described to have been carefully examined rectified, otherwise the observations will be good for no-.) First clamp the lower horizontal limb firm in any ion, and direct the telescope to one of the objects to be ved, moving it till the cross wires and object coincide; clamp the upper limb, and by its tangent screw make the ection of the wires nicely bisect the object. Now, read ie two verniers, the degrees, minutes, and seconds of r) one, which call A, and the minutes and seconds only e other, which call B, and take the mean of the reading

$$
\begin{aligned}
\text{A} &= 142^\circ\ 36'\ 30'' \\
\text{B} &= 37'\ \ 0'' \\
\hline
\text{Mean} &= 142^\circ\ 36'\ 45''
\end{aligned}
$$

release the upper plate, and move it round until the

them; the last has a greater magnifying power, showing the contact of the images much better. The adjustment for distinct vision is obtained by sliding the tube at the eye-end of the telescope in the inside of the other. This also is the means of adapting the focus to suit different eyes. In the inverting telescope are placed two wires, parallel to each other, and in the middle of the space between them the observations are to be made, the wires being first brought parallel to the plane of the sextant, which may be judged of with sufficient exactness by the eye. When observing with this telescope, it must be borne in mind, that the instrument must be moved in a contrary direction to that which the object appears to take, in order to keep it in the field of view.

Four dark glasses, of different depths of shade and colour, are placed at K, between the index and horizon glasses; also three more at N, any one or more of which can be turned down to moderate the intensity of the light before reaching the eye, when a very luminous object (as the sun) is observed. The same purpose is effected by fixing a dark glass to the eye-end of the telescope; one or more dark glasses for this purpose generally accompany the instrument. They, however, are chiefly used when the sun's altitude is observed with an artificial horizon, or for ascertaining the index error; as employing the shades attached to the instrument for such purposes would involve in the result any error which they might possess. The handle, which is shown at O, is fixed at the back of the instrument. The hole in the middle is for fixing it to a stand, which is useful when an observer is desirous of great steadiness.

OF THE ADJUSTMENTS.

The requisite adjustments are the following:—The index and horizon glasses must be perpendicular to the plane of the instrument, and their planes parallel to each other when the index division of the vernier is at 0° on the arc, and the optical axis of the telescope must be parallel to the plane of the instrument.

TO EXAMINE THE ADJUSTMENT OF THE INDEX GLASS.

Move the index forward to about the middle of the limb, then, holding the instrument horizontally with the divided

telescope to some well-defined object, and make the verti- wire bisect it; then turn the axis end for end, an operation ch of course inverts the telescope, and if the object be not r bisected by the vertical wire, correct half the deviation the collimating screws at the eye-end of the telescope, and other half by giving motion in azimuth to the instrument; this must be repeated till the adjustment is satisfactorily ›mplished.

'inally, For the zero of altitude. Take the altitude or :ession of an object with the vertical sector in reversed po- ns, half the sum will be its true altitude or depression; to this let the verniers be set. Again, carefully direct the scope to the object, making the bisection by the screws :h retain the index in a horizontal position, and finally, ect the level by the adjusting screws at one of its ends.

THE METHOD OF OBSERVING WITH THE THEODOLITE.

'he instrument being placed exactly over the station from ıce the angles are to be taken, by means of the plumb- suspended from its centre, it must be set level by the pa- l plate-screws *b*, *b*, &c., bringing the telescope over each alternately; one must be unscrewed while its opposite one 'ewed up, until the two spirit levels on the vernier plate ily keep their position in the middle of their tubes while ıstrument is turned quite round upon its staff-head, when l be ready for commencing operations—(supposing the tments before described to have been carefully examined :ectified, otherwise the observations will be good for no- .) First clamp the lower horizontal limb firm in any on, and direct the telescope to one of the objects to be ved, moving it till the cross wires and object coincide; clamp the upper limb, and by its tangent screw make the ection of the wires nicely bisect the object. Now, read ıe two verniers, the degrees, minutes, and seconds of :r) one, which call A, and the minutes and seconds only ɜ other, which call B, and take the mean of the reading

$$
\begin{array}{rl}
\text{A} = & 142^\circ\ 36'\ 30'' \\
\text{B} = & \ 37'\ \ 0'' \\
\hline
\text{Mean} = & 142^\circ\ 36'\ 45''
\end{array}
$$

release the upper plate, and move it round until the

contact by moving the tangent screw, and call the minutes and seconds denoted by the vernier the reading on the arc. Next, place the index about the same quantity to the right of zero, or on the arc of excess, and make the contact of the two images perfect as before, and call the minutes and seconds on the arc of excess* the reading off the arc; and half the difference of these numbers is the index error; additive when the reading on the arc of excess is greater than that on the limb, and subtractive when the contrary is the case.

EXAMPLE.

Reading on the arc,	. .	31′ 56″
,, off the arc,	. .	31′ 22″
Difference,	. .	0′ 34″
Index error,	. =	— 0′ 17″

In this case, the reading on the arc being greater than that on the arc of excess, the index error = 17 seconds, must be subtracted from all observations taken with the instrument, until it be found, by a similar process, that the index error has altered. One observation on each side of zero is seldom considered enough to give the index error with sufficient exactness for particular purposes. It is usual to take several measures each way, "and half the difference of their means will give a result more to be depended on than one deduced from a single observation only on each side of zero." A proof of the correctness of observations for index error is obtained by adding the above numbers together, and taking one-fourth of their sum, which should be equal to the sun's semi-diameter, as given in the Nautical Almanac. When the sun's altitude is low, not exceeding 20° or 30°, his horizontal, instead of his perpendicular diameter, should be measured (if the observer intends to compare with the Nautical Almanac, otherwise there is no necessity); because the refraction at such an altitude affects the lower border (or limb) more than the upper, so as to make his perpendicular diameter appear less than his horizontal one, which is that given in the Nautical Almanac;

* When reading off the arc of excess, the vernier must be read backwards, or from its contrary end.

in this case the sextant must be held horizontally. Atmospheric refraction operates only on an object in a vertical plane; an azimuthal angle is not in the least effected by it, whereas an altitude is considerably increased from its effects.

TO MAKE THE LINE OF COLLIMATION OF THE TELESCOPE PARALLEL TO THE PLANE OF THE SEXTANT.

This is known to be correct when the sun and moon, having a distance of 90° or more, are brought into contact just at the wire of the telescope which is nearest the plane of the sextant, fixing the index and altering the position of the instrument to make the objects appear on the other wire; if the contact still remains perfect, the axis of the telescope is in proper adjustment; if not, it must be altered by moving the two screws which fasten to the up-and-down piece the collar into which the telescope screws. This adjustment is not very liable to be deranged.

Having now gone through the principle and construction of the sextant, it remains to give some instructions as to the manner of using it.

It is evident that the plane of the instrument must be held in the plane of the two objects, the angular distance of which is required. It must therefore be held in a vertical plane when altitudes are to be taken, but in a horizontal or oblique plane when horizontal or oblique angles are to be measured. As this adjustment of the plane of the instrument is rather difficult and troublesome to the beginner, he need not be surprised or discouraged although his first attempts may not answer his expectations. The sextant must be held in the right hand, and as slack as is consistent with its safety; for, in grasping it too hard, the hand is apt to be rendered unsteady.

When the altitude of an object, the sun for instance, is to be observed, the observer, having the sea horizon before him, must turn down one or more of the dark glasses or shades, according to the brilliancy of the object; and directing his sight to that part of the horizon immediately beneath the sun, and holding the instrument vertically, he must, with the left hand, slightly slide the index forward until the image of the sun reflected from the index glass appears in contact with the horizon, seen through the unsilvered part of the horizon glass; then clamp it firm, and gently turn the tangent screw to make the contact of the upper or lower limb of the sun and the ho-

rizon perfect, when it will appear a tangent to his circular disc.* If an artificial horizon is employed, the two images of the sun must be brought into contact with each other; but this will be explained when speaking of that instrument. To the angle read off apply the index error, and then add or subtract the sun's semidiameter, as given in the Nautical Almanac, according as the lower or upper limb is observed, to obtain the apparent altitude of the sun's centre. Before we can use this observation for determining the time, the latitude, &c., it must be further corrected for refraction and parallax, to obtain the true altitude, subtracting the former and adding the latter; and when the sea horizon is employed, a quantity must also be subtracted for the dip, which is unnecessary when the altitude is taken by means of an artificial horizon.

Tables for obtaining the above corrections may be found in Mr Baily's Astronomical Tables, &c., in the Requisite Tables, or in any modern work on navigation.

EXAMPLE.

Observed altitude of sun's lower limb, ...	70° 28′ 12″
Index error additive, ...	0 0 14
Apparent altitude, ...	70° 28′ 26″
† Refraction = 40″.20 *minus* parallax in altitude 4″ =	0 0 36 . 20
	70° 27′ 49″.80
Add sun's semi-diam. = 16′ 4″. 12 *minus dip* of horizon, for 20 feet elevation, 4′ 4″ =	+ 12′ 0″. 12
True altitude of the sun's centre,	70° 39′ 49″.92

If the observer is ignorant of the precise moment of the object's being on the meridian, he should, by a slow and gradual motion of the tangent screw, keep the observed limb in contact with the horizon, so long as it continues to rise; and immediately on the altitude's appearing to diminish, cease from observing, and the angle then read on the instrument will be the meridian altitude.

* If the observer knows his latitude approximately, he may find the meridional altitude nearly, to which he may previously set his instrument, when he will not only find his object more easily, but have only a small quantity to move the index to perfect the observation.

Take from the Nautical Almanac the declination of the object, and, if it be of the same name with the latitude, add it to the co-latitude; if of a different name subtract it. The sum or difference will be the meridian altitude nearly.

† An observation of a star requires no correction for either parallax or semidiameter.

The accuracy of the observation will much depend on the clearness of the horizon, which will always be ill defined when the object is surrounded by clouds, which will descend with it in altitude; and the varying state of the atmosphere near the horizon will materially affect the correction for the dip, which must render the observation less correct than that which would be obtained by the aid of an artificial horizon. The young practitioner would find it much to his advantage, in acquiring an expert and easy manner of using the inverting telescope, were he daily to observe the meridian altitude of the sun, or other objects with it, instead of a plain tube.

After what has been advanced, little need be said about observing lunar distances, whether of the moon and the sun, or the moon and a fixed star or planet, except that the instrument must be held in the plane of the two objects; and it is generally preferable to direct the telescope to the fainter object, particularly if a star, as it can be more easily kept in view, when seen directly, than by reflection. If the brighter object is to the left, the sextant must be held with the face downwards.

The enlightened limb of the moon is always to be brought into contact with the sun or star, even though the moon's image is made to pass beyond the sun or star before the desired contact can be obtained.

Perhaps the best method of taking a lunar distance is, not to attempt to make the contact perfect by the tangent-screw; but, when the nearest limbs are observed, make the objects overlap each other a little when they are receding, or leave a small space between them when they are approaching, and wait till the contact is perfect, and the reverse when the furthest limbs are observed.

The altitudes of the two objects should be observed at the same instant as the distance, and the time noted by a chronometer or watch. This would require several observers, but one person may take them all by having recourse to the following method:—"First, observe the altitude of the sun or star; secondly, the altitude of the moon; then any number of distances; next, the altitude of the moon; and lastly, the altitude of the sun or star, noting the times of each by a watch. Now, add together the distances and times when they were observed, and take the mean of each; and, in order to reduce the altitudes to the mean time, make the following proportion:—As the difference of times between the observations is to the dif-

ference of their altitudes, so is the difference between the time that the first altitude was taken and the mean of the times at which the distances were observed, to a fourth number, which, added to, or substracted from, the first altitude, according as it is increasing or decreasing, will give the altitude reduced to the mean time."

The angular distances of terrestrial objects are measured by the sextant in the same manner as those of celestial ones; but if the objects are not in the same horizontal plane, a reflecting instrument will not give their horizontal angular distance; but this may be obtained nearly by measuring their angular distances from an object in or near the horizon, which subtends a great angle with both; and the sum, or the difference of the angles so measured, will be nearly the required horizontal angle.

Of the sextant, it has been said, that it is in itself a portable observatory; and it is, doubtless, one of the most generally useful instruments that has ever been contrived, being capable of furnishing data, to a considerable degree of accuracy, for the solution of a numerous class of the most useful astronomical problems; affording the means of determining the time, the latitude and longitude of a place, &c., for which, and many other purposes, it is invaluable to the land-surveyor, as well as the navigator.

TROUGHTON'S REFLECTING CIRCLE.

FIG. 43 represents this instrument, which, in principle and use, is the same as the sextant. It has three vernier readings, A, B, C, moving round the same centre as the index glass, E, which is upon the opposite face of the instrument. One of the verniers, B, carries the clamp and tangent screw; D represents the microscope for reading the verniers; it is similar to the one used in reading the sextant, and is adapted to each index bar, by slipping it on a pin placed for that purpose, as shown in the figure. The horizon glass is shown at F. The barrel, G, contains the screws for giving the up-and-down motion to the telescope; it is put in action by turning the milled head under the barrel. H is the telescope, adapted to the instrument in a manner similar to that of the sextant. I and J are two handles

fixed parallel to the plane of the circle, and a third handle, K, is screwed on at right angles to that plane, and can be transferred to the opposite face of the instrument, by screwing it into the handle, I. The use of this extra handle is for convenience in reading and in holding the instrument, when observing angles that are nearly horizontal. It can be shifted according as the face of the instrument is held upwards or downwards. The requisite dark glasses are attached to the framework of the circle, to be used in the same manner and for the same purposes as those of the sextant. With respect to the adjustments and application of this instrument, we cannot do better than use the words of the inventor, Mr Troughton, contained in a paper which he calls

"DIRECTIONS FOR OBSERVING WITH TROUGHTON'S REFLECTING CIRCLE.

"Prepare the instrument for observation by screwing the telescope into its place, adjusting the drawer to focus, and the wires parallel to the plane, exactly as you do with a sextant; also set the index forwards to the rough distance of the sun or moon, or moon and star; and holding the circle by the short handle, direct the telescope to the fainter object, and make the contact in the usual way. Now read off the degree, minute, and second, by that branch of the index to which the tangent screw is attached; also the minute and second shown by the other two branches. These give the distance taken on three different sextants; but as yet it is only to be considered as half an observation; what remains to be done is to complete the whole circle by measuring that angle on the other three sextants; therefore, set the index backwards nearly to the same distance, and reverse the plane of the instrument by holding it by the opposite handle, and make the contact as above, and read off as before what is shown on the three several branches of the index. The mean of all six is the true apparent distance, corresponding to the mean of the two times at which the observations were made.

"When the objects are seen very distinctly, so that no doubt whatever remains about the contact in both sights being perfect, the above may safely be relied on as a complete set; but if, from the haziness of the air, too much motion, or any other cause, the observations have been rendered doubtful, it will be advisable to make more; and if, at such times, so many

readings should be troublesome, six observations and six readings may be conducted in the following manner:—Take three successive sights forwards, exactly as is done with a sextant, only take care to read them off on different branches of the index; also make three observations backwards, using the same caution: a mean of these will be the distance required. When the number of sights taken forwards and backwards are unequal, a mean between the means of those taken backwards and those taken forwards will be the true angle.

"It need hardly be mentioned, that the shades, or dark glasses, apply, like those of a sextant, for making the objects nearly of the same brightness; but it must be insisted on that the telescope should, on every occasion, be raised or lowered, by its proper screw, for making them perfectly so.

"The foregoing instructions for taking distances apply equally for taking altitudes by the sea or artificial horizon, they being no more than distances taken in a vertical plane. Meridian altitudes cannot, however, be taken both backwards and forwards the same day, because there is not time: all, therefore, that can be done, is to observe the altitude one way, and use the index error; but even here you have a mean of that altitude and this error taken on three different sextants. Both at sea and land, where the observer is stationary, the meridian altitude should be observed forwards one day and backwards the next, and so on alternately from day to day. The mean of latitudes deduced severally from such observations will be the true latitude; but in these there should be no application of index error, for that being constant, the result would in some measure be vitiated thereby.

"When both the reflected and direct images require to be darkened, as is the case when the sun's diameter is measured, and when his altitude is taken with an artificial horizon, the attached dark glasses ought not to be used. Instead of them, those which apply to the eye-end of the telescope will answer much better; the former, having their errors magnified by the power of the telescope, will, in proportion to this power and those errors, be less distinct than the latter.

"In taking distances, when the position does not vary from the vertical above 30 or 40 degrees, the handles which are attached to the circle are generally most conveniently used; but in those which incline more to the horizontal, that handle which screws into a cock on one side, and into the crooked handle on the other, will be found more applicable.

"When the crooked handle happens to be in the way of reading one of the branches of the index, it must be removed for

the time, by taking out the finger screw which fastens it to the body of the circle.

"If it should happen that two of the readings agree with each other very well, and the third differs from them, the discordant one must not on any account be omitted, but a fair mean must always be taken.

"It should be stated that, when the angle is about 30 degrees, neither the distance of the sun and moon, nor an altitude of the sun, with the sea horizon, can be taken backwards; because the dark glasses at that angle prevent the reflected rays of light from falling on the index glass, whence it becomes necessary, when the angle to be taken is quite unknown, to observe forwards first, where the whole range is without interruption; whereas, in that backwards, you will lose sight of the reflected image about that angle. But in such distances, where the sun is out of the question, and when his altitude is taken with an artificial horizon (the shade being applied to the end of the telescope), that angle may be measured nearly as well as any other; for the rays incident on the index glass will pass through the transparent half of the horizon glass, without much diminution of their brightness.

"The advantages of this instrument, when compared with the sextant, are chiefly these:—The observations for finding the index error are rendered useless, all knowledge of that being put out of the question, by observing both forwards and backwards. By the same means the errors of the dark glasses are also corrected; for, if they increase the angle one way, they must diminish it the other way by the same quantity. This also perfectly corrects the errors of the horizon glass, and those of the index glass very nearly. But what is still of more consequence, the error of the centre is perfectly corrected by reading the three branches of the index; while this property, combined with that of observing both ways, probably reduces the errors of dividing to one-sixth part of their simple value. Moreover, angles may be measured as far as 150 degrees, consequently, the sun's double altitude may be observed when his distance from the zenith is not less than 15 degrees; at which altitude the head of the observer begins to intercept the rays of light incident on the artificial horizon; and, of course, if a greater angle could be measured, it would be of no use in this respect.

"This instrument, in common with the sextant, requires three adjustments:—First, The index glass perpendicular to the plane of the circle. This being done by the maker, and not

liable to alter, has no direct means applied to the purpose; it is known to be right, when, by looking into the index glass, you see that part of the limb which is next you reflected in contact with the opposite side of the limb as one continued arc of a circle; on the contrary, when the arc appears broken, where the reflected and direct parts of the limb meet, it is a proof that it wants to be rectified. The second is, to make the horizon glass perpendicular. This is performed by a capstan screw, at the lower end of the frame of that glass, and is known to be right, when, by a sweep of the index, the reflected image of any object will pass exactly over, or cover the image of that object seen directly. The third adjustment is for making the line of collimation parallel to the plane of the circle. This is performed by two small screws, which also fasten the collar into which the telescope screws to the upright stem on which it is mounted; this is known to be right, when the sun and moon, having a distance of one hundred and thirty degrees, or more, their limbs are brought into contact, just at the outside of that wire which is next to the circle, and then examining if it be the same, just at the outside of the other wire. Its being so is the proof of adjustment."

THE PRISMATIC COMPASS.

THE use of this little instrument is to measure horizontal angles only, and, from its portability, it is particularly adapted for military surveying, or where but little more than a *sketch map* of the country is required. It is also very useful in filling in the detail of a map where all the principal points have been correctly fixed by means of the theodolite; and for this purpose it has been extensively employed by the gentlemen engaged on the Ordnance Survey. It may likewise be used for determining approximately the direction of the true meridian, the variation being determined by comparing the observed azimuth of a celestial object with its true azimuth deduced from an observation made for the purpose.

In fig. 44, A represents the compass-box, and B the card, which, being attached to the magnetic needle, moves as *it* moves, round the agate centre *a*, on which it is suspended. The circumference of the card is usually divided to 15′ of a degree, but it is doubtful whether an angle can be measured by it even to that degree of accuracy: *c* is a prism, which the

observer looks through in observing with the instrument. The perpendicular thread of the sight-vane, E, and the divisions on the card, appear *together* on looking through the prism; and the division with which the thread coincides, when the needle is at rest, is the magnetic azimuth of whatever object the thread may bisect. The prism is mounted with a hinge joint, D, by which it can be turned over to the side of the compass-box, that being its position when put into the case. The sight-vane has a fine thread stretched along its opening in the direction of its length, which is brought to bisect any object by turning the box round horizontally. The vane also turns upon a hinge joint, and can be laid flat upon the box, for the convenience of carriage. F is a mirror, made to slide on or off the sight-vane, E; and it may be reversed at pleasure, that is, turned face downwards: it can also be inclined at any angle by means of its joint, *d;* and it will remain stationary on any part of the vane, by the friction of its slides. Its use is to reflect the image of an object to the eye of the observer when the object is much above or below the horizontal plane. When the instrument is employed in observing the azimuth of the sun, a dark glass must be interposed, and the coloured glasses represented at G are intended for that purpose—the joint upon which they act allowing them to be turned down over the sloping side of the prism box.

At *e* is shown a spring, which being pressed by the finger at the time of observation, and then released, checks the vibrations of the card, and brings it more speedily to rest. A stop is likewise fixed at the other side of the box, by which the needle may be thrown off its centre; which should always be done when the instrument is not in use, as the constant playing of the needle would wear the point upon which it is balanced; and upon the fineness of the point much of the accuracy of the instrument depends. A cover is adapted to the box, and the whole is packed in a leather case, which may be carried in the pocket without inconvenience.

The method of using this instrument is very simple:—First, Raise the prism in its socket, *b*, until you obtain distinct vision of the divisions on the card; and standing at the place where the angles are to be taken, hold the instrument to the eye, and looking through the slit, *c*, turn round till the thread in the sight-vane bisects one of the objects whose azimuth, or angular distance from any other object, is required; then, by touching the spring, *e*, bring the needle to rest, and the division on the card which coincides with the thread on the vane will be the azimuth, or bearing of the object from the north or

south points of the magnetic meridian. Then turn to any other object, and repeat the operation; the difference between the bearing of this object and that of the former will be the angular distance of the objects in question. Suppose the former bearing to be 40° 30′, and the latter 10° 15′, both east, or both west, from the north or south, the angle will be 30° 15′.

The divisions are generally numbered 5°, 10°, 15°, &c. round the circle, to 360°. A stand can be had with the instrument, if required, on which to place it when observing, instead of holding it in the hand.

THE ARTIFICIAL HORIZON.

When the altitude of a celestial object is to be taken at sea, the observer has the natural (or sea) horizon as a line of departure; but on shore he is obliged to have recourse to an artificial one, to which his observations may be referred. This consists of a reflecting plane parallel to the natural horizon, on which the rays of the sun or other object falling, are reflected back to an eye placed in a proper position to receive them; the angle between the real object and its reflected image being then measured with the sextant, is double the altitude of the object above the horizontal plane.

Various natural, as well as artificial, reflecting surfaces have been made by mechanical arrangements, to afford the means of obtaining double angles; such as pouring water, oil, treacle, or other fluid substances, into a shallow vessel; and to prevent the wind giving a tremulous motion to its surface, a piece of thin gauze, talc, or plate glass, whose surfaces are perfectly plane and parallel, may be placed over it when used for observation. But the most accurate kind of artificial horizon is that in which fluid quicksilver forms the reflecting surface, the containing vessel being placed on a solid basis, and protected from the influence of the wind.

Fig. 45 represents an instrument of this kind. The mercury is contained in an oblong wooden trough, placed under the roof A, in which are fixed two plates of glass, whose surfaces are plane and parallel to each other. This roof effectually screens the surface of the metal from being agitated by the wind; and when it has its position reversed at a second observation, any error occasioned by undue refraction at either plate of glass will be corrected.

Another and more portable contrivance for an artificial horizon is represented in fig. 46, which consists of a circular plate of black glass, about two inches diameter, mounted on a brass stand, half an inch deep, with three foot screws, *a*, *b*, *c*, to set the plane horizontal; the horizontality being determined thus by the aid of a short spirit level, *d*, having under the tube a face ground plane on which it lies in contact with the reflecting surface, place the level on the glass in a direction parallel to the line joining two of the three foot screws, as *a* and *b*, then move one of these screws till the bubble remains in the middle of the tube in both the reversed positions of the level, and the plate will be horizontal in that direction; then place the level at right angles to its former position, and turn the third foot screw back or forwards, till the bubble again settles in the middle of its tube, the former levelling remaining undisturbed, and the plane will then be horizontal. This instrument, from its portability, is extremely convenient for travellers, as, when packed in its case, it can be carried in the pocket without being any incumbrance.

When an artificial horizon is used, the observer must place himself at such a distance that he may see the reflected object as well as the real one; then, having the sextant properly adjusted, the upper or lower limb of the sun's image (supposing that the object) reflected from the index glass, must be brought into contact with the opposite limb of the image reflected from the artificial horizon, observing that when the inverting telescope is used, the upper limb will appear as the lower, and *vice versa;* the angle shown on the instrument, when corrected for the index error, will be double the altitude of the sun's limb above the horizontal plane; to the half of which, if the semi-diameter, refraction, and parallax be applied, the result will be the true altitude of the centre.

EXAMPLE.

Observed angle, . .		122° 25′ 50″.00
Index error, . .	—	17″.05
		2)122° 25′ 32″.95
Appt. alt. . . .		61° 12′ 46″.47
Semi-diameter, . .	+	15′ 46″.91
Parallax, . . .	+	4″.00
Refraction, . .	—	34″.40
True alt. of sun's centre,		61° 28′ 2″.98

When the contact is formed at the lower limb, the images will separate shortly after the contact has been made, if the altitude be increasing; but, if the altitude be decreasing, they will begin to overlap; but when the contact is formed at the upper limb, the reverse takes place. An observer, if in doubt as to which limb he has been observing, should watch the object for a short time after he has made the observation.

EXAMPLES IN NAUTICAL ASTRONOMY.

If a vertical circle pass through a star (or any point in the heavens), the arch of that circle intercepted between the star and the horizon is called the *altitude* of the star; and the arch of the horizon intercepted between the said vertical and the meridian is called the *azimuth* of the star. Vertical circles are also called *circles of azimuth.*

The altitude of a star above the horizon is greatest when it is on the meridian, and a star has equal altitudes when it is at equal distances from the meridian on either side, or when its eastern and western azimuths are equal.

If a meridian circle pass through any star, the arch of that circle intercepted between the star and the equator is called the *declination* of the star; and if it pass through the zenith of any place, the arch intercepted between that zenith and the equator is called the *latitude* of the place.

The latitude of any place is equal to the altitude or elevation of the pole above the horizon of that place.

The elevation of the pole at any place is found by observing one of the stars which are so near it as not to set when it passes the meridian above, and again when it passes the meridian under, the pole; that is, by observing its greatest and its least altitudes above the horizon. Half the sum of the two altitudes of the star is the elevation of the pole, or the latitude of the place.

If the polar distance of a star be equal to the latitude, the star will never set at that place.

OF TIME.

The motion of the sun is not equable; it takes sometimes more and sometimes less than a mean solar day to complete its revolution. When the sun is upon the meridian, we say it is *apparent noon;* and all time deduced immediately from the sun is called *apparent time*, and is that portion of an *actual* revolution which the sun has passed through since it left the meridian.

In order to convert apparent into mean solar time, the *equation* of time must be applied to it; the equation of time being the accumulated errors of the sun (if the expression may be allowed) from mean solar time. The result will be the time which should be indicated by a solar clock or chronometer.

To convert degrees into time, and the contrary.—The following method is the most expeditious and independent of tables:—

As 15° degrees give one hour,
1° degree will give 4^m, minutes of time,
1′ minute ... 4^s, seconds, and
1″ second ... 4^{ths}, thirds.

If, then, the seconds of space be multiplied by four, the result will be *thirds* of time. In like manner, the minutes will produce *seconds*, and the degrees *minutes* of time.

Let it be required to convert 57° 49′ 54″ *into time.*—The operation will stand thus:—

$$\begin{array}{r} 57^\circ\ 49'\ 54'' \\ 4 \\ \hline 3^h\ 51^m\ 19^s\ 36^{ths} \end{array}$$

To convert time into degrees, &c.—The following method will be found the most convenient. Multiply the time by ten, and add to this half the product, the sum will be the corresponding degrees, &c.

Let it be required to convert $3^h\ 51^m\ 19^s\ 36^{ths}$ *into degrees, &c.*

$$\begin{array}{rrrrl} h. & m. & s. & ths. & \\ 3 & 51 & 19 & 36 & \\ & & & 10 & \\ \hline 38 & 33 & 16 & 0 & \\ 19 & 16 & 38 & 0 & \\ \hline 57^\circ & 49' & 54'' & 0''' & \text{corresponding space.} \end{array}$$

All the calculations in the *Nautical Almanac*, with the exception of the eclipses of Jupiter's satellites, are made for the instant of apparent noon at Greenwich. It becomes necessary, when any of the data contained in the *Almanac* are required for a given time at ship, to find the corresponding time at Greenwich, as follows:—

Turn the longitude of the ship into time, and add it to the given time if the longitude be west, but subtract it from the given time if the longitude be east. The result will be the given time at *Greenwich*.

If the result of the addition should exceed twenty-four hours, the excess will be the time from noon of the following day. In subtracting, twenty-four hours must be added if necessary to the time at ship, and the remainder will be the Greenwich time from noon of the preceding day.

EXAMPLES.

What is the apparent time at Greenwich, when the sun is upon the meridian in 76° west longitude?

76°	0′		ship's longitude,
	4		
$5^{h.}$	$4^{m.}$	$0^{s.}$	longitude west in time,
0	0	0	apparent time at ship,
5	4	0	apparent time at Greenwich.

Given the ship's longitude on the 15th of January, 105° 13′ west, and the apparent time, $6^{h.}$ $5^{m.}$; required the apparent time at Greenwich?

105°	13′		longitude west,
	4		
$7^{h.}$	$0^{m.}$	$52^{s.}$	longitude west in time,
6	5	0	apparent time at ship,
13	5	52	apparent time at Greenwich, 15th Jan.

Given the ship's longitude on the 13th August, 96° 7′ east, and the apparent time, $5^{h.}$ $7^{m.}$ $30^{s.}$; required the apparent time at Greenwich?

96°	7′		
	4		
$6^{h.}$	$24^{m.}$	$28^{s.}$	longitude east in time,
5	7	30	apparent time at ship,
22	43	2	apparent time at Greenwich, 12th August.

Given the ship's longitude on the 15th of January, 105° 13′ west, and the apparent time, $20^{h.}$ $5^{m.}$ $7^{s.}$; required the apparent time at Greenwich?

$7^{h.}$	$0^{m.}$	$52^{s.}$	longitude west in time,
20	5	7	apparent time at ship,
3	5	59	apparent time at Greenwich, 16th January.

TO DETERMINE THE LATITUDE OF A PLACE

FROM OBSERVATION.

THE terrestrial latitude of any station is its distance, in geographical degrees and parts of a degree, from the equator; which distance, being considered parallel to a corresponding arc in the heavens, is assumed to be equal to a celestial arc extending from the zenith of the station to the celestial equator or equinoctial line: any observation, therefore, of a heavenly body, which will afford data for determining this arc, will be competent to determine the latitude required.

The nature of the observation will depend on the properties and powers of the instrument made use of, and on the opportunity of using it with advantage. In an observatory supplied with fixed instruments, meridian altitudes or zenith distances of the sun, or, what is better, of a known star, afford the readiest as well as the most correct means of deducing the latitude, by direct measurement of the arc in question, or of its complement; but at sea, where there is a natural horizon, an instrument that measures by reflection, and that is also of a portable construction, such as the sextant and reflecting circle, is the only kind that can be employed with success.

When the celestial equator and observed object are on the same side of the zenith, the measured arc denoted by the zenith distance, when corrected for refraction, and for parallax if necessary, *plus* or *minus* the declination, according as it is north or south, will give the latitude, or arc contained between the zenith point and celestial equator; or, which is the same thing in other terms, the observed altitude, properly corrected, by changing the sign of the declination, will give the co-latitude. This may be called the solar method, as being applicable in all cases, both by sea and land, when this luminary is observed. It is, however, equally applicable to observations of stars that do not pass beyond the zenith towards the elevated pole, provided their declinations be known.

The second method of determining the latitude by observation, is by means of a known star passing the meridian be-

tween the zenith and elevated pole. In this case we have the latitude *equal* to the declination *minus* the meridian zenith distance; or *equal* to the altitude *minus* the polar distance (co-declination): but when the star is observed in passing the meridian below the pole, the co-latitude is *equal* to the zenith distance *minus* the polar distance; or *equal* to the altitude *plus* the polar distance.

And, thirdly, when a circumpolar star is observed on the meridian, both above and below the pole, at an interval of twelve siderial hours, half the sum of the two altitudes, respectively corrected for refraction, will be the latitude, without reference to the declination, which circumstance renders this an independent method, except so far as refraction is concerned, and may be relied on in an observatory. If these direct methods be repeated with different stars, and give the same result when compared together, it may be safely concluded that the instrument is good, and that the latitude is correctly deduced. When the star is observed according to any of these methods, at a short distance from the meridian, either before or after, as well as on the meridian, for the extermination of errors, the additional observations must of course be reduced to the meridian, by the proper tables, before a mean is taken for the true meridian observation.

All these direct methods are founded on the consideration, that the arc contained between the pole and nearest horizon is equal to the arc extending from the zenith to the celestial equator, which are each equal to the latitude of the place; and that therefore the arcs from the pole to the zenith, and from the equator to the other horizon, are equal to the co-latitude: hence it becomes a matter of indifference, as to accuracy, which of these four arcs be determined by observation. The most convenient of the four may therefore be measured.

These are the methods of ascertaining the latitude of a place, which are either differential or indirect, but which, under certain circumstances, may be resorted to.

First, when the latitude of one station or object has been well determined, the latitude of another station within sight may be inferred by a trigonometrical operation; secondly, the pole star may be observed at any point of its diurnal arc, and may be reduced to the meridian by tables given for that purpose; thirdly, the *true* latitude as well as the horary angle, may be computed from the observed altitudes, and the interval of time elapsed, provided the latitude be nearly known.

For other methods of determining the latitude by stars and the moon, see books on navigation.

Example I.

At a station near Calcutta, the altitude of the sun's upper limb was observed (by means of an artificial horizon) to be 138° 36′ 47″.00, the refraction being 59″.40, parallax 5″, the sun's semi-diameter 15′ 52″.70, and his declination, 1° 23′ 8″. Required the latitude?

The mean of several altitudes, . . =	138°	36′	47″	.00
Half the altitude, ...	69	18	23	.50
Refraction 59″.40, *minus* the parallax 5″, =			54	.40
	69	17	29	.10
Subtract sun's semi-diameter, because upper limb was observed, ...		15	52	.70
True altitude of the sun's centre, .	69	1	36	.40
N. declination, change the sign and subtract .	1	23	8	.12
Co. Latitude, . . =	67	38	28	.28
	90			
Latitude North, . . . =	22	21	31	.72

Or in this manner—

Altitude of sun's centre, . .	69°	1′	36″	.40
	90			
North zenith distance, . . =	20	58	23	.60
North declination, add . .	1	23	8	.12
As before, latitude N. . . =	22	21	31	.72

Example II.

The meridian altitude of Aldebaran, 1834, was 56° 30′, the zenith being north of the star, and the height of the eye 15 feet. Required the latitude?

Observed altitude, .	56°	30′	0″
Depression of the horizon, .	—	3	42
Apparent altitude, .	56	26	18
Refraction, . .	—	0	38
True meridian altitude, .	56	25	40
	90		
Zenith distance north, .	33	34	20
Declination north, . .	16	8	44
Latitude north, . .	49	43	4

Example III.

The observed meridian altitude of Sirius was 80° 4′, the zenith being south of the star, declination 16°·28·9 S., and height of the eye 20 feet. Required the latitude?

Observed altitude,	80°	4′	00″
Dip, ...	—	4	17
Apparent altitude, . . .	79	59	43
Refraction, ...	—	0	10
True altitude, . . .	79	59	33
	90		
Zenith distance south, .	10	00	27
Declination south, . .	16	28	9
Latitude south, . . .	26	28	36

Example IV.

The observed altitude of the Polar star, when on the meridian below the pole, was 30° 7′, the height of the eye being 18 feet. Required the latitude?

Observed meridian altitude,	30°	7′	0″
Dip, ...	—	3	50
	30	3	10
Refraction, ...	—	1	40
Star's true altitude, .	30	1	30
Polar distance, . +	1	38	2
Latitude north, . .	31	39	32

Example V.

The observed altitude of Capella, when on the meridian below the pole, was 10° 2′, the height of the eye being 17 feet. Required the latitude?

Observed meridian altitude,	10°	2′	0″
Dip, ...	—	3	57
	9	58	3
Refraction, ...	—	5	21
Star's true altitude, .	9	52	42
Polar distance, . +	44	11	33
Latitude north, . .	54	4	15

EXAMPLE VI.

The meridian altitude of the planet Jupiter was observed, on the 5th of February 1823, to be 50° 25′, the zenith being to the north of the planet, the ship's longitude 24° west, and the height of the eye 18 feet. Required the latitude?

	h.	m.
Time at ship nearly, . .	6	24
Longitude west, 24° in time, .	1	36
Greenwich time nearly,	8	00
Jupiters declination at Greenwich, 1st Feb., north, . .	18°	44
Jupiter's declination, 7th Feb., north, . . .	18°	49
Variation of declination in six days,	0	5

Hence, by proportion, the declination of Jupiter, the 5th Feb. at $8^{h.}$, is 18° 47′ 37″ north.

Observed altitude of Jupiter, .	50°	25′	0″
Dip. . .	0	4	4
Apparent altitude, .	50	20	56
Refraction, . —	0	0	48
True meridian altitude, .	50	20	8
	90		
Zenith distance, north .	39	39	52
Declination north, .	18	47	37
Latitude north, . .	58	27	29

The importance of repeatedly ascertaining the latitude on approaching land at night is well known to every seaman; any method, therefore, by which he might ascertain his latitude hourly if necessary, under such circumstances, cannot but be truly valuable.

The following method, by Mr Littrow, is valuable for its simplicity and accuracy. The data required are, the observed altitudes of the pole star, and the corresponding apparent time at the ship, or place, from which the latitude may be found, in the following manner:—

With the apparent time of observation and the longitude, find the corresponding time at Greenwich, for which take the right ascension of the sun from the *Nautical Almanac*, and add it to the apparent time at the ship. The sum, rejecting

24 hours, if necessary, will be the siderial time, or the right ascension of the meridian.

From the right ascension of the meridian subtract the right ascension of the Pole star, the remainder will be the distance of the Pole star from the meridian in siderial time.

Enter the annexed table, and seek this time in one of the columns, entitled "Distance from the meridian," and take out the corresponding first correction, subtracting the annual variation if necessary; take out also the logarithm of A.

Apply the first correction with the sign at the top of the column in which the distance from the meridian was found to the true altitude of the pole star.

To the logarithm of A add the logarithmic tangent of the altitude, the sum will be the logarithm of a further correction in seconds, which being always added to the number before found, the result will be the latitude.

The sun's right ascension for the corresponding time at Greenwich may be taken from the *Nautical Almanac*, with sufficient accuracy for the present purpose, by estimation.

Example.

On the 25th of June 1824, in longitude 16° west, the altitude of the Pole star was observed to be 47° 5′ at 12h. 14m. mean time, the height of the eye being 20 feet. Required the latitude?

	h. m. s.		° ′ ″		
Mean time, . .	12 14 00	Obs. alt.	47 5 0		
Longitude in time added,	1 4 00	Dip of hor.—	4 17		
Mean time at Greenwich,	13 18 00		47 0 43		
Equation of time, .	0 2 17	Refraction,—	0 54		
App. time at Greenwich,	13 15 43	True alt.	46 59 49	tan.	10·0455
Sun's right ascension,	6 19 0	1st correc.—	15 43	log. A.	1·9098
Right ascen. of the merid.	19 34 43		46 44 6		
Rt. ascen. of the Pole star,	0 58 1				
			+ 1 30=90″	log.	1·9553
Star's dist. from the merid.	18 36 42				
		Lat. req.	46 45 36 N.		

The mean of several altitudes ought always to be taken in preference to one.

TABLE FOR FINDING THE LATITUDE BY THE ALTITUDE OF THE POLE STAR.

Distances from the meridian.								First correction.			Annual variation	Log of A'
−		+		+		+						
h.	min.	h.	min.	h.	min.	h.	min.	°	′	″	″	
0	0	12	0	12	0	24	0	1	37	48	19.45	
	10		50		10		50	1	37	42	19.41	9.2009
	20		40		20		40	1	37	26	19.37	9.8021
	30		30		30		30	1	36	58	19.28	0.1529
	40		20		40		20	1	36	19	19.15	0.4009
	50		10		50		10	1	35	30	18.97	0.5921
1	0	11	0	13	0	23	0	1	34	28	18.80	0.7475
	10		50		10		50	1	33	17	18.54	0.8777
	20		40		20		40	1	31	54	18.30	0.9897
	30		30		30		30	1	30	21	17.99	1.0871
	40		20		40		20	1	28	38	17.62	1.1733
	50		10		50		10	1	26	45	17.26	1.2503
2	0	10	0	14	0	22	0	1	24	42	16.83	1.3195
	10		50		10		50	1	22	29	16.41	1.3819
	20		40		20		40	1	20	7	15.92	1.4387
	30		30		30		30	1	17	36	15.42	1.4903
	40		20		40		20	1	14	55	14.90	1.5377
	50		10		50		10	1	12	7	14.36	1.5809
3	0	9	0	15	0	21	0	1	9	9	13.74	1.6205
	10		50		10		50	1	6	5	13.15	1.6569
	20		40		20		40	1	2	52	12.51	1.6899
	30		30		30		30	0	59	32	11.84	1.7205
	40		20		40		20	0	56	6	11.17	1.7483
	50		10		50		10	0	52	33	10.45	1.7735
4	0	8	0	16	0	20	0	0	48	54	9.73	1.7965
	10		50		10		50	0	45	10	8.98	1.8173
	20		40		20		40	0	41	19	8.22	1.8361
	30		30		30		30	0	37	26	7.45	1.8527
	40		20		40		20	0	33	27	6.66	1.8675
	50		10		50		10	0	29	25	5.85	1.8803
5	0	7	0	17	0	19	0	0	25	19	5.04	1.8915
	10		50		10		50	0	21	10	4.21	1.9007
	20		40		20		40	0	16	59	3.38	1.9081
	30		30		30		30	0	12	46	2.54	1.9139
	40		20		40		20	0	8	32	1.70	1.9181
	50		10		50		10	0	4	16	0.85	1.9207
6	0	6	0	18	0	18	0	0	0	0	0.00	1.9215

TO DETERMINE THE TIME BY MEANS OF AN OBSERVATION OF THE SUN OR A STAR TAKEN AT A DISTANCE FROM THE MERIDIAN.

To perform this, the altitude of the sun or a known star must be taken either at sea, or on shore, with an artificial horizon and sextant, from which the time, at the moment when the observation was made, may be found by calculation. It is a case in spherical trigonometry, where the three sides are given to find an angle opposite to one of them.

To solve the problem it is necessary that the latitude of the place, the declination of the observed object, and its altitude, should be known, as the complements of these three arcs constitute the sides of the given triangle; and it is the angle at the pole, subtended by the zenith distance, which is to be found.

NOTE.—Subtract the latitude from 90° to find the co-latitude. The polar distance is found by subtracting the sun's declination from 90° when the latitude and declination are both north or both south, or by adding it to 90° when one is north and the other south.

RULE TO FIND THE TIME.

Add together the zenith distance and the polar distance, and from their sum subtract the co-latitude, and note the remainder.

Then add together the zenith distance and co-latitude, and from their sum subtract the polar distance, and note the second remainder.

Then to the log. sin. of half the first remainder add the log. sin. of half the second remainder; their sum, adding 20 to the index, call A.

Add together the log. sin. of the co-latitude and the log. sin. of the polar distance, and subtract their sum from the sum denoted A, half the remainder will be the log. sin. of half the hour angle in space, which must be converted into time and multiplied by 2 to obtain the apparent hour angle. To this apply the equation of time, as directed in the *Nautical Almanac*, and the result will be the mean hour angle sought; the difference between which and the time indicated by chronometer will be its error, either too fast or too slow, according as it exceeds, or is less than the mean time deduced from observation.

EXAMPLE.

In latitude 51° 31′ 8″ N., at $9^h\ 28^m\ 45^s$ A. M. mean time by a chronometer, several altitudes of the sun were observed by a reflecting circle and artificial horizon; after the corrections for index error, refraction, parallax, and sun's semidiameter had been made, the true altitude of the sun's centre was found to be 31° 6′ 56″·43, making the zenith distance = 58° 53′ 3″·57, the co-latitude 38° 28′ 52″, and the sun's polar distance from the *Nautical Almanac*, and corrected for time before noon = 88° 56′ 45″. Required the apparent time and error of the watch?

Zenith distance, . . .	58° 53′ 3″.57	
Polar distance, . . .	88 56 45	
	147° 49′ 48″.57	
Co-latitude, ...	38 28 52	
First remainder,	2)109° 20′ 56″.57	
Half, . . .	54° 40′ 28″.28	Sine 9.911626
Zenith distance, . . .	58° 53′ 3″.57	
Co-latitude, ...	38 28 52	
	97° 21′ 55″.57	
Polar distance, . . .	88 56 45	
Second Remainder,	2)8° 25′ 10″.57	
Half, . . .	4° 12′ 35″.28	Sine 8.865748
		A 38·777374
Co-latitude, 38° 28′ 52″	sin. 9·793969	
Polar dist., 88 56 45	9.999926	
	19·793895	19·793895
		2)18·983479
Sin. ½ hour angle = 18° 4′ 32″	=	9·491739
Multiply by	2	
Whole hour angle	= 36° 9′ 4″	
	$= 2^h\ 24^m\ 36^s.3$	
Equation	+ 6 39 .7	
	$2^h\ 31^m\ 16$	= mean hour angle.

hence, $9^h\ 28^m\ 44^s$ = time of observation.

9 28 45 = time by chronometer.

Method 2d. From the Requisite Tables.

If the sun's declination and co-latitude of the place be one north and the other south, take their difference; but if both north or both south, take their sum for the meridian altitude.

From the natural sine of the sun's meridian altitude take the natural sine of the sun's true altitude, then add together the log. co-secant of the co-latitude, the log.-secant of the declination, and the logarithm of the difference of the natural sines.

The sum of these three logarithms being found in the table of *rising*, the corresponding time will be the apparent time from the nearest noon, as before.

EXAMPLE I.

Co-latitude,	= 38° 28′ 52″	. . . . Co-secant,	0.20603
Declination,	= 1 3 15	. . . . Secant, .	0.00007
Meridian altitude,	= 39° 32′ 7″	. Nat. sine, 63655	
True altitude,	= 31 6 56.43	Nat. sine, 51676	
	Difference,	. . 11979 Log.	4.07842
Apparent time, .	2^h 24^m $36^s.3$	Log. rising, .	4.28452

EXAMPLE II.

Latitude north, .	40° 5′ 0″	. . . . Secant, .	0.11628
Co-latitude, . .	49° 55′ 0″		
Declination north,	22 9 17	. . . . Secant, .	0.03330
	72° 4′ 17″	Nat. sine, 95144	
True altitude, .	30 2 0	Nat. sine, 50050	
		45094 Log.	4.65412
Apparent time, .	4^h 34^m $42^s.2$	Log. rising, .	4.80370

HAVING GIVEN THE OBSERVED ALTITUDE OF A FIXED STAR, ITS DECLINATION, AND THE LATITUDE, TO FIND THE APPARENT TIME AND LONGITUDE OF THE SHIP.

RULE.

Find the hour angle, or the distance of the star from the meridian, as in the last example. If the star is to the *west* of the meridian, add to this the star's right ascension, but if to the *east* of the meridian subtract it, and the result will be the

right ascension of the meridian. With the mean time at Greenwich by chronometer, find the apparent time at Greenwich, and the sun's right ascension for that time. Subtract the sun's right ascension from the right ascension of the meridian, and the remainder will be the apparent time at the ship; the difference between which and the apparent time at Greenwich, deduced from the chronometer, will be the longitude in time.

Example.

1834. In latitude 21° 12′ north, the mean of several observed altitudes of the star Antares, when eastward of the meridian, was 30° 42′, the height of the eye 16 feet, and the mean of corresponding times by chronometer $9^h\ 35^m\ 43^s$, which chronometer was too fast on a particular day for Greenwich time, $1^m\ 5^s$ at noon, and gaining daily $5^s{\cdot}4$. Required the apparent time and longitude of the ship? The observation was made 38 days after the error and rate of the chronometer had been determined.

	h.	*m.*	*s.*
Star's right ascension, .	16	18	31·2
Sun's right ascension, .	4	19	17·0
Star's declination, . .	16	28	30
Dip of horizon, . .	0	3	50
Refraction, . .	0	1	55

	h.	*m.*	*s.*
Time by chronometer,	9	35	43
Fast on given day for Greenwich,	—	1	5
	9	34	38
Accumulated gain in 38^d $9^h\ 35^m\ 43^s$ . .	—	3	27·5
Mean time at Greenwich, . .	9	31	10·5
Equation of time,	+	3	9·2
App. time at Greenwich,	9	34	19·7

Mean of obsd. altitudes,	30°	42′	0″
Dip of horizon,	—	3	50
	30	38	10
Refraction, . .	—	1	55
Star's true alt. . .	30	36	15

North latitude, . .	21°	12′		Secant, . . .		0.03043
Co-latitude, . . .	68	48				
Star's declination north,	16	28	30	Secant, . . .		0.01821
Meridian altitude, . .	85	16	30	Nat. sine,	996602	
True altitude, . .	30	36	15	Nat. sine,	509104	
					487498 Log.	4·68797
				Logarithmic rising,		4·73661

Apparent time, ...	4^h	11^m	$48^s.6$	
Star's right ascension, . .	16	18	31 . 2	
Right ascension of meridian, .	12	6	42 . 6	
Sun's right ascension at $9^h\ 34^m\ 19^s.7$,	4	19	17 . 0	
Apparent time at ship, . .	7	47	25 . 6	
Apparent time at Greenwich, .	9	34	19 . 7	
Longitude in time, . . .	1	46	54 . 1	
Multiply by . .				10 to turn it into space.
	17	49	1 . 0	
Add half, . .	8	54	30 . 5	
Longitude west, . . .	26°	43′	31″ 5‴	

Remark.—The logarithmic rising is the logarithmic versed sine of the hour angle, but with an index of four, in consequence of the natural sines being considered as whole numbers instead of decimals, as is usual, in which case the index would have been nine. The logarithmic versed sine may therefore be used instead of the logarithmic rising.

TO FIND THE RATE OF A CHRONOMETER BY A TRANSIT INSTRUMENT.

The best method, when on shore, of determining the rate of a chronometer, is by a portable transit instrument. For this purpose, it is by no means necessary that the instrument should be accurately in the meridian, but then the observations must be confined to the stars. The axis, however, should be carefully levelled, and the middle wire (which must be truly vertical) adjusted to a certain mark, previous to each observation. It is not *absolute* but *comparative* time that is required. Now, if the time of the passage of a certain star should be observed every night by a chronometer, and the chronometer keep mean time, it should lose $3^m\ 55^s{\cdot}91$ between any two successive transits of the same star. If the loss is less than this, the chronometer is *gaining* the difference daily; if, on the contrary, the loss is more than $3^m\ 55^s{\cdot}91$, the difference is the rate which the chronometer *loses* daily. The observations should be continued for at least a week, and the mean taken for the rate. It must be remembered that this rate is for a siderial day, but may be readily found for a solar day, if thought necessary, by simple proportion.

ON LONGITUDE.

The different methods of determining the longitude of a given place may be divided into *terrestrial* and *celestial*.

The first terrestrial method is by means of trigonometrical measurement, having previously determined the longitude of some object not very remote. In Great Britain, where the longitude of various church steeples and of elevated stations have already been ascertained by the general survey of the United Kingdom, which is still in operation, these determined land-marks afford facilities, in every county, of obtaining the longitude of any observatory, or temporary station, that may be within sight of one or more of them.

The second of these methods is that in which signals are used, which may be practised with considerable success when the distance from the known station, or point of departure, does not exceed twenty or thirty miles, according as the face of the country may be favourable to such mode of transmitting time from one place to another. When this plan is adopted, it is necessary that the exact time should be known at both stations, the longitude of one of which is supposed to be known; for ascertaining which two portable transit instruments may be recommended, for previous or subsequent regulation and correction of the clocks or chronometers used. A rocket, or succession of rockets, sent up at certain instants, previously agreed upon, will be proper signals when they burst at one station, and are observed at the other. The absolute time of such bursting will be the same at both stations, within a *quantity*, depending on the velocity of *light*, not appreciable at a practicable distance.

If a rocket cannot be seen at the whole distance, an elevated spot may be chosen between the two stations, where it may be viewed from the extreme stations at the same time; or a repetition of signals from several intermediate eminences may be observed; and then the difference of the observed times, taken at the two extreme stations, or the mean of the several differences, will give the difference of longitude in solar time, without regard to the bearing of the stations or other computation.

Of course, a clear evening must be mutually agreed upon, and the bearing of one place from the other must be so far known as to guide tne position of the telescopes to be used on the occasion, which should have large fields of view, and consequently but small magnifying powers. This method has been used with satisfactory success.

The third and last terrestrial method of comparing the longitude of one station with that of another, is by the conveyance of several chronometers, the rates of which are known; for when the time taken at the first station is conveyed to the second, and compared with the exact time there obtained, the difference between the two, when due allowance is made for the rates during the interval, will, as in the case of signals, give the difference of longitude, and at any unlimited distance.

This method has been practised with success by Dr J. L. Tiarks, who, under the employ of our government, conveyed several chronometers from England to the isle of Madeira, the longitude of a station in which he thus determined, as a convenient place for checking the marine rates of chronometers when vessels on long voyages touch at that island, which rates frequently differ from those previously ascertained on shore. As the vessel which conveyed the chronometers was actuated by steam, which occasioned frequent shocks in its motion, the ingenious German contrived a table that was suspended in gimbals; two out of three parallel frames turned on pivots at right angles to each other, and were loaded by a heavy weight, hanging down below the table, and lashed to it by diagonal lashings. Four hollow tubes were screwed down to the cabin deck, which contained each a spiral spring, on which the four feet of the outermost frame rested, and where the lower ends of the four diagonal lashings were fastened that held the feet in their places. This contrivance kept the surface of the table so level, and at the same time so free from the effect of the shocks from the engine, as well as from the vessel's motion occasioned by the sea, that the compartments for holding the chronometers might have been dispensed with; for their boxes had no tendency to slide from the position given them. The vibrating balances were not acted upon in any way that lengthened or shortened their vibrations, or that otherwise affected their going.

When several chronometers are used, an individual one is chosen as the standard, by which the indications of the rest are compared, in obtaining a mean of the whole; but, in making these comparisons, method and practice are indispensable.

The chronometer is of most general use as a nautical machine, to preserve the time given to it at the first meridian; for *the solar time at ship*, determined by a single observation of a celestial body, and compared with the corrected time of the chronometer, will give a difference equal to the longitude expressed in solar time—east when the time at the ship is greater, but west when smaller, than the Greenwich time.

The celestial or astronomical methods of obtaining the longitude may be divided into six—viz. (1.) by observing an immersion or emersion of one of Jupiter's satellites; (2.) by a lunar eclipse; (3.) by the moon's passage over the meridian compared with that of a known star, preceding or following her; (4.) by measured lunar distances; (5.) by an occultation of a star by the moon; and (6.) by an occultation of the sun, or solar eclipse. With respect to the first mode, by Jupiter's satellites, though it is not so accurate as some of the others, it is often used in observations; and as these phenomena frequently occur, they are previously computed for Greenwich time, and published annually, three or four years in advance, in the Nautical Almanac; and when several of these occurrences have been observed with a good telescope, at any particular place, the mean of the difference between the computed Greenwich time and the corresponding times of observation will give the longitude, east or west of Greenwich, according as the observed times are greater or less than the computed times given in the Almanac. The principal objection to this method is, that an immersion or emersion cannot be observed at sea, where a knowledge of the longitude is of daily importance.

LUNAR DISTANCES.

In the practical application of the *lunar method* of determining the longitude, the mode of making the necessary observations is first to be considered. As an oblique arc and two altitudes are required to be measured at the same moment to ensure complete success, and as a repetition of such co-temporary measures, taken at successive intervals, and as near as may be equidistant, will conduce to accuracy, it has been found most convenient to employ four persons; the first to observe the oblique arc, or apparent lunar distance, with the sextant or repeating circle; the second to measure the altitude of the moon; the third to measure that of the sun or star with a similar instrument; and the fourth to note down the exact time by the chronometer, at each instant when the observer taking the distance gives the notice; or, what will be more correct, the equidistant times may be announced by the person watching the chronometer, while the observers keep their *respective contacts* correct by the *tangent screws*. But if so many skilful persons are not present, both altitudes may be taken by a single observer, both before and after the angular distance is measured, and the corresponding times put down; from which the altitudes due to the mean time of all the distances may be

inferred by computation. The method of putting it into practice will be found with the description and use of the sextant, page 50.

When the sun or star is at a proper distance from the meridian, the time may be computed from its observed altitude, provided the latitude be known, with which the time shown by the chronometer may be compared as a check on the resulting longitude; otherwise the time must be determined by a preceding or subsequent observation.

It is very important that the contact, in measuring a lunar distance, be, as nearly as can be estimated, at that diametrical line of the telescope's field of view that lies parallel to the face of the sextant or circle; for at any other part, to the right or left of that line, the measured distance will be always too great, however properly the adjustment for parallelism of the telescope's collimation may have been previously made. The measure taken at the centre of the field will therefore always be the safest to adopt; and as this is the smallest measure that can be taken of a long arc, a practised observer will seldom have occasion to entertain a doubt of its accuracy; but without such precaution, there may be an error of some minutes in the measured distance, which, in spite of all the Nautical Tables, will be charged on the longitude. The apparent altitudes of the sun and moon are those which are corrected for the effects of refraction, parallax, dip, index error, and semidiameter, by the tables in the usual way; but that of a star requires not the correction for either parallax or semidiameter. The apparent distance is the measured arc corrected for the index error, and for the moon's semidiameter at the time when a star is the second object; but when the sun is the object, the sum of the semidiameters of the sun and moon must be applied with the index error.

Since the effect of refraction is to elevate a heavenly body, and that of parallax to depress it, the sun is more elevated by the former than depressed by the latter; but the moon, on the contrary, is more depressed by the latter than elevated by the former; and these effects vary at different altitudes. The first question therefore that arises, which is indeed the only difficult one to a common observer, is, how the apparent distance of the moon from the sun, or star, is affected by a reduction of their apparent to their true altitudes, as seen in a right line from the centre of the earth, instead of from a point on its surface, through a refracting medium?

To reduce the apparent to the true angular distance, is a problem that has employed the pens of many mathematicians;

and various tables have been computed and published, for the express purpose of clearing the apparent distance from the effect of parallax and refraction, by such rules and computations as have appeared best adapted for nautical usage. The following valuable method by Dr Thomas Young, taken from the Journal of Science, Literature, and Arts, will serve to explain what has been said on the subject.

Rule.

1. Add together the apparent altitude of the star, the apparent altitude of the moon, and the apparent distance, and from half the sum subtract the apparent altitude of the star, and call the result the remainder.

2. To the proportional logarithm of the moon's horizontal parallax, add the logarithmic secant of the moon's altitude, the sum will be the proportional logarithm of the parallax in altitude.

3. Add together the logarithmic secant of the half sum, *the* sine of the apparent distance, the co-secant of the remainder, the constant logarithm 9·6990, and the proportional logarithm of the horizontal parallax, the sum will be the proportional logarithm of the diminution of parallax; the difference between which, and the parallax in altitude, is the *parallactic correction*, which is to be subtracted from the observed distance, if the parallax in altitude is the greater, but if otherwise, to be added.

4. To the logarithmic sine of the moon's altitude add the proportional logarithm of the parallactic correction, and subtract the sum from the proportional logarithm of the horizontal parallax; the difference will be the tangent of an angle, which is the *refractional* distance for the moon. Consider as zenith distances both this angle and the difference between it and the apparent distance, and find the corresponding refractions in a table of refraction. These will be the first and second *refractional corrections;* the first of which is to be applied to the apparent distance, with a contrary sign to that of the parallactic correction, and the second to be added when the refractional distance for the moon is less than the observed distance; otherwise the second refractional correction must be subtracted.

5. Add together the proportional logarithm of the diminution of parallax, that of the sum of the parallactic correction and the parallax in altitude, the logarithmic tangent of the apparent distance and the constant logarithm 1.5870, the sum will be the proportional logarithm of the correction for obliquity, which is always additive.

XAMPLE.—*Taking the apparent distance* 43° 36′, *the apparent altitude of the moon* 90° 38′, *that of the star* 11° 17′, *the moon's horizontal parallax* 54′ 42″, *and its proportional logarithm* .5173; *to find the true distance.*

1.	2. PARALLAX IN ALTITUDE.	3. PARALLACTIC CORRECTION.	4. REFRACTIONAL CORRECTION.	5. COR. FOR OBLIQUITY.
* Star's apparent altitude, 11° 17′ 0″		Const. log., 9.6990		Const. log., 1.5870
☽'s apparent altitude, . 9 38 0	Secant, . . 0.0612	. . . .	Sine, ... 9.2236	. . . .
Apparent distance, . . 43 36 0	. . . .	Sine, . . 9.8886	. . . .	Tangent, 9.9788
Sum, . . . 64° 31′ 0″				
½ Sum, . . 32 15 0	. . . .	Secant, . 0.0728	. . . .	. . . .
Remainder, . . 20 58 0	. . . .	Co-secant, 0.4462	. . . .	. . . .
Horizontal parallax, . . 54′ 42″	P. log., . . 0.5173	P. log., . 0.5173	. . . .	. . . .
Parallax in altitude, . .	53′ 55″ P. log. 0.5785			
	Dimin. of parallax,	48′ 1″ P. log. 0.5739	. . . .	P. log., . 0.5739
	Parallax in altitude,	53 55		
Diff. =	parallactic correction,	5′ 54″	P. log., . . . 1.4844	
Parallactic correction +	parallax in altitude,	59 49	. . . .	P. log., . 0.4784
			Sum, . 0.7080	
			Hor. Par. P. log. 0.5173	
Refractional distance for ☽		32° 48′	Tangent, . . 9.8093	
Apparent distance, ...		43 36		
Difference, ...		10° 48′		26″ P. log., 2.6181

For zenith distance, . 32° 48′ tab. refraction, = 37″.5 } Refractional corrections.
. . . . 10 48 . . . . = 11 .0 }

Apparent distance, . . .	43° 36′ 00″
Parallactic correction, . .	— 5 54
1st Refractional correction,	+ 0 37.5
2d Refractional correction, .	+ 0 11.0
Correction for obliquity, .	+ 0 26.0
True distance, . .	43° 31′ 20″.5

Having the true central distance between the moon and the sun or a fixed star, and the apparent time at the ship, to find the longitude.

RULE.

Among the true distances of the moon's centre from the sun or fixed stars, in the *Nautical Almanac*, find those two distances which are next less and greater than the true given distance, which true distance place under it. Take the difference between the true distance and the first of the two distances taken from the *Almanac*, and also the difference between the two distances. From the proportional logarithm of the first difference subtract the proportional logarithm of the second difference, the remainder will be the proportional logarithm of a portion of time, which, added to the time corresponding to the first of the two differences taken from the *Nautical Almanac*, will give the apparent time of observation at Greenwich.

Take the difference between this time and the apparent time of observation at the ship, which, being converted into degrees, will give the longitude of the ship: east, if the time at the ship be greater than at Greenwich, otherwise west.

EXAMPLE.

The true distance of the moon's centre from *Aldebaran*, the 20th March 1823, was 35° 50′ 24″, the corresponding apparent time at ship being 7^h 5^m 21^s. What was the ship's longitude?

True distance, . . .	35°	50′	24″				
Distance at 6 hours from Nautical Almanac,	35	44	33	0°	5′	51″	P. Log. 1·4881
Distance at 9 hours from Nautical Almanac,	37	26	15	1	41	42	P. Log. 0·2480
Time from midnight, . .	0^h	10^m	21^s	. . .			P. Log. 1·2401
	6	0	0				
Apparent time at Greenwich,	6^h	10^m	21^s				
Apparent time at ship, . .	7	5	21				
Longitude east in time, .	0^h	55^m	0^s	= 13°	45′	0″	

THE METHOD OF OBTAINING MECHANICALLY THE EXACT MEASURE OF A FUNDAMENTAL BASE LINE.

Before we can undertake to delineate a coast, or make a correct trigonometrical survey of a harbour, it is absolutely necessary that the precise distance between two objects or stations should be determined, whence a series of triangles (the angles of which must be stations conspicuously visible from each other) may be carried along the coast, and through that part of the country which is to be surveyed. The first step, therefore, to obtain mechanically the exact measure of a *base*, or one side of a triangle by which the others may be computed, is a task involving the greatest difficulty, and has given rise to the most delicate experiments which ingenuity can devise or expense command; and as a proof of how far art and indefatigable zeal will at all times enable us to surmount the difficulties presented by nature, we have only to view—which we cannot do without inexpressible delight and wonder—the late operations in Ireland, where a *base line* was measured parallel to the shore of Loch Foyle, between seven and eight miles long, and its greatest error is supposed not to have exceeded two inches. But we cannot look for so great a degree of nicety in the operations of the marine surveyor, with his limited means on a rough and rugged coast; yet as the just determination of the length of the other sides of the triangles, formed by straight lines joining the summits of hills and other remarkable objects, mainly depends upon the correctness and true measurement of a *base* line, it will be necessary to bestow on it the greatest care and labour. It is a well-known fact that all bodies, in a certain degree, expand by heat, and contract by cold; therefore, numerous expedients have been devised to avoid errors from this cause, in the substances

used in measuring the *base*, such as deal rods, glass rods, and steel chains of a peculiar construction. These were the methods adopted in the trigonometrical survey of Britain, also in that which has been going on in Ireland; it has been the work of many years, and great has been the labour and ingenuity exerted in the operation.

The measuring wheel is an instrument much used in measuring roads and distances, where expedition is more an object of consideration than correctness; indeed, the instrument cannot be relied on where great exactness is required, as the measure obtained by it must be too great where the ground is at all uneven, by traversing the hills and declivities of the road. The same objection may, in a certain degree, be used against the measuring chain, for it is liable to errors: First, by expanding or contracting; secondly, if care be not taken, the links will ride; and, thirdly, the great difficulty of laying it straight, all of which must be carefully observed and guarded against, by an examination of the links, and by propping up and removing as much as possible the different curves formed in it by the inequalities of the ground, when, for most nautical purposes, it will be found sufficiently correct. Deal rods may not be so convenient for carrying, but that material has been found to be less liable to change from the effect of the atmospheric action than most other bodies, and being so easily obtained, it will be found a useful article as a check on the chain; and by the assistance of a spirit level, you may approximate to the measurement of a line in a horizontal plane. Poles of 20 feet or upwards will be found a good size. The most important consideration is to select the two stations that are to form the extremes of the *base*. It must be fixed, if possible, on a perfectly smooth plane—the greater the extent the better; and if a *remarkable object* can be conveniently taken as one extreme, it would answer well to point out the spot in case it should be found necessary to go over the operations again. Let the direction of the base be such as to form triangles free, if possible, from any material inequality in their angles; for instance, such a triangle as ABC (fig. 47) would be altogether an improper triangle, for it would, in consequence of the acuteness of the angles at A and B, be impossible to determine with precision the position of C from observations made at those extremes, as a small error in either of the acute angles would produce a great error in the sides CB and AC. If, therefore, this precaution be attended to, and the position of the base line be such that angles taken between objects and its extremes,

such as ABK, BAK (fig. 47), may not be too acute or too obtuse, not less than ABK = 35°, or above BAK = 110°, but to form in the commencement of the survey, as nearly as possible, equiangular and equilateral triangles, similar to AGB (fig. 47), it will then be found that the accuracy and determination of the calculated sides will not differ perceptibly from what they would have been had it been possible to obtain their length by actual measurement. When this arrangement has been made, and several of the most prominent and remarkable objects have been thus fixed by trigonometrical calculations, as we recede from the base along the coast, it will be practicable to use as *bases* the sides of large triangles, formed by those objects which have already been determined. The angles measured between any three of them at the same instant, by two observers, or one only, if expert, with a sextant from a ship (either at anchor or under weigh), will give the position of the ship, or a rock, at the time the angles were taken. The method of protracting and calculating the sides of such triangles, is fully demonstrated in the surveying problems. If the observations are to be made from a rock or on shore, the theodolite is the fittest instrument, as will be seen by the description of it given in page 39, because it gives the angles at once reduced to the plane of the horizon.

HOW TO MEASURE A BASE LINE ON SHORE.

Suppose it were determined to measure a base line on the island ABCD (fig. 48).—Having prepared the measuring pole, &c. before described, or a steel chain of suitable length, a number of poles 8 or 10 feet long, with white flags on them, which is more easily distinguished at a distance than any other colour, and a few coils of deep sea lead line to stretch along the ground for a straight line to place the poles by; then supposing A to be the point from whence you commence the measurement, send a man on to P, which call, at a rough estimation, 800 yards from A, let him fix a long pole in the ground, with a flag on it, in such a position that it may be in a line with a tree or any conspicuous object at the farther end of the plane, in the direction you intend your *base* to run. You must guide him, in fixing the pole, by standing at a distance from A; then, with one eye closed, look when the pole at P is in a direct line with the pole at A and the tree, and then, by a preconcerted signal, desire him to fix it firmly in the ground; then between

A and P set up other poles, with little flags on them, as at *a*, *b*, *c*, about 200 yards apart, so that when the eye is placed as before, a little from A, the edges of the poles may coincide exactly with each other. In the next place, stretch along the lead line, and if it touch the sides of each of the poles, when *hauled taut*, you may be sure that they are in a direct line with each other. The accuracy of the operation, in a great measure, depends upon fixing, with great nicety, the poles upright, which can only be done by standing back a short distance from A, so that the poles may cover each other; then the other poles, if not seen either to the left or right, must, as is evident, be in the line required. Thus continue the line, by erecting another long pole 800 yards from P, as at S; then proceed in the same manner as before, until you have reached the other extreme of your intended *line*, near the tree at C.

Having now obtained the true direction of your *base*, next proceed to actual measurement, previously examining your chain by a well-divided rod. As a two-foot scale would be *too* short, and involve an error by repeated changing, a rod of six or eight feet long would do better. Then run the chain along by the side of the lead line, and carefully attend to the adjustment of the links; go over the same again with the long measuring pole, noticing most particularly the number of times the pole is carried forward; two poles would be preferable to one, and a wooden picket might be stuck in the ground at the end of each pole, as a mark to count by, before it be shifted. It would be satisfactory to go over the work a second or even a third time, before you commence taking your angles. If there should be a sensible declivity in the plane ABCD, it will be necessary to find its amount by the spirit level or by the theodolite, and then calculate for the horizontal distance instead of the hypothenusal line measured. If some obstruction should intervene so as to prevent the measurement of a long base line, such as a hill at H, directly interposed between the two points A and B (fig. 49), in which direction it might be required to carry out the *base;* in such a case let two or more lines, if necessary, be measured, such as AC, CB, and the angles they make with each other be carefully observed with a theodolite; then having the two sides, AC and CB, and the included angle, BCA, find trigonometrically the side BA; the process would be exactly the same if there were more sides than two, but poles ought to be set up at the angular points, and the theodolite cannot be set with too much precision over the point, where

the sides meet, as a trifling error in these angles would materially affect the distance deduced therefrom.

Various methods are employed for measuring a base line at sea. Every seaman is familiar with heaving the log-line to find the rate of the ship. But this, though sufficient for ordinary purposes, is by no means accurate enough for the surveyor. A better method is to use a strong cord, thoroughly wet, whose length must be carefully measured, with cork floats attached to it to keep it on the surface of the water. One end of this cord is made fast by a heavy piece of lead, which acts as an anchor, while the other end is carried out by a boat in the direction of the base line. When the cord is stretched to its full length, another anchor is dropped from the end of the boat to keep this end of the cord in its place. Another boat then takes in the end of the cord which was first anchored, and carries it forward in the same manner till it is again stretched to its full length. In this way the whole of the base line is measured. The same precautions must be used here as in measuring a base on shore to keep the boats exactly in the proper direction; and it will be necessary, immediately after the process is completed, to measure the cord again with the greatest care, lest it may either have stretched or contracted during the operation. This method can only be practised with success under favourable circumstances, as when the sea is calm and smooth, without any current or tide to prevent the cord from lying in a straight direction.

But the method most frequently employed on board surveying ships, is by the transmission of sound, which, though far from being very accurate, is found in general to answer the purpose, and is attended with least trouble and difficulty. On account of its utility, and being so generally adopted, we shall enter somewhat fully into the detail of this method.

ON THE VELOCITY OF SOUND IN AIR AND WATER.

(From the Encycolpædia Metropolitana, by Sir John Herschel.)

Sound is not *instantaneously* conveyed from the sounding body to the ear; it requires time for its propagation. This is a matter of the most ordinary observation. We hear the blows of a hammer at a distance a very sensible interval of

time after we see them struck. The report of a gun is always heard later than the flash is seen, and the interval is longer the more distant the gun. We estimate the distance of a thunder storm by the length of the interval between the lightning and the thunder-clap, which often arrives when we have ceased to expect it. The report of the meteor of 1783 was heard at Windsor Castle, ten minutes after its disappearance. This is probably the longest interval yet observed.

A great multitude of experiments have been made to determine the precise velocity of sound. The earlier results differ more than might have been expected, from the influence of several causes not immediately obvious, but chiefly from want of due attention to the influence of the wind. It is evident from the mechanical concussion attending loud noises, that sound consists in a motion of the air itself, communicated along it by virtue of its elasticity, as a tremor runs along a stretched rope. If, then, the whole body of the air were moving in a contrary direction with the velocity of sound, it would never make its way against the stream at all; and, on the other hand, when the wind blows from the sounding body direct towards the ear, it is equally clear that the velocity of the wind itself will be added to that of sound in still air. If a stone be thrown into a still lake, the waves spread with equal rapidity in all directions, in circles whose centre is the stone. If into a running river, they still form circles, but their centre is carried down the stream; and, in point of fact, the wave arrives opposite to a point of the bank *above* the place where the stone fell, later than a point at the same distance below it, in proportion to the rapidity of the stream. Hence all experiments on the velocity of sound ought to be made, if possible, either in calm weather, or in a direction at right angles to that of the wind.

The assumption of 1300 feet per second, for the velocity of sound, by Roberts (*Phil. Trans.* 1694), and the inaccurate determinations of Mersenne, Bayle, and Walker, (*Phil. Trans.* 1698), which give respectively 1474, 1200, and 1305 feet (the latter by a mean of twelve experiments, disagreeing no less than 370 feet, *inter se*), scarcely deserve more mention than the rude guesses of Roberval and Gassendi, which differ by an amount nearly equal to the whole quantity to be measured; the former fixing it at 560 feet, the latter at 1473. The first experiments which appear to have been made with any degree of care, were those instituted by the Florentine Academy, *Del cimento*. It was observed in these, that in a

distance equal to 5739 English feet, the sound of a harquebuss arrived five seconds after the flash; and repeating the experiment at half the distance, they found exactly half the time to be required. This gives for the velocity of sound 1148 feet per second.

Cassini, the elder, Picard and Roëmer, from experiments made at a distance of 1280 toises, as related by Duhamel in the *Hist. de l'Acad. Par.*, assign 1172; while Flamsteed and Halley, from a series of observations at the Royal Observatory, the origin of the sound being three miles distant, concluded the velocity to be 1142 feet per second.

In a paper communicated to the Royal Society in 1708, by Dr Derham, the subject of the velocity of sound is investigated more fully and distinctly than had before been done, and with some degree of attention to a variety of circumstances which appear likely to influence its propagation. These are chiefly—

1. The direction and velocity of the wind.
2. The amount of barometric pressure.
3. The temperature of the air through which the sound is conveyed.
4. Its hygrometrical state of moisture and dryness.
5. The actual weather, whether fog, rain, snow, sunshine, &c.
6. The nature of the sound itself, whether produced by a blow, a gunshot, the voice, a musical instrument, its pitch, quality, and intensity.
7. The original direction impressed on the sound by turning, for instance, the muzzle of a gun one way or the other.
8. The nature and position of the surface over which the sound is conveyed, whether smooth or rough, horizontal or sloping, moist or dry, &c.

To all these circumstances, except the wind, Derham attributes no effect; and in fact none of them, except the temperature of the air, have been ascertained to exercise any material influence on the velocity; though many, indeed all, have a very powerful one on its intensity, or the loudness of the sound, as it reaches the ear from a given distance. The quantity of aqueous vapour, indeed, ought (as we shall see) to affect the velocity, but in a degree only appreciable in the most delicate experiments. Derham concludes, from the whole of his observations, that sound is propagated at the rate of 1142 feet per second, agreeing with the result of Flamsteed and Halley, and with that of the Florentine academicians; and as

the distances of the stations employed were considerable, in one case amounting to upwards of twelve miles, this determination appears deserving of some reliance. The temperature, unfortunately, was not registered; so that the experiment loses much of its value from the impossibility of applying with certainty the requisite correction.

In 1737-1738, the Academy of Paris directed a reinvestigation of the subject; and Messrs Cassini, de Thury, Maraldi, and La Caille, who were at that time engaged in the triangulation of France, were charged with the conduct of the experiments; an account of which, by Cassini, is to be found in the volumes of the *Historie de l' Acad.* for the latter year, and for 1739. Their observations were carefully made, and the distance of the stations was considerable (from 2931 to 16079 toises). In these experiments we find the first example of observations so disposed as to eliminate in some measure the disturbing effect of the wind. To apprehend how this may be done, let us suppose a current of wind to blow *uniformly* with any velocity from one station, A, to another, B, at any distance, and at these two stations let shots be fired. The sound of the shot fired at A will then be accelerated, and that of the signal at B will be retarded, in traversing the interval, by equal quantities; and consequently (since the velocity of sound is very much greater than that of the most violent wind), the time in which the sound runs over the line AB will be diminished, and that in which it traverses B A increased, by equal quantities, so that the mean will be unaffected by the wind's velocity.

It is evident, however, that any want of uniformity in the rate of the wind will destroy, so far as it goes, the precision of the result so obtained; and that, in consequence, if the corresponding signals are not *strictly simultaneous*, so as to make the sound traverse the same identical portion of the aerial current, a great part of the advantage of this mode of experimenting is lost. M. Arago has indeed remarked, that even in that case, if the wind be very irregular, and in sudden gusts, it will still affect the result. To conceive which we will suppose a gust of wind to arrive suddenly at the station A, at the moment of firing the signals both at A and B. The sound which proceeds in the direction AB, as it runs quicker than the wind, will leave it behind, and be propagated at every point of AB in still air, before the agitation of the wind has had time to reach it. On the other hand, the sound from B will meet the wind; and during the latter part of its course at least, will be

propagated in a moving atmosphere. Still, it will be observed, that it can be only the *latter part* of its course which can be thus affected, less, at all events, than one-tenth of the whole space; and the effect during that tenth being to retard the sound by one-tenth, at most of *that interval*, will produce a total effect, not exceeding a hundredth of the whole time of traversing AB; and consequently, will affect the mean of the two deduced velocities, by a quantity not exceeding a two-hundredth part of its value, or about five feet per second.

One very material difficulty in the way of former observers (Benzenberg excepted) lay in the want of adequate means of measuring with precision intervals of time to a minute fraction of a second. This difficulty was obviated in the experiments of the French commissioners, by the use of the stop-watch of Breguet, and the *chronograph* of Rieussec—a species of watch, one of whose hands performs a revolution per second, and can be made to touch with its extremity the dial-plate at any instant, and leave there a dot, without interrupting its motion of rotation, by the sudden pressure of a small lever; to effect which it carries with it a drop of printer's ink in a peculiar and ingenious species of dotting pen. In the Dutch experiments, a clock with a conical pendulum was used, capable of determining intervals to the hundredth of a second, by suddenly suspending the motion of the index, without stopping the clock. By the use of these instruments, it was found practicable to ascertain the interval between the sight of the flash and the arrival of the report of a gun with such precision as to destroy all material error in the result which might arise from this cause; an improvement of great importance, when we consider that an error of a single tenth of a second in the measure of time is equivalent to 110 feet in that of distance.

The close agreement of the results of these experiments is a convincing proof of their accuracy. The French philosophers state 331·05 met. = 1086·1 feet, as the velocity of propagation of sound in air of the temperature of freezing water; while the Dutch experimenters make it 332·05 met. = 1089·42 feet in perfectly dry air of the same temperature. The latter seems to deserve the preference, if only from the circumstance of the signals from which it is deduced having been strictly simultaneous, the guns at the two extremities of the line (9 miles in length) having been fired at the same second of time, while in the former series this exact coincidence was not obtained.

We subjoin a list of the results arrived at in the various determinations above enumerated, with their dates, the distances

[illegible] &c. to bring the whole subject under [illegible].

The [illegible] between such of the results in the following [illegible] to the standard or freezing temperature, [illegible] and the first determination of Cassini at [illegible] their extreme discrepancy being less than [illegible] of the whole amount, and their mean [illegible] precisely with the result of Moll, [illegible] we may therefore adopt 1090 feet without [illegible] as no doubt within a yard of the [illegible] within a foot. The reduction to the [illegible] has been made (when not performed by the [illegible] on the supposition that every additional [illegible] temperature, on Fahrenheit's [illegible] to the velocity.

[illegible] in round numbers, that sound, [illegible] temperature, travels at the rate [illegible] per second; and that at 62° Fahrenheit [illegible] temperature of the British [illegible] feet in eight seconds, $12\frac{3}{4}$ [illegible] miles in a minute, or 765 miles in an hour, [illegible] of the diurnal velocity of the [illegible]

[illegible] if a gun be fired at the [illegible] the meridian of any station, the sound [illegible] west of it at the precise [illegible] on its meridian.

[illegible] of pressure, its effect is to increase [illegible] but since at the same time it [illegible] the same ratio, the mass [illegible] are increased alike, and [illegible] unaltered. The velocity [illegible] no change by this alteration. [illegible] temperature, under a constant [illegible] the air, and either renders it [illegible] more rare with the same [illegible] of temperature, the moving [illegible] decreases; and [illegible] the resulting motions must [illegible] The velocity of sound, then, ought to be greater in [illegible]

A TABLE CONTAINING THE VELOCITY OF SO

OBSERVERS' NAMES.	Date of determination. A.D.	Distance of stations in feet.	Velocity in Eng. ft. per sec.	
Mersenne,	...	...	1474	
Florentine Academicians,	1660	5906	1148	Mo
Roberval,	...	...	560	
Gassendi,	...	...	1473	
Boyle,	...	...	1200	*Ess*
Roberts,	1694	...	1300	No
Walker,	1698	variable 600 to 2370	1305	By
Cassini, Picard, Roëmer,	...	8186	1172	Dul
Cassini, Huygens,	...	9239	1151	Mol
Flamsteed, Halley,	...	15840	1142	As
Derham,	1704	5280 to 63360	1142	
	1738	18744 to 102824	1106	Nea
			1093	Do.
Cassini, Lacaille,	1739	144124	1110	Bet
Bianconi,	1740	78740	1043	
La Condamine,	1740	67400	1112	At
La Condamine,	1744	129360	1175	At
Millington,	...	...	1130	Cit
J. F. Mayer,	1778	3412	1105	
G. E. Muller,	1791	8330	1109	
Pictet,	...	...	1130	Cit
Espinosa and Bauza,	1794	53626 to 14071	1222·23	At
Benzenberg (Dusseldorff),	1809	29764	1093	At
Arago, Matthieu, Prony, Bounaid, Humbolt,	1822	61064	1086·1	At
Moll, Vanbeek, Kuytenbrouner,	1823	57839	1089·42	In
Gregory,	1823	various 2700 to 13460	1088·5	M
Myrbach,	1822	32615	1092·1	M
Goldingham (Madras),	1821	29547	1089·9	H
		13932	1079·9	H
		Mean	1086·7	

EXPERIMENTS IN WATER TO DETERMINE THE VELOCITY OF SOUND.

A series of experiments on the velocity of sound in *sea water* was instituted by M. Beudant, at Marseilles, by striking a bell at a distance under water. The mean result gave 4921 feet per second for the velocity.

A more careful and more exact determination was undertaken and executed in 1826, by M. Colladon, in the lake of Geneva. After trying various means for the production of the sound, as the explosion of gunpowder, blows on anvils, and bells, the latter were preferred, as giving the most instantaneous, and, at the same time, most intense sound, the blow being struck about a yard below the surface by means of a metallic lever. The experiments were all made at night, to avoid the interference of extraneous sounds, and for the better observing of the signals made at each blow by the flash of gunpowder.

The result of his experiments (the water being at the temperature of 46° 6′ Fahrenheit) gave the velocity of sound in the water of the lake of Geneva, 1435 metres = 4708 feet per second.

Having gone through the account of the experiments for the determination of the velocity of sound, the following examples will, it is presumed, clearly point out how the distance between two places is to be ascertained, by noting the interval of time between the flash and report of a gun.

April 4, 1827.—In latitude 13° 6′ N., light airs and cloudy weather, a gun was fired on board the surveying ship at anchor, whilst the time was noted by two chronometers on board the assistant vessel, also at anchor. The mean of several experiments gave the elapsed time (carefully noted by three observers) between the flash and report = 27¼ seconds. Then supposing the velocity of sound to be 1142 feet per second, the work will stand thus:—

```
    1142
    27.25
   ------
    5710
   2284
  7994
 2284
 --------
= 31119.50 feet, distance of the vessels
           from each other.
```

But if 1090 feet be taken as the velocity of sound, then

```
      1090
     27.25
    ------
      5450
     2180
    7630
   2180
   ----------
   29702.50 feet.
```

It will be seen that a difference of 52 feet in the velocity of sound produces no more than an error equal to the elapsed time multiplied by that difference = 1417 feet.

Example II.

April 5, 1827.—In latitude 12° 55′ N., light breeze, thermometer being at 80° Fahrenheit, the vessels again anchored to measure a base line, the elapsed time between the flash and report was 26¼ seconds.

At 32° Fahrenheit the velocity of sound is 1090 feet per second; therefore, allowing 1·14 feet for each degree of temperature above the freezing point, the velocity of sound at 80° will be found thus:—

```
                 80°
                 32°
                 ---
                 48°
    Multiply by 1.14
                ----
                 192
                 48
                48
                -----
                54.72

    Then to     1090
    Add           54.72
                -------
Velocity of sound = 1144.72 at temperature of 80° F.
Elapsed time,   .     26.25
                   --------
                    572360
                   228944
                  686832
                 228944
                 ------------
                 30048.9000 distance between the vessels.
```

It may be remarked, that the accuracy of the result depends much more upon the *time* which elapses between observing the flash and hearing the report, than upon the number assumed as the velocity of sound; for an error in time, if even so small a quantity as the tenth of a second, must produce an error in distance of not less than 109 feet.

To illustrate this, take the preceding example, and suppose that $\frac{1}{4}$ of a second is lost in noting the interval, then

1144·72	
26	elapsed time
686832	
228944	
29762·72	feet distance.

Hence the error upon the whole amounts to 286·18 feet.

From these remarks, it is obvious that the chances of error in measuring a base line by this method are very great; and, indeed, until some means be devised for taking the time more accurately than any we now possess, this method can at best be considered but an approximation to the truth.

We have already alluded to a method of finding the latitude and longitude of places on the earth's surface by trigonometrical calculation, when the latitude and longitude of some other place not very remote is known, and we shall now show how this is to be done.

Let ABC (fig. 50) be a triangle, of which the side BC = 6083·3 feet, the angle ABC = 60° 15′, ACB = 49° 52′, and consequently BAC = 69° 53′; and let the longitude of B be 24° 15′ 30″ west, and its latitude 56° 58′ north: also let the azimuth of B from the meridian, or the angle BAS, be 44° 12′. It is required to find the latitude and longitude of A?

In the triangle ABC, of which the side BC = 6083·3 and all the angles are known, find the side AB.

As sine BAC 69° 53′	9·9726629
Is to sine ACB 49° 52′ . . .	9·8834039
So is BC 6083·3	3·7841392
	13·6675431
To AB 4953·14	3·6948802

Through the point A draw the meridian line NS, and from the point B draw BD perpendicular to NS, meeting it in D; then

AD is the difference of latitude, and BD the difference of longitude of A and B.

In the right-angled triangle ABD, of which the side AB = 4953·14, and the angle BAD = 44° 12′ are given, find the sides BD and AD.

To find BD.		To find AD.	
As radius . . .	10·0000000	As radius . . .	10·0000000
Is to sine A 44° 12′	9·8433356	Is to cos. A 44° 12′	9·8554650
So is AB 4953·14 .	3·6948802	So is AB 4953·14 .	3·6948802
	13·5382158		13·5503452
To BD 3453·15 .	3·5382158	To AD 3550·95 . .	3·5503452

To reduce the difference of longitude BD = 3453·15 thus found to degrees, it will be necessary to find the length of a degree of longitude in the given latitude 56° 58′, which may be done by the following proportion:—As radius is to the length of a degree upon the equator, so is the cosine of the given latitude to the length of a degree in that latitude. Having in this way found the length of a degree of longitude, reduce it to feet, and by that number divide 3453·15, which will give the difference of longitude in degrees or parts of a degree.

For reducing the difference of latitude AD = 3550·95 to degrees, it will in general be sufficient to consider all the degrees of latitude as equal, and to divide the given difference of latitude by 69·15 miles = 365110 feet. But as the earth is not a perfect sphere, it may be necessary, for the sake of greater accuracy, to make a correction for the compression at the poles, which is generally reckoned = $\frac{1}{300}$.

To facilitate these processes, the following table, computed from a formula of Mr Baily, which shews the length of a second of longitude and latitude in English feet at the earth's surface, will be found very convenient. In this table the equatorial diameter of the earth is assumed = 7924 miles; a degree of longitude, therefore, at the equator will be equal to 69·15 miles = 365110 feet, and hence a second of time at the equator =1521·3 feet.

Lat.	Second of Longitude.	Second of Latitude.	Lat.	Second of Longitude.	Second of Latitude.
°	FEET.	FEET.	°	FEET.	FEET.
0	101.42	101.42	35	83.17	101.75
1	101.40		36	82.15	
2	101.36		37	81.10	
3	101.28		38	80.02	
4	101.17		39	78.92	
5	101.03	101.43	40	77.80	101.84
6	100.87		41	76.65	
7	100.67		42	75.48	
8	100.44		43	74.29	
9	100.18		44	73.07	
10	99.89	101.45	45	71.83	101.93
11	99.57		46	70.57	
12	99.22		47	69.29	
13	98.84		48	67.99	
14	98.43		49	66.66	
15	97.99	101.49	50	65.32	102.02
16	97.52		51	63.95	
17	97.02		52	62.57	
18	96.49		53	61.17	
19	95.44		54	59.75	
20	95.36	101.54	55	58.30	102.11
21	94.74		56	56.84	
22	94.09		57	55.37	
23	93.41		58	53.87	
24	92.70		59	52.36	
25	91.97	101.60	60	50.84	102.19
26	91.21		61	49.30	
27	90.43		62	47.74	
28	89.62		63	46.17	
29	88.77		64	44.58	
30	87.90	101.67	65	42.98	102.26
31	87.01		66	41.37	
32	86.09		67	39.74	
33	85.14		68	38.10	
34	84.17		69	36.45	

To apply this table to the preceding example.—Enter the table with the latitude of B, 56° 58′, and take out the length of a second both of latitude and longitude, and by these divide the difference of latitude and longitude already found, viz. BD = 3453·15, AD = 3550·95, and the quotients will be the difference of longitude and latitude required. Thus :—

$$\frac{\text{BD} = 3453.15}{\text{A sec. of long. from tab. } 55.42} = 62''.309 = 1'\ 2''.309 \text{ diff. of longitude.}$$

$$\frac{\text{AD} = 3550.95}{\text{A sec. of lat. from tab. } 102.11} = 34''.775 \text{ difference of latitude.}$$

Hence

Long. of B =	24° 15′ 30″	West.
Difference,	1 2.309	East.
Long. of A =	24° 14′ 27″.691	West.
Lat. of B =	56° 58′ 00″	North.
Difference,	34.775	North.
Lat. of A	56° 58′ 34.775	North.

It will now be understood how the measurement of a base line ought to be conducted, either on shore or on the water; but it is no less important that the direction of the base with regard to the true meridian should be laid down. To accomplish this in a scientific manner would require the aid of a transit, or azimuth and altitude instrument, as the nice determination of a meridian line is the first consideration on erecting an observatory; but for nautical purposes, so great a degree of refinement need scarcely be observed. It is the general practice, when on survey, to take a set of angles between every object right round the compass, every day at sun rise and sun set; commencing, in the first place, with the angle between the sun's centre and a well-defined object on land, with a sextant, when the sun's semi-diameter is on the horizon, which must be carefully guessed at, making due allowance for the great refraction at that time. When his lower limb is just touching the horizon will be the proper time to take the angle, because his semi-diameter is then, in reality, on the horizon, being elevated so much above it by the effect of refraction. His amplitude being found by methods which will be hereafter given, the true bearing of the fixed object is thus obtained; consequently, all the other objects will, in like manner, hold a relative position with regard to the true north; and in this way may the angle between the extremity of the base and the meridian be found. We may also apply the sun's azimuth to determine the direction of the base, by measuring the angular distance between the sun and one of its extremes; but when a sextant is employed for that purpose, the altitude of the sun ought to be small. The horizon must be unobstructed by land, when a true bearing, or the sun's amplitude, is sought.

VARIATION OF THE COMPASS.

To find the variation of the compass by observed magnetic amplitude of the sun, or of a star.

THE solution of this problem depends upon finding the hypothenuse of a right-angled spherical triangle, in which one side (the declination of the sun or star) and an angle (the co-latitude) are given.

RULE.

From the logarithmic sine of the declination (the index being increased by 10) subtract the logarithmic cosine of the altitude, the remainder is the logarithmic sine of the true amplitude, to be reckoned north or south, according to the name of the star's declination; then, if the true and observed amplitudes be both north, or both south, their difference is the variation; but if one is north, and the other south, their sum is the variation. And to know if it be east or west, let the observer look directly towards the compass representing the true amplitude; then, if the observed amplitude is to the left of the true, the variation is easterly, but if to the right, it is westerly.

Remark.—The magnetic bearing of the object must be taken when its apparent altitude is equal to the depression of the horizon.

EXAMPLES.

The latitude being 18° 40′ *north, the star Sirius was observed to set* 24° 10′ *to the southward of the west point of the compass. Required the variation?*

Declination of Sirius,	16° 27′ S.	sine	19.45206
Latitude, . . .	18° 40′	co-sine	9.97653
True amplitude, W.,	17° 23′ S.	sine	9.47553
Mag. amplitude, W.,	24° 10′ S.		
Variation, . . .	6° 47′		

which is east, because the observed amplitude is to the left hand of the true.

Example II.

In latitude 48° 20′ *north, the star Rigel was observed to set* 9° 50′ *to the northward of the west point of the compass. Required the variation?*

Declination of Rigel, .	8° 25′ S.	sine	19.16545
Latitude, ...	48° 20′	co-sine	9.82269
Star's true amplitude W.,	12° 43′ S.	sine	9.34276
Magnetic amplitude, W.,	9° 50′ N.		
Variation, ...	22° 33′		

which is west, because the observed amplitude is to the right hand of the true amplitude.

Example III.

Required the sun's true amplitude in latitude 21.12 *N., when his corrected declination is* 16° 2′ *S.?*

Sun's declination, . .	16° 2′ S	sine	19.44122
Latitude, ...	21° 12′ N	co-sine	9.96957
True amplitude,	W. 17° 14′ S.	sine	9.47165

To find the variation of the compass, having the true altitude of the sun, or of a fixed star, and its magnetic azimuth, the latitude of the ship and the declination being known.

To compute the true azimuth, we have given the three sides of a spherical triangle, viz. the co-latitude, the zenith distance, and the polar distance, to find the angle at the zenith between the zenith distance and the co-latitude.

Rule.

Add together the polar distance, the zenith distance, and the co-latitude, and take half the sum; the difference between which and the polar distance call the remainder. Subtract the sum of the log. sines of the zenith distance and co-latitude from the sum of the log. sines of the half sum and remainder (increasing their index by 20), and half of this result will be the co-sine of half the true azimuth; to be reckoned from the north in north latitude, but from the south in south latitude.

Then, if the true and observed azimuths are on the same side of the meridian, their difference is the variation, but if on different sides, their sum is the variation. Now, if the observed azimuth is to the left of the true, the variation will be easterly, but if to the right, the variation will be westerly.

EXAMPLE.

In latitude 48° 50′ *north, the true altitude of the sun's centre was* 22° 2′*; the declination being* 10° 12′ *south, and its magnetic bearing N.* 161° 32′ *E. Required the variation?*

Polar distance,	100° 12′		
Zenith distance,	67 58	Sine	9.9670637
Co-latitude, .	41 10	Sine	9.8183919
Sum, . .	209° 20′		19.7854556
½ Sum, .	104 40	Sine	9.9856129
Remainder,	4 28	Sine	8.8914209
			38.8770338
			2)19.0915782
69° 25′ 40″ . .			9.5457891 Co-sine ½ sun's true azimuth.
2			

N. 138° 51′ 20″ E., sun's true azimuth.
N. 161 32 E., observed magnetic azimuth.

Variation, 22° 40′ 40″ West, because the observed azimuth is to the right of the true azimuth.

The arithmetical complements of the sines of the zenith distance and co-latitude might have been used instead of the log. sines, in which case half the sum of the four log. sines would have given the co-sine of half the sun's true azimuth. The arithmetical complement of a logarithm is found by subtracting each figure of the logarithm from 9, except the last figure, which is to be subtracted from 10, beginning at the left hand figure. Thus:—

9.99999910
9.8183919

0.1816081

METHOD II.

Add together the polar distance, the zenith distance, and the co-latitude, and call their sum A. To the log. sine of half A *minus* the zenith distance, add the log. sine of half A *minus* the co-latitude, and their sum (the index being increased by 20) call B.

To the log. sine of half A add the log. sine of half A *minus* the polar distance, and call the sum of these two logs. D.

From B subtract D, and divide the remainder by 2, the quotient will be the log. tangent of half the object's true azimuth, as before.

Take last Example.

Polar distance, . . .	100° 12′		
Zenith distance, . .	67 58		
Co-latitude, ...	41 10		
	2)209° 20′	. . . .	A
$\frac{1}{2}$ A =	104° 40′		
$\frac{1}{2}$ A *minus* the zen. dist., =	36 42	Sine 9·7764289	
$\frac{1}{2}$ A *minus* the co-latitude,	63 30	Sine 9·9517912	
		39·7282201 . .	B
$\frac{1}{2}$ A =	104° 40′	Sine 9·9856129	
$\frac{1}{2}$ A *minus* the polar dist.,	4 28	Sine 8·8914209	
		18·8770338 . .	D
		2)20·8511863 =	B — D
69° 25′ 40″ 2	. . . .	10·4255931 =	tan. $\frac{1}{2}$ sun's true az.
138° 51′ 20″ = sun's azimuth, as before.			

To find the meridian line, or that in which the plane of the meridian intersects the horizon of any place.

When a star is on the east side of the meridian, and at any altitude, bisect the star with the middle wire of the telescope, and clamping it tight, observe what point or object on the horizon is in the same *vertical* circle, or has a corresponding azimuth with it.

Then turning the instrument steadily round, without putting the telescope in the slightest degree out of the vertical, cover the instrument up, and set a person to guard it, until the star shall have arrived at the same altitude again on the west side of the meridian. Previous to its arrival at the central wire as before, carefully examine the level of the instrument; and, after observation, mark any other point on this side of the meridian which may appear in the same vertical with the star; then a line bisecting the arch subtended by these two points will be the meridian line. But to ensure greater ac-

curacy, and to prevent the chance of losing the evening observation, when one altitude only is observed, which may happen by the sudden formation of a cloud over the star at the moment he is about to enter the field of the telescope, it is advisable to note several altitudes on both sides of the meridian, and arrange them in corresponding pairs, beginning with the two smallest altitudes, which will be the first in the morning opposite to the last in the evening, and so on to the two greatest. By this arrangement, should one altitude be lost in consequence of intervening clouds, or otherwise, there will still be several pairs left to complete the work. Another advantage attending this method is, that any errors arising from the imperfections of the instrument will probably counteract each other; and should it be found that the meridian deduced from any pair of equal altitudes does not correspond exactly with that deduced from every other pair, a mean of the whole may be taken, which will give the meridian very near the truth.

The application of equal azimuths to find the true meridian.

These must be opposed to each other in pairs just in the same manner as corresponding altitudes, the first in the morning to the last in the evening, and so on of the rest. Then deducting the one from the other, and applying half the difference between the two to the smallest number in each pair, it will give a number of degrees, minutes, and seconds, in which, if all the observations were perfect, the whole six pair would coincide; and if they do not, the fair mean deduced from among them will approach nearly to the truth.

To that mean point, deduced from these observations, the instrument must now be turned, and fixed there till the proper correction can be applied to it; and direct the telescope down to the horizon each way, and note what distinct object, either to the north or south, coincides with one of the perpendicular wires. If no such object be found, set up a mark each way, or either way, to which keep the instrument directed till the correction can be investigated which is requisite on account of the change of the sun's declination during the interval between the morning and evening observations, as any alteration in his declination will affect the azimuth deduced in this way as it does the hour. This correction is the greatest about the time of the equinoxes, on account of the rapid change of the sun's declination at that time. When a star's azimuth is observed, no such correction is requisite.

To find the correction for change of sun's declination between the observations.

RULE

To the log. of half the change of declination, add the log. secant of the latitude, and the log. co-secant of half the interval of time converted into degrees, &c., the sum *minus* 20, will be the log. of the correction in seconds of space.

When the sun is advancing towards the elevated pole, the middle point or meridian, as found by equal altitudes, will be too much to the west of the true meridian by the amount of this correction, and *vice versa* when he is receding from the elevated pole; therefore, the telescope being shifted in azimuth by the quantity thus computed, will be correctly placed in the meridian.

EXAMPLE.

On February 28, 1834, when the sun had equal altitudes, his azimuth was found to be 130° 10′ 15″, and 32° 36′ 15″; therefore the middle point, or reading of the approximate meridian, was 81° 23′ 15″. The interval of time between the observations was five hours, the half of which converted into arc = 37° 30′. The sun's hourly change of declination = 56″·77; therefore the change for half the interval = 141″·92, (approaching the north pole). The latitude of the place, = 51° 28′ 39″. Required the correction to be applied to the middle point to obtain the direction of the true meridian?

Half the change of declination, .	141″.92	Log.	2·1510436
Latitude of the place, ...	51° 28′ 39″	Sec.	0.1065913
Half interval of time between observations, ...	37° 30′	Co-sec.	0.2155529
	297″.29	Log.	2·4731868
	= 4′ 57″.29		
Reading of the middle point, ...			81° 23′ 15″
Correction, ...			— 4′ 57″·29
Reading of the instrument when set to the true meridian,			81·18·17·71

If dependance can be placed upon the time shown by a watch or chronometer, compute the time of the meridional passage of the pole star, or a star near the pole, either above or below it, then point the telescope of the instrument to the star, and bisect it at the exact moment, when, if the adjustments of the instrument are perfect, the telescope will be in

the plane of the meridian nearly; or we may mechanically fix a meridian line, by suspending two plummets at a distance apart, and one of them so contrived that its position may be easily changed from side to side; then (having previously computed the time of the pole star's passage over the meridian) proceed to the moveable plummet, and at the exact moment when the star is on the meridian, move the inner plumb line until the star and the two lines are made to coincide precisely in the same line with each other: the two plummets will then be in the meridian line; and if they were made to swing in water, it would materially check the vibrations which might be occasioned by the air.

To find the true meridian by a circumpolar star.

If the observer can depend with confidence on the time shown by chronometer, let him bisect a circumpolar star when at its greatest eastern or western elongation, then read off the azimuth, and note some remarkable object with respect to the star's azimuth, or erect a pole with a lantern on it at some distance. Having thus obtained its azimuth, the angle between it and other objects can be taken at leisure.

If the time cannot be relied upon, compute the azimuth of the star when at its greatest elongation in the following manner:—

From the log. sine of the polar distances, subtract the log. co-sine of the latitude; the remainder will be the log. sine of the azimuth required.

To compute the time when a circumpolar star will attain its greatest elongation, before or after its meridional passage, either east or west.

Add together the log. tangent of the polar distance and the log. tangent of the latitude, their sum, rejecting ten from the index, will be the log. co-sine of the hour angle (in space); which, divided by fifteen, will be the siderial time when a star attains its greatest elongation before or after it passes the meridian at its upper culmination; therefore, having the time of the meridional passage, the time of its greatest elongation will be known.

Having the time of the star's greatest elongation, follow it in its slow motion until it is stationary as before, and mark the azimuth, or if the star can be seen all night, a line bisecting the horizontal interval between its greatest eastern and western elongation, will be the direction of the true meridian. To make use

of the last methods, it is necessary that the polar distance of the stars should be less than the co-latitude of the place of observation to determine the direction of the meridian.

Having determined the direction of the true meridian by either of the foregoing methods, it remains to be shown in which manner the angle between the extreme of a *base line* and the meridian line may be most correctly laid down. To those who have had experience in protracting lines and angles, it will be well known that the side of a triangle can be more easily and correctly protracted than either of its angles, even admitting the aid of the most delicate and well-constructed protractor. It is from this cause, therefore, that a decided preference must always be given to the computed, instead of the protracted sides, in the triangulation of a survey; but as it is absolutely necessary to lay down the angle between the magnetic direction of the *base line* and the true meridian, the utmost exactness must be observed, and the following method will be found the most accurate and convenient:

Let AB (fig. 51) be the base line, NS the true meridian, and let the angle BAN be 50° 26′. From any diagonal scale, whose accuracy can be depended upon, take the radius AE = 10, and describe the arc EN. From the same scale take AC, exactly equal to the half of EN or 5, and describe the arc CD. Join the points AE and CD. It has already been demonstrated (Property i. page 5) that the chord of an arc is double the sine of half that arc; therefore EN = 2 sin. $\frac{1}{2}$ BAN to the radius AE, and CD = 2 sin. $\frac{1}{2}$ BAN to the radius AC. But since AC = $\frac{1}{2}$ AE, CD = $\frac{1}{2}$ EN (Euc. vi. 4), therefore CD = sin. $\frac{1}{2}$ BAN to the radius AE. If, then, AE be assumed equal to the radius of the tables, CD is the natural sine of half the azimuthal angle BAN. In the Table of Natural Sines, find the sine of $\frac{1}{2}$ BAN = 25° 13′, and take that distance from the same scale as AE; then, with that distance, from the centre C, intersect the arc CD in the point D, and through D draw ADN, which will be the meridian line. It will be observed that in practice the arc EN is not necessary, and that all that is requisite is to take AC = 5, with it to describe the arc CD, and, from the same scale as AC is measured, to make the line CD equal to the natural sine of half the given angle.

The above method of laying off arcs or angles by means of their chords, affords the means of dividing a circle into degrees and minutes with considerable precision. And as it may be convenient to the surveyor to be able to construct a scale for himself, instead of using a protractor, which, at best, is not a

very accurate method, we will point out the manner in which this is to be done.

Describe a circle with a radius equal to 5, that is, equal to half the radius of the tables, which is assumed equal to 10; then, since (Property ii. page 5) the chord of 60° is equal to the radius of the circle, with the radius 5 step round the circle, making a slight mark at each step; this will divide the circle into six equal parts, each containing 60°.

Next, from the Table of Natural Sines take the sine of 15°, which, to radius 5, as was shown above, will be equal to the chord of 30°. With this distance step round the circle, beginning at the same point as before, and this will bisect each 60°, or divide the circle into arcs of 30° each.

In the same manner, by taking the natural sines of 10°, 7° 30′, 3° 45′, 3° 20′ and 3°, the circle may be divided into arcs of 10°, 5°, 1°, 30′, 10′, which last division is almost as far as it is possible to go. Any arc less than 10′ must be guessed at, and probably it will be safer to estimate so small a quantity by the eye than to carry the subdivisions to a greater degree of minuteness.

SURVEYING PROBLEMS.

TO FIND THE POSITION OF THE SHIP FROM TWO PROTRACTED ANGLES MEASURED BETWEEN THREE FIXED OBJECTS.

PREVIOUS to entering into a demonstration of the various problems connected with Marine Surveying, it will be necessary to understand the method of finding a segment of a circle that shall contain any given angle.

CASE I.

Let it be required to find a segment of a circle which shall contain an angle of 90°.

Bisect the straight line AB in G (fig. 52), then with G as a centre, and radius GA or GB, describe the semicircle ADB. From A and B draw any straight lines, AD, BD, to meet in a point D, in the circumference. Then by (Euclid iii. 31) the angle ADB is a right angle, and by a line of chords will be found equal to the required angle.

CASE II.

When the segment is required to contain an angle less than a right angle, suppose 60°.

At the point A, on the straight line AB (fig. 53), make an angle, BAC, equal to the given angle 60°. From A draw AF at right angles to AC, bisect AB in G, and from G draw GF perpendicular to AB and meeting AF in F; join FB. Then, because AG is equal to GB, and GF common to the two triangles AGF, BGF, and the angles AGF, BGF are equal, being right angles (Euclid i. 4), AF is equal to FB, and a circle described with F as a centre, and FA as a radius, will pass through the point B. Let this be the circle ADB. And because AC is perpendicular to AF, AC touches the circle at the point A (Euclid iii. 18), and since AC touches the circle and AB cuts it (Euclid iii. 32), the angle CAB is equal to any angle in the alternate segment of

the circle; it will therefore be equal to the angle ADB. But CAB is equal to the given angle, viz. 60°, therefore ADB is also equal to 60°, and ADB is the segment required.

From this proposition it is manifest that since CAF is a right angle, and CAB, by construction, made equal to the given angle, BAF = ABF is the complement of the given angle. If, therefore, from the extremities of the line AB, straight lines AF, BF, be drawn, making with it angles BAF, ABF, each equal to the complement of the given angle, the point F where these lines meet will be the centre of the segment containing an angle equal to the given angle.

Case III.

When the segment is required to contain an angle greater than a right angle, suppose 120°.

At the point A (fig. 54) on the straight line AB, make an angle, BAC, equal to the given angle 120°.

From A draw AF at right angles to AC, bisect the line AB in G, and draw GF perpendicular to AB, meeting AF in the point F; join FB. Then, as in the preceding case, AF is equal to FB, and a circle described from F as a centre, with radius FA, will pass through the point B. And because AC is at right angles to AF (Euclid iii. 18) AC touches the circle at A. But AB is drawn cutting the circle, therefore (Euclid iii. 32) the angle CAB is equal to any angle in the alternate segment ADB. But CAB is equal to the given angle 120°, therefore ADB is also equal to 120°.

In this case, since the angle CAB is equal to the given angle, and CAF is a right angle, FAG = FBG is the difference between 90° and the given angle. If, therefore, at the extremities of the line AB, straight lines AF, BP, are drawn, making with it angles FAG, FBG, each equal to the difference between 90° and the given angle, the point F where these lines meet will be the centre of a segment, ADB, containing an angle equal to the given angle. It will be seen that when the given angle is less than 90°, the segment ADB (fig. 53) must be greater than a semicircle, and consequently the centre F must be within the segment. The angles BAF, ABF, must in that case be made on the same side of the line AB on which the angle ADB will fall. But when the given angle is greater than 90°, the segment ADB (fig. 54) is less than a semicircle, and the centre F is without the segment. In this case the angles BAF, ABF, must be made on the opposite side of the line AB from that on which the angle ADB falls.

Having given the method of finding a segment of a circle to contain a given angle, the next thing is to shew that two objects will always subtend the same angle in whatever part of a segment an observer may be, providing that circle passes through the objects and the position he stands in; and if the angle measured between them be less than 90°, the place of observation must be at some point in the greater segment of the circle, as in ADB (fig. 53). But if the measured angle subtended by the objects be greater than 90° (fig. 54), the place of observation must be somewhere in the lesser segment; and they will subtend the same angle in whatever part of their respective segments the observer may be. This will be evident on an examination of fig. 55.

Let ADB be a segment of a circle, containing 70°, and AGB be a segment of the same circle, containing 110°, and suppose an observer at N to have first measured the angle ANB, and thence to have proceeded to D and M; then by (Euclid iii. 21) the three angles, ANB, ADB, AMB, being in the same segment, are equal to each other, and by a scale of chords will be found each equal to 70°; for the same reason, AOB, AGB, in the lesser segment, are also equal to each other, and by a scale of chords they will be found to measure an angle equal to 110° each.

As the mere measurement of an angle subtended by any two objects by a sextant would not enable us (unless a reference to the meridian were made) to fix our position, and thence to calculate our distance from them, seeing that the observer may have the same angle in every part of the segment; the next consideration is to point out the method of determining the station from which angles may have been measured, by the intersection of lines drawn from three objects, whose distances from each other must be known; *and that station will be either within or without the triangle*, or *in one of the sides*, formed by drawing lines from one object to the other; or it will be in a right line with *two points*, or *two angles of the triangle:* in which case they are said to be on, or in one with each other, and afford great facility in determining the station-point.

When the distance between two remarkable objects is known, and protracted according to their magnetic bearing, we may find our position and distance from them in the following easy manner :—

Let AB (fig. 56) represent two fixed points, distant 20 miles from each other; at any point, P, with a good compass

take the correct bearing of one or both the objects; if one only, then measure the angular distance between them with a sextant or a theodolite. In this case, suppose A was found to bear from P, by a good magnetic needle, N. 8° W., and the angular distance, by a sextant, was found to be 51°, then, on a sheet of paper, lay down a line, AB, equal to 20 from a scale of equal parts; at A make an angle, BAP, equal to 82°, being the difference between the magnetic bearing of A from P, and a right angle; then 180° *minus* (82° + 51°) will leave the angle ABP = 47°. Protract the angle ABP and draw out the line, and it will meet the line AP, in the point P, the station required; the side BP will measure 24·8, and AP 18 miles, nearly. The calculation is so evident as not to require any explanation.

This method of finding the station of the observer is exceedingly simple, and affords a ready method of fixing different points both on land and on water, and may be extensively employed by using the useful little instrument called a prismatic compass, fully described in the article on surveying instruments, page 62.

In the next place, we will suppose that the mutual distances of three fixed objects on land are known, and, as in the first case, we will take them in a straight line with each other, and after finding the station of the ship by different methods of protraction suitable to each case, we will proceed to illustrate them separately, by such examples as occur in actual Marine Surveying.

PROBLEM I.

Having given the three points A B C *(fig.* 57), *in a straight line, whose distances from each other are,* AB = 3, BC = 3·75, *and the angles subtended by these objects from a certain position as follows, viz. by* AB = 52°, *and by* BC 27°. *Required the station whence the angles were observed, and the distance from the three points respectively?*

METHOD I.—GEOMETRICAL CONSTRUCTION. (Fig. 57).

Join AB, BC; then on AB, as in case second (fig. 53), describe a segment to contain the subtended angle 52°, and on BC in like manner describe a segment that shall contain an angle of 27°, and the point P, where these segments intersect each other, shall be the station whence the angles were measured; then join AP, BP, CP, and their measure from the scale of equal parts from which AB, BC were taken, will be the distances required.

Method II.—Geometrical Construction. (Fig. 58).

When the sum of the observed angles APB, BPC, is less than 90°, find a segment ADC (Case 2, fig. 53), that will contain the sum of the observed angles, = 79°, and in that greater segment the station must be, because the whole angle subtended by the *extreme* objects is less than 90°. Next describe a segment on BC that shall contain the angle subtended by those two objects, namely 27°; and the point P, where it intersects the other segment, will be the station sought; and PA, PB, PC, the distances as before. This method only differs from the last in having found a segment capable of containing the sum of the observed angles, instead of finding segments to contain them separately. For a demonstration of this problem we refer to case second, (fig. 53.)

Method III.—Geometrical Construction. (Fig. 59).

At A, on the opposite side of the straight line AC from that on which the observer is, make an angle CAG, equal to the angle subtended by BC, viz. 27°; and at the point C, on the same side of AC, make an angle ACG, equal to the angle subtended by AB, viz. 52°, and let AG and CG meet in the point G; then, about the triangle AGC describe a circle AGCB (Euc. iv. 5), join GB, and produce it to meet the circumference in P, which will be the station of the observer; join PA, PB, PC, and they will be the distances required.

Demonstration.

By construction the angle GAC was made equal to the angle subtended by BC = 27°; now, by (Euc. iii. 21), the angle GPC is equal to GAC, because they stand in the same segment, GAPC, therefore GPC is the angle subtended by BC = 27°. Again, the angle GCA was made, by construction, equal to the angle subtended by AB = 52°; now, by the same proposition, the angle GPA is equal to to the angle GCA, for they stand in the same segment APCG; therefore GPA is the angle subtended by AB = 52°, consequently P is the station from which the angles were measured, and PA, PB, PC, the distances required.

PRACTICAL EXAMPLES TO ILLUSTRATE THE LAST PROBLEM.

Example I. (Fig. 60.)

Being on survey on the coast of Tenasseriem in the Bay of Bengal, and suddenly shoaling out of seven into four fathoms, two observers with the sextant at the same instant measured

the angles between three fixed objects in a straight line with each other, as follows:—The hill B *to the left of the hill at* C, 50° 27′, *the hill at* A *to the left of* B, 30° 18′, *and the distance by chart from* A *to* B *was ten miles, and from* B *to* C *eight miles. Required the position of that part of the shoal from which the angles were measured?*

GEOMETRICAL CONSTRUCTION. (Fig. 61.)

By either of the methods (suppose method 3d) lay down the distances AB = 10, BC = 8, from a scale of equal parts, then make an angle CAG equal to the angle subtended by BC, namely, 50° 27′, and an angle ACG equal to the angle subtended by AB, namely, 30° 18′, produce the lines that contain these angles until they meet in G; describe a circle that will pass through the points A, G, and C (Euclid iv. 5); then join GB, and produce it to meet the circumference in P, which will be the station required. Join PA and PC, and their measure from the same scale of equal parts will be the distances from those objects.

To find the distances by logarithmic calculation.

In the triangle AGC we have given the angle GAC = 50° 27′, the angle GCA = 30° 18′, consequently, the angle AGC = 99° 15′ (being their supplement), and the side AC = 18. To find the side AG.

As sine AGC 99° 15′	. .	9.9943156
Is to sine ACG 30° 18′	. .	9.7028849
So is AC 18	. .	1.2552725
		10.9581574
To AG 9.20114	. .	0.9638418

Then in the triangle AGB are given AG = 9.20114, AB = 10, and the included angle GAB = 50° 27′. To find the angles ABG, AGB.

As (AB + AG) 19.20114	. . . . .	1.2833270
Is to (AB — AG) .79886	. . . . .	$\bar{1}$.9024707
So is tang. ½ (ABG + AGB) 64° 46′ 30″	.	10.3268893
To tang. ½ (ABG — AGB) 5° 2′ 49″	.	10.2293600
		8.9460330

AGB = 69° 49′ 19″
ABG = 59 43 41
180 00 00

ABP = 120° 16′ 19″

In the triangle APB, we have the angle ABP = 120° 16′ 19″, being the supplement of ABG; consequently all the angles in this triangle are known, namely, APB = 30° 18′, and BAP = 29° 25′ 41″, and the side AB = 10. To find the sides PA, PB.

To find PA.			To find PB.		
As sine APB	30° 18′	9.7028849	As sine APB	30° 18′	9.7028849
Is to sine ABP	120° 16′ 19″	9.9363440	Is to sine PAB	29° 25′ 41″	9.6913736
So is AB	10 . . .	1.0000000	So is AB	10 . .	1.0000000
		10.9363440			10.6913736
To PA = 17.1182	.	1.2334591	To PB	9.7384 .	0.9884887

Lastly, In the triangle PBC are given the angle CBP = 59° 43′ 41″, the supplement of ABP, and BPC = 50° 27′, consequently, BCP = 69° 49′ 19″, and the side BC = 8, to find the side PC; or in the whole triangle APC, the three angles and sides AP, AC are known; therefore find the side PC.

In the triangle BPC, to find PC.			In the triangle APC, to find PC.		
As sine BPC	50° 27′	9.8870934	As sine APC	80° 45′	7.9943156
Is to sine CBP	59° 43′ 41″	9.9363340	Is to sine PAC	29° 25′ 41″	9.6913736
So is BC	8	0.9030900	So is AC	18 . .	1.2552725
		10.8394240	To PC 8.96047	. .	10.9466461
To PC 8.96047	.	0.9523306			0.9523305

ANOTHER EXAMPLE IN THIS CASE.

In the Bay of Bengal, near the mouth of Rangoon River, wishing to determine the position of a shoal, I anchored the boat upon it, and with a sextant measured the angles between three fixed objects on shore, as follows:—(Fig. 60) A left of B 19°, and B left of C 25°. By my chart of the coast, I found by careful measurement with compasses, that the distance between the two conspicuous objects A and B was = 3.626, and between B and C = 8.374 miles. From these angles and distances, it is required to find the position of the boat with regard to the shore, and the distances from the three objects.

The construction will be exactly the same as fig. 61, only the station-point P will not fall in the same part of the circumference. We will therefore use the same letters and lines in this explication.

First, In the triangle GAC, we have by construction the two observed angles, namely, GAC = 25° and GCA = 19°, consequently 180° *minus* their sum will leave the angle AGC = 136°. With these angles, and the side AC = 12 miles, find the side AG = 5.624, and CG = 7.3 miles.

Then, in the triangle AGB are given AB = 3.626, AG = 5.624,

and the included angle GAC = 25°. With these two sides and the angle, find the angle ABG = 121° 45′ 7″, and consequently the angle AGB = 33° 14′ 53″.

Again, In the triangle ABP, the angle PBA = 58° 14′ 53″, being the supplement of ABG, and APB = 19°, and the side AB = 3·626 are given. Therefore find the side AP = 9·4711, and BP = 10·86.

Lastly, In the triangle APC, the angle PAC = 102° 45′ 7″, APC = 44°, ACP = 33° 14′ 53″, and the sides AP = 9·4711, AC = 12. To find the side PC = 16·8485 miles.

PROBLEM II.

In this case two of the fixed objects are supposed to be in the same straight line with the observer, or they are said to be on with each other, and the angle measured between them (when exactly in that position) and the third object, either to the right or left, is given, to find the station-point P. *Their mutual distances must of course be known.*

EXAMPLE. (Fig. 62.)

Let A and B be two fixed objects distant from each other 13 miles, and suppose an observer at a position in the same straight line when they were on with each other measured the angular distance between *them* and the other object C to the right, and found it to be 25°, the distance from B to C = 13·5, A to C = 19. Required the station-point P when the angle was taken?

GEOMETRICAL CONSTRUCTION. (Fig. 62.)

Construct a triangle ABC, having the proportions given, from B let fall upon AC a perpendicular BG, then by the 4th case of oblique-angled triangles, find the segments of the base AG, GC. Next find the angle BAC = 44° 30′, from which angle subtract the observed angle = 25°, then shall the remainder be equal to the angle ACP 19° 30′, because, by (Euclid i. 32) the exterior angle BAC is equal to the two interior and opposite angles ACP, APC of the triangle PAC. On that side of the line AC next to the observer, lay off an angle equal to the angle ACP just found, produce the side until it meet the line AB produced in P, which is the station whence the angle was observed, and PC, PB will be the distances from the objects measured on the

same scale of equal parts from which the sides of the triangle were taken.

Example I. (Fig. 63.)

On the coast of Arracan, in the Bay of Bengal, the soundings suddenly decreased; at this moment the angle between a hill* A *and a hill* C *to the right was found with a sextant to be* 30° 25′, *whilst a high peak at* B *happened at the time to be exactly on with* A, *the distances of the objects from each other being* A *to* C = 16½, A *to* B = 11, *and* B *to* C = 13 *miles. From these observations it is required to find the station of the ship when the angle was measured.

Geometrical Construction. (Fig. 64.)

With a scale of equal parts construct a triangle ABC having its sides respectively equal to the proportions given. Then from the point B let fall upon AC the perpendicular BG, and find the segment of the base AG, and angle BAC.

LOGARITHMIC CALCULATION.

To find the segments.		To find the angle BAC.	
As AC 16·5	1·2174839	As AB 11 .	1·0413927
Is to (BC + BA) 24	1·3802112	Is to AG 6·7955 .	0·8322214
So is (BC — BA) 2	0·3010300	So is radius .	10·0000000
	1·6812412		10·8322214
To (CG — GA) 2·9091	0·4637573	To cos. BAC 51° 50′ 47″	9·7908287
½ AC = 8·25		180°	
½ (CG — GA) = 1·4545		CAP = 128° 9′ 13″	
CG = 9·7045			
AG = 6·7955			

From the exterior angle BAC 51° 50′ 47″, subtract the observed angle 30° 25′, and the remainder will be the other interior angle ACP = 21° 25′ 47″.

Then in the triangle PAC, we have angle APC = 30° 25′, angle PAC = 128° 9′ 13″, angle ACP = 21° 25′ 47″, and the side AC = 16·5. To find the sides AP, PC.

To find the side AP.		To find the side PC.	
As sine P 30° 25′	9·7043947	As sine P 30° 25′	9·7043947
Is to sine ACP 21° 25′ 47″	9·5627206	Is to sine PAC 128° 9′ 13″	9·8956166
So is AC 16·5 .	1·2174839	So is AC 16·5 .	1·2174839
	10·7802045		11.1131005
To AP = 11·9072 .	1·0758098	To PC 25·6275 .	1·4087058

EXAMPLE II.

Being on the same coast, and wishing to fix the vessel in a position until I measured several angles, I ran until I brought the peak at A *on with the peak at* B. *(See last figure). I then, with a sextant, measured the angle between the summit of a quoin-shaped hill at* C *and the peak at* A, *which was* $= 17° 47' 19''$; *the distance of one object from the other as follows:—*A *to* C $= 12$, A *to* B $= 8$, *and* B *to* C $= 7\frac{1}{3}$ *miles. Required the station whence I took the angle?*

Construct the figure as in last example; then with the three given sides of the triangle ABC, find the segments of the base, and the angle BAC $= 35° 34' 38''$, the supplement of which PAC $= 144° 25' 22''$, and the exterior angle BAC $=$ $35° 34' 38''$ *minus* the observed angle $= 17° 47' 19''$, leaves ACP $= 17° 47' 19''$. Hence we have given the three angles of the triangle APC and the side AC to find the remaining sides, namely, AP $= 12$, and PC $= 22{\cdot}85$ miles.

PROBLEM III.

When the station from which the angle is observed is in one of the sides of the triangle formed by the three objects.

Let ABC (fig. 65) be the three hills or peaks, and an observer from some station in the line AC measures an angle subtended by one of the other sides (suppose AB) and finds it to be $48° 30'$, it is required to find the point in that side from which he took the angle.

GEOMETRICAL CONSTRUCTION. (Fig. 65).

Construct a triangle ABC, having its sides AB $= 12$, BC $= 14$, and AC $= 19\frac{1}{2}$ miles. Then at any point in AC, as at G, make an angle AGB $=$ to the observed angle $48° 30'$, through B draw a line BP parallel to FG, and it will intersect the line AC in P, the station required. BP and FG being parallel to each other, the exterior angle FGA (Euc. i. 29) is equal to the interior and opposite angle APB, which is therefore equal to the observed angle, and P the station from which it was observed.

EXAMPLE I. (Fig. 66.)

Being in the entrance of a bay on the coast of Tenasseriem, and exactly on the straight line drawn from two hills or peaks, A *to* C, *I measured the angle subtended by* C *and a conical hill* B, *at the inner extreme of the bay, which was* = 98° 30′. *At what point in the side* AC *must the ship have been to obtain such an angle? The distance from* A *to* B = 3½, C *to* B = 4½, *and* A *to* C = 5 *miles.*

GEOMETRICAL CONSTRUCTION. (Fig. 67.)

From a scale of equal parts construct a triangle, having its side AB = 3·5, CB = 4·25, and AC = 5 miles. Then make an angle, CAO, equal to the observed angle, viz. 98° 30′, and draw BP parallel to AO, cutting the side AC in P. Then shall the point P be the station of the ship when the angle BPC was measured.

To find the distances by logarithmic calculation.

First, From the point B let fall a perpendicular BG, dividing the side AC of the triangle ABC into two segments, AG, GC. Then to find these segments, and the angle at C, say—

As AC 5	0·6989700
Is to (CB + BA) 7·75	0·8893017
So is (CB — BA) ·75	$\bar{1}$·8750613
	0·7643630
To (CG — GA)· 1·1625	0·0653930
½ AC = 2·5	
½ (CG — GA) = 0.58125	
CG = 3·08125	

In the triangle BGC, we have the two sides BC, GC, and an angle opposite to one of them. To find the other angle at C.

As BC 4·25 . . .	0·6283889
Is to GC 3·08125 . .	0·4887270
So is radius, . .	10·0000000
	10·4887270
To cosine C 43° 31′ 52″	9·8603381
Obs. angle = 98 30	
142° 1′ 52″	
180	
Supp. PBC = 37° 58′ 8″	

To find the side BP.

As sine BPC 98° 30′	9·9952033
Is to sine C 43° 31′ 52″	9·8380606
So is BC 4·25	0·6283889
	10·4664495
To BP 2·9597 . .	0·4712462

To find the side PC.

As sine BPC 98° 30′	9·9952033
Is to sine PBC 37° 58′ 8″	9·7890400
So is BC 4·25	0·6283889
	10·4174289
To PC 2·6438	0·4222256
AC = 5·	
PA = 2·3562	

Example for Practice.

In the entrance of a great river, and on a line drawn from a church in the town at A, *and a very high peak on the right side of the river at* C, *I measured the angle subtended by the church steeple* A, *and another hill a short distance up the river at* B, *which was* 107° 56′ 13″, *the distance from* A *to* B = 8, B *to* C = $7\frac{1}{5}$, *and* A *to* C = 12 *miles. Required the position of the ship when the angle was taken?*

As in last example (fig. 67), with the three sides of the triangle, namely, AB = 8, BC = 7·2, and AC = 12, find the segments of the base, and the angle BAC = 35° 34′ 38″.

Then in the triangle APB all the angles are known, namely, BPA = 107° 56′ 13″, being the observed angle, BAC = 35° 34′ 38″, consequently the supplement ABP = 36° 29′ 9″. With these angles and the side AB = 8 find the side AP = 5, PB = 4·892, therefore the side PC = 7 miles.

PROBLEM IV.

When the station from which the angles are measured falls within the triangle formed by the objects, which may happen in a deep bay, river, *or amongst* islands.

Let ABC (fig. 68) be three such objects, and suppose an observer to be within the triangle (formed by straight lines joining the objects), and that he found by observation that the angle subtended by BA was 98° 40′, and the angle in like manner measured between A and C was 110° 30′, the distances between the hills, as follows:—A to B = 12, B to C = 14, and A to C = 15 miles. From these distances and angles let it be required to find the station whence they were taken.

Geometrical Construction—Method I. (Fig. 68.)

By a scale of equal parts construct a triangle ABC, having its sides equal to 12, 14, and 15, respectively. Then, on the sides AB and AC, describe segments that shall contain the angles subtended by those sides (see Cases ii. and iii., figs. 52 and 53), namely, on the side AB a segment that shall contain the subtended angle 98° 40′, and on AC a segment that shall contain 110° 30′, then shall these two segments intersect each other within the triangle at P, which is the station required; and BP, AP, PC, measured on the same scale from which the sides of the triangle was taken, will give the distances of the several objects.

GEOMETRICAL CONSTRUCTION—METHOD II. (Fig. 69.)

At B, outside the triangle ABC, make an angle equal to 69° 30′, being the supplement of the measured angle APC, and at C, also outside the triangle, make an angle equal to 81° 20′, which is the supplement to the angle APB; these sides produced will meet at the point G. Next find the centre of a circle which, when described, shall pass through the three points B, C, and G, then draw a line from A to G and it will cut the circle in P, the station required, and BP, AP, PC, will be the distances, as before.

DEMONSTRATION.

Because GCB, GPB, are in the same segment GCPB, they are equal (Euc. iii. 21.) But GCB is by construction = 81° 20′, the supplement of the first observed angle, therefore GPB is also equal to 81° 20′, the supplement of the first observed angle. Now, APB is the supplement of GPB, therefore APB is equal to 98° 40′, the first observed angle.

Again, because CPG, CBG, are in the same segment CPBG, they are equal (Euc. iii. 21.) But CBG is, by construction, = 69° 30′, the supplement of the second observed angle, therefore CPG is also equal to 69° 30′, or the supplement of the second observed angle. Now, APC is the supplement of CPG, therefore APC is equal to 110° 31′, the second observed angle. Hence, P is the point of station, and AP, PC, PB, the distances required.

EXAMPLE. (Fig. 70.)

On survey amongst some islands near St Matthew's, on the coast of Tenasseriem, and wishing to determine the position of a coral reef not visible above water, I anchored a boat upon it, and amongst a set of angles the two following were taken: namely, B, *a high-pointed rock, on the extreme of an island to the right of another peak* A, 116° 40′; C, *a bluff summit to the right of* B, 112° 30′, *the distance between* B *and* A = 5·75, B *and* C = 7·5, A *and* C = 8·25 *miles. It is required to find the position of the reef on which the boat was anchored.*

GEOMETRICAL CONSTRUCTION. (Fig. 71.)

Construct the triangle ABC, having its sides AB = 5·75, BC = 7·5, and AC = 8·25, make the side AC the base, then at C, outside the base, make an angle ACG = 60° 20′, being the supplement of the angle subtended by AB; and at A make an angle

GAC equal to 67° 30′, which is the supplement of the angle subtended by BC; produce the sides, and they will meet in a point at G. Find the centre O, and describe a circle that will pass through the points A, C, and G, then draw a line from B to G and it will cut the circle in P, the station of the boat required; and PA, PC, PB, will be the distances of the different objects.

To find the distances by logarithmic calculation.

First, From the point B in the triangle ABC let fall the perpendicular BD, which will divide the side AC into two segments AD, DC. Then—

To find the segments.

As AC .	8·25	0·9164539
Is to (BC+BA)	13·25	1·1222159
So is (BC—BA)	1·75	0·2430380
		1·3652539
To (CD—DA)	2·8106	0·4488000
½ AC . . =	4·125	
½ (CD—DA) =	1·4053	
CD =	5·5303	
AD =	2·7197	

To find the angle BAC.

As BA 5·75 . .	0·7596678
Is to AD 2·7197 .	0·4345210
So is radius, . . .	10·0000000
	10·4345210
To cos. BAC 61° 46′ 17″	9·6748532
GAC = 67 30	
BAG = 129° 16′ 17″	
180	
ABG + AGB = 50° 43′ 43″	

Next, in the triangle AGC the angles ACG, GAC are known, being by construction made equal to the supplements of the two observed angles, APB, BPC, consequently the whole angle AGC, = 49° 10′, is likewise known, being their supplement, and we have the side AC = 8·25 opposite to this angle, with which find the side AG.

As sine AGC, 49° 10′,	9·8788748
Is to sine ACG, 63° 20′	9·9511590
So is AC 8·25, . . .	0·9164539
	10·8676129
To AG 9·74402, . .	0·9887381

To find ABG.

As (AG+AB) 15·494,	1·1901636
Is to (AG+AB) 3·994,	0·6014081
So is tan ½ (B+G) 25° 21′ 51″	9·6758413
	10·2772494
To tan ½ (B—G) 6° 58′ 2″	9·0870858
ABG, 32° 19′ 53″	

Then, in the triangle APB, the angle APB = 116° 40′, being one of the observed angles, and the angle ABG or ABP = 32° 19′ 53″, consequently the angle BAP = 31° 00′ 7″; therefore—

To find AP.

As sine APB 116° 40′	9·9511590
Is to sine ABP 32° 19′ 53″	9·7282038
So is AB 5·75, .	0·7596678
	10·4878716
To AP 3·4412, .	9·5367126

To find BP.

As sine APB 116° 40′,	9·9511590
Is to sine BAP 31° 0′ 7″,	9·7118638
So is AB 5·75, . .	0·7596678
	10·4715316
To BP 3·3142, .	9·5203726

Then, in the triangle BDC, right-angled at D, the side BC = 7·5, the side DC = 5·5303; with these sides, and an angle opposite to one of them, find the angle DCB.

As BC 7·5	0·8750613
Is to CD 5·5303, . .	0·7427487
So is radius, . . .	10·0000000
	10·7427487
To cos. DCB 42° 29′ 31″	9·8676874

The angle DCB = 42° 29′ 31″ + BAC = 61° 46′ 17″ = 104° 15′ 48″, subtracted from 180°, will give the whole angle ABC = 75° 44′ 12″, from which subtract ABG = 32° 19′ 53″, and we obtain the angle PBC = 43° 24′ 19″. Then,

To find the side PC.

As sine BPC 112° 30′	9·9656153
Is to sine PBC 43° 24′ 19″	9·8370544
So is side BC 7·5 . .	0·8750613
	10·7121157
To the side PC 5·5782	0·7465004

Example for Practice.

Wishing to determine the position of a sunken rock, in a deep and extensive bay, I placed a boat over it, and measured the following angles—namely, the angle subtended by AC *was observed to be* 123° 45′, *that subtended by* CB 132° 22′, *consequently the angle which* AB *subtends* = 103° 53′. *The distance from* A *to* C *was known to be* = 12, B *to* C = 9, *and* A *to* B = 6 *miles. It is required to find the distance of the boat from each object.*

Geometrical Construction. (Fig. 71.)

Construct a triangle, having its sides equal to 12, 9, and 6 respectively; then, by either of the two last methods, complete the figure, and find the segments of the base, and the angle BAC = 46° 34′.

Then, in the triangle AGC all the angles and one side are given. Thus, by construction, the angle ACG = 76° 7′, and CAG = 47° 38′, therefore AGC must be = 56° 15′, and the side AC = 12; hence the side AG will be found = 14·011.

Next, in the triangle BAG, the whole angle GAB is found by adding the angle BAC 46° 34′ to the angle CAG 47° 38′, which is = 94° 12′. With this included angle GAB, and the sides AB = 6 AG = 14·011, find the angle ABG = 63° 18′ 21″.

Again, in the triangle PAB, all the angles are given, name-

ly, ABP = 63° 18′ 21″, APB = 103° 53′, being the supplement of the observed angles, and PAB = 12° 48′ 39″, and the side AB = 6; therefore, find AP = 5·523, and PB = 1·372.

Lastly, in the triangle CPB, the two sides, CB = 9, and BP = 1·372, and the angle CPB = 132° 22′, opposite to one of them, are known; therefore, find the angles BCP = 8° 9′, PBC 39° 29′, and the side PC = 8, nearly.

PROBLEM V.

When the three fixed objects form a triangle, and the station whence the angles are measured is outside the triangle.

This problem admits of two cases.

First, when the vertex of the triangle is turned from the observer (see fig. 72), and, secondly, when it is towards him, (see fig. 75); both of which cases often occur on survey. However, it will be seen that the protraction and calculation are precisely the same in both.

CASE I. (Fig. 72.)

When the vertex of the triangle is turned from the observer.

EXAMPLE.

Let A, B, C, *be three fixed objects forming a triangle, with its vertex,* B, *farthest from the observer. Then, suppose he measured the angle between* A *and* B, *and found it* = 32° 30′, *and the angle between* B *and* C = 28° 35′, *the distance between* A *and* C = 9, *between* A *and* B = 6, *and between* B *and* C = 7 *miles. Required his position and distance from each object when he took the angles, by the different methods of protraction?*

GEOMETRICAL CONSTRUCTION—METHOD I. (Fig. 72.)

Having constructed a triangle ABC, with the given proportions from a scale of equal parts, make the side AC (which subtends the greatest angle) the base, and at C, on that side of AC which is most remote from the observer's station, make an angle ACD, equal to the angle subtended by AB, namely, 32° 30′, and at A make an angle CAD, equal to the angle subtended by BC, namely, 28° 35′; produce the sides, and they will meet at the point D; then describe a circle, so that it may pass through the points A, D, and C, and the station from which the angles were observed is in that circle; join BD, and produce the line until it cuts the circle in P, and that will be the station sought. Join CP, PA, and they will be the distances required.

Demonstration.

By Euc. iii. 21, the angles APD and ACD are equal, for they stand in the same segment, APCD; but, by construction, the angle ACD was made equal to the observed angle subtended by AB: APD is therefore equal to the first observed angle.

Again, the angles CPD and CAD are equal, for they stand in the same segment, DAPC; but, by construction, CAD was made equal to the observed angle subtended by BC; CPD is consequently equal to the second observed angle, and P the station whence they were taken, and PC, PA the distances required.

When the sum of the two observed angles is less than a right angle.

Geometrical Construction—Method II. (Fig. 73.)

Construct a triangle, ABC, having the same proportions and using the same angles as in last example; then at the points A and C, and on the side next to the observer, make angles ACG, GAC, each equal to 28° 55′, being the complement of the sum of the observed angles subtended by BC, BA, and the point in which the lines AG, CG meet will be the centre of the circle ANC; or, in other words, by Case II. (fig. 53), describe on AC a segment, containing an angle equal to the sum of the observed angles, 32° 30′ + 28° 35′ = 61° 5′. Next, at the points B and C make the angles OBC, OCB each equal to 61° 25′, the complement of the angle subtended by the side BC, namely 28° 35′ (which is, in other words, describing a segment, &c. as before, Case II. fig. 53); and where the lines OB, OC intersect each other will be the centre of the circle BMR, and the point P, where the circles cut each other, is the point from which the angles were taken.

Remark.—A segment may be described on the side AB, with FB or FA, as radius, instead of the side BC: the result will be exactly the same; for it will be seen that the segment BHA cuts the other segments in the point P also. But the required segment (in this method) must always be constructed so as to fall towards the observer; for, if the complement of the angle 28° 35′ had been laid off on the remote side of the line BC, instead of the inner or nearest side, the segment of the circle, BMR, would have fallen inside the triangle ABC, cutting two of the objects, B and C; which would be similar to Problem IV. where the observer is situated within the triangle formed by the objects.

This method of construction will be found simple, and more

accurate than Method I. fig. 72, in some instances; for suppose (see fig. 72) the observed angles should be nearly equal to, or the same as, the angles ACB, CAB, made by the objects themselves, in that case, in making the angles ACD, CAD respectively equal to the observed angles, the lines CD, AD would either coincide with the lines CB, AB, or would fall so near to them that it would render the protraction extremely doubtful and uncertain, for the line BD could not be drawn with any certainty. Should the observed angles be much greater than the angles contained by the sides of the triangle formed by the objects themselves, the point D would fall outside the triangle; and, providing it was at a distance from the point B, the protraction would be the same, and with care might be in some degree trusted to.

When the observer is close in with the objects, and the sum of the observed angles is greater than a right angle.

EXAMPLE.

Let A, B, C *be three objects, the distance from* A *to* B = 3, B *to* C = 6, *and* A *to* C = 8 *miles, the angle taken between* B *and* C = 91° 30′, *and between* A *and* B = 29° 10′. *Required the position from which they were taken?*

GEOMETRICAL CONSTRUCTION. METHOD III. (Fig. 74.)

Construct a triangle, ABC, having its sides respectively equal to the proportions given; then on BC, one of the sides of the triangle, describe a segment, containing the sum of the observed angles, 120° 40′ (see Case III. fig. 54); then on the other side, AB, describe a segment that will contain the angle subtended by that side, namely 29° 10′ (Case II. fig. 53); the point P, where these two segments intersect each other, will be the station, and PB, PC, PA the distances required.

CASE II. (Fig. 75.)

Where the vertex of the triangle formed by the objects falls towards the observer, which will occur in practice when the hill or conspicuous object at B is situated on a point or jutting headland, whilst the two objects A and C are at a distance inland.

EXAMPLE.

Let A, B, C, *be three objects so situated as to have the object* B, *forming the vertex of the triangle, towards the observer; then suppose he takes the angular distance between* A *and* B = 25°

20′; and between B *and* C = 36° 40′, *and call the distance from* A *to* C = 7, A *to* B = 4, B *to* C = 6 *miles. What was the station of the ship to have obtained such angles, and what was her distance from each object?*

GEOMETRICAL CONSTRUCTION. (Fig. 75.)

Construct the triangle as before, only let the vertex, B, fall towards your station; then at C, on the opposite side of the line AC (as in fig. 72), make an angle equal to the angle subtended by A and B = 25° 20′, and at A, on the same side also, make an angle equal to the angle subtended by B and C = 36° 40′; produce the sides that contain these angles, and they will meet at the point D; find the centre, and describe a circle that will pass through the points A, D, and C, and the station will be somewhere in the greater segment of this circle, because the whole angle subtended by A and C is less than a right angle. Draw a line from the intersection of the lines at D through B, and it will cut the circle in P, the station, and PB, PA, PC will be the distances required.

DEMONSTRATION.

By Euc. iii. 21, the angles APD and ACD are equal, for they are in the same segment, APCD; but, by construction, the angle ACD was made equal to the observed angle subtended by AB = 25° 20′, APD is therefore equal to the first observed angle.

Again, the angles CPD and CAD are equal, for they stand in the same segment, DAPC; but, by construction, the angle CAD was made equal to the observed angle subtended by BC; CPD is consequently equal to the second observed angle, and P the station whence the angles were taken, and PC, PA, PB the distances required.

EXAMPLES TO ILLUSTRATE Figs. 72 AND 73.

On survey near Tavoy river, on the Tenasseriem coast, I measured the angles between three remarkable peaks, as follows, (fig. 76):—Peak B *to the right of peak* A, *26° 27′; peak* C *to the right of peak* B, *34° 12′; the distance from* A *to* B = 5, *from* B *to* C = 6, *and from* A *to* C = 7 *miles. Required the position of the ship?*

GEOMETRICAL CONSTRUCTION—METHOD I. (Fig. 77).

Construct a triangle, ABC, with the above proportions, and at C, on the furthest side of AC from the observer, make an

angle equal to the angle measured between A and B = 26° 27′, and at A, on that side also, make an angle equal to the angle measured between B and C = 34° 12′; draw out the sides that contain these angles and they will meet at D; then describe a circle, so that it may pass through the points A, D, and C, and the station will be found somewhere in the greater segment; from B draw a line to pass through D, and it will cut the segment at P, which is the station sought, and PA, PB, PC the distances required.

Geometrical Construction—Method II. (Fig. 78).

Construct the triangle ABC as before, then on AC describe a segment, ANC, containing the sum of the observed angles, namely, 60° 39′ (Case II, fig. 53); then on the side AB describe a segment, ABM, that will contain the angle measured between the two objects at the extremes of that side, namely, 26° 27′; then shall the point of intersection, P, of these two segments ANC, ABM be the station, and PA, PB, PC the distances as before.

Logarithmic Calculation. (Fig. 77).

Let fall from the point B the perpendicular BH; then in the triangle ABC, with the three sides AB = 5, BC = 6, and AC = 7, find the segments of the base AH, HC.

To find the segments.

As AC	7	0.8450980
Is to (AB + BC) .	11	1.0413927
So is (AB—BC) .	1	0.0000000
		1.0413927
To (HC—AH)	1.5714	0.1962947
½ AC .	= 3.5	
½ (CH—HA)	= .7857	
CH .	= 4.2857	
AH .	= 2.7143	

To find the angle BAC.

As AB .	5	0.6989700
Is to AH	2.7143	0.4336578
So is radius, . . .		10.0000000
		10.4336578
To cos. BAH =	57° 7′ 17″	9.7346878

In the triangle ADC, the angle ACD = 26° 27′, CAD = 34° 12′, consequently the supplement of these angles ADC = 119° 21′; with these angles and the side AC = 7 find the sides AD and DC.

To find the side AD.

As sine ADC 119° 21′ .		9.9403381
Is to sine ACD 26° 27′ .		9.6487665
So is AC .	7 . .	0.8450980
		10.4938645
To AD .	3.5771	0.5535264

To find the side DC.

As sine ACD 26° 27′ .		9.6487665
Is to sine CAD 34° 12′		9.7498007
So is AD 3.5771 . .		0.5535264
		10.3033271
To DC 4.514 . . .		0.6545606

In the triangle DAB, the sides AD, BA are given, and the angle BAD = 22° 55′ 17″ being the difference between BAC and DAC, therefore find the angle ABD.

As (AB+AD) 8·5771	. . .	0·9333405
Is to (AB—AD) 1·4229,	. . .	0·1521744
So is tan. ½ (D+B) 78° 32′ 21″		10·6930596
		10·8452340
To tan. ½ (D—B) 39° 13′ 39″		9·9118935

ADB = 117 46 00
ABD = 39 18 42

From the exterior angle ADB, = 117° 46′ of the triangle PDA, subtract the interior angle APD = 26° 27′, and the remainder will be the other interior angle DAP = 91° 19′. This angle *plus* the angle BAD = 22° 55′ 17″, will give the whole angle BAP = 114° 14′ 17″. In the triangle BAP all the angles are known, and the side AB = 5; therefore, find the sides AP, PB.

To find the side AP.

As sin. APB	26° 27′	9·6487665
Is to sin. PBA	39° 18′ 43″	9·8017755
So is AB	5 .	0·6989700
		10·5007455
To AP	7·1118	0·8519790

To find the side PB.

As sin. ABP	39° 18′ 43″	9·8017755
Is to sin. BAP	114° 14′ 17″	9·9599223
So is AP	7·1118	0·8519790
		10·8119013
To PB	10·236	1·0101258

By Euc. iii. 22, the opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles; therefore the angle DCP is equal to 88° 41′, being the supplement of the angle DAP; hence in the triangle DPC we have the angle DPC = 34° 12′, DCP = 88° 41′, consequently CDP = 57° 7′, being the supplement of these angles, and the side DC = 4·514.

To find the side PC.

As sin. DPC	34° 12′	. .	9·7498007
Is to sin. CDP	57° 7′	. .	9·9241644
So is DC	4·514	. .	0·6545606
			10·5787250
To PC	6·7441	. .	0·8289243

Example for Practice.

Being at anchor on a coast, and bad weather coming on, I was obliged to cut and run; but previous to cutting, I had taken angles between three remarkable objects as follows (Fig. 76):

B, a hill to the right of another hill A, 19°; C, a high peak to the right of B, 25°, and the distances, by my chart, between A and B = 8, between B and C = 7½, and between A and C = 12 miles. From these observations, I am to find my position off shore when I cut from my anchor.

Construct a triangle, ABC, whose three sides are AC = 12, BC = 7½, AB = 8, then complete the geometrical construction by either of the foregoing methods, suppose that described (fig. 77.)

The three sides of the triangle A B C are given, with which find the segments of the base and angle BAC = 35° 34′ 38″.

In the triangle ADC all the angles are known, namely DAC = 25°, DCA = 19°, consequently, ADC = 136°, and the side AC = 12; therefore find side AD = 5.624, and side DC = 7·3.

Again, in the triangle BAD, the side AB = 8, side AD = 5·624, and the angle BAD = 10° 34′ 38″, (being the difference between BAC = 35° 34′ 38″, and DAC = 25°); hence the angle ADB will = 146° 45′ 7″. If from this exterior angle ADB of the triangle DAP we subtract the opposite interior angle APD = 19°, we shall obtain the angle DAP = 127° 45′ 7″; and to this if we add the angle BAD = 10° 34′ 38″, we shall get the whole angle BAP = 138° 19′ 45″.

In the triangle PAB, therefore, all the angles are known, namely, BAP = 138° 19′ 45″, APB = 19°, and ABP = 22° 40′ 15″, being the supplement of BAP, APB, and the side AB = 8; hence find the side AP = 9·471, and the side BP = 16·337 miles.

Then in the triangle PDC, the side DC = 7·3, the angle DCP = 52° 14′ 53″, being the supplement of its opposite angle DAP = 127° 45′ 7″, in the quadrilateral figure, DAPC, inscribed in the circle (Euc. iii. 22); therefore, PDC = 102° 45′ 7″, being the supplement of DPC = 25° + DCP = 52° 14′ 53″. Hence the side PC will be found = 16·85 miles.

Example II.

Suppose I wanted to lay down on a chart the position of a spit of sand on which I had placed a buoy, and that three fixed objects, A, B, C (as in last example) were in sight, and with a sextant from a boat I found the angle subtended by BC to be = 33° 45′, and that subtended by AB = 22° 30′, the distance from A to C = 12, B to C = 9, A to B = 6 miles. What distance was I from each object?

The calculation is exactly the same as in the preceding example. The side AP will be found = 14, side PB = 15·64, and the side PC = 10·6 miles.

An EXAMPLE to illustrate fig. 75, where the vertex falls towards the observer.

Surveying off the northern extreme of an island, and wishing to lay down on my chart the position of a sunken rock, I measured the angle (fig. 79) *between a peak at* B *(on the extreme point of the island), and another hill at* C, *to the right of* B, *which was* 20° 30′, *and a peak at* A *to the left of* B, 40° 28′, *the distance from* A *to* B = 6, B *to* C = 5, *and* A *to* C = 8 *miles. Required the position of the rock, and distance from the objects?*

GEOMETRICAL CONSTRUCTION. (Fig. 80.)

Construct a triangle, ABC, having the vertex, B, towards the observer; make the side AC = 8 from a scale of equal parts, BA = 6, and BC = 5: then, as in fig. 75, at the point C make an angle, ACD = 40° 28′, equal to the angle subtended by AB; and at A, in like manner, make an angle, CAD = 20° 30′, being equal to the angle subtended by BC. Next draw out the lines containing these angles, and they will meet in D; then describe a circle that shall pass through the three points, A, D, C; join DB, and produce it, and it will cut the circle in P; join PA, PC, then shall P be the station from which the angles were measured, and PA, PB, and PC the distances sought.

To find the distances by logarithmic calculation.

From the point B, in the triangle ABC, draw the perpendicular BH upon the side AC; then, with the three sides, AC, AB, BC, find the segments of the base, AH, HC, and the angle BAC.

As . . AC 8	0.9030900	As BA 6	0.7781513
Is to (AB + BC) 11	1.0413927	Is to AH 4·6875 . .	0.6709413
So is (AB — BC) 1	0.0000000	So is radius	10.0000000
	1.0413927		10.6709413
To (AH — HC) 1·375	0.1383027	To cos. BAC 38° 37′ 29″	9.8927900

½ AC . . = 4
½ (AH — HC) = ·6875

AH = 4·6875
HC = 3·3125

In the triangle ADC all the angles are known, and the side AC = 8, thence find the sides AD and DC.

To find AD.		To find DC.	
As sin. ADC 119° 2′	9.9416791	As sin. ADC 119° 2′	9.9416791
Is to sin. ACD 40° 28′	9.8122484	Is to sin. DAC 20° 30′	9.5443253
So is AC 8 . . .	0.9030900	So is AC 8 . . .	0.9030900
	10.7153384		10.4474153
To AD 5.93826 .	0.7736593	To DC 3.20432 . .	0.5057362

In the triangle DAB, the sides AB, AD, and the included angle DAB = 59° 7′ 29″ (being the sum of DAC = 20° 30′ + BAC = 38° 37′ 29″), are known. Therefore find the angles ABD, ADB.

To find ABD.	
As (AB + AD) 11.93826	1.0769411
Is to (AB — AD) 0.06174	$\bar{2}$.7905666
So is tan. ½ (ADB + ABD) = 60° 26′ 15″ .	10.2462533
	9.0368199
To tan. ½ (ADB — ABD) = 0° 31′ 21″ .	7.9598788

ADB = 60° 57′ 36″
ABD = 59 54 54

In the triangle APD the angle ABP = 120° 5′ 6″, being the supplement of ABD; the angle APB = 40° 28′, being one of the observed angles; hence find side AP.

As sine APB 40° 28′	9.8122484
Is to sine ABP 120° 5′ 6″	9.9375580
So is AB 6 . . .	0.7781513
	10.7153093
To PA 7.99946 . .	0.9030609

In the triangle ADC the whole angle ADC = 119° 2′ *minus* the angle ADB = 60° 57′ 36″, leaves the angle PDC = 58° 4′ 24″, and DPC, or BPC, the other observed angle, = 20° 30′.

To find the side PC.		To find the side PB.	
As sin. DPC 20° 30′	9.5443253	As sin. APB 40° 28′	9.8122484
Is to sin. PDC 58° 4′ 24″	9.9287673	Is to sin. BAP 19° 26′ 54″	9.5223877
So is DC 3.20432 .	0.5057362	So is BA 6 . . .	0.7781513
	10.4345035		10.3005390
To PC 7.76566 . .	0.8901782	To PB 3.07815 .	0.4882906

EXAMPLE II. (Fig. 81.)

Sailing through some straits, and rapidly decreasing my soundings, I hove the vessel to, and took angles between B *(a pointed hill on an island, nearly right a-head) and* A, *a conical-shaped hill at a distance, on an island a little to the larboard bow, which measured* 19°. *Then between* B *and* C, *(a sugar-loaf-shaped hill) on an island at a distance, and a little on the starboard bow, the angle measured* 25°. *The distance between* A *and* C *I found by the chart to be* = 12, B *to* A = 8, *and* B *to* C = 7⅓. *What was the position of the shoal with regard to the objects?*

GEOMETRICAL CONSTRUCTION. (Fig. 82.)

Construct a triangle having the side AC = 12, AB = 8, and BC = 7·2, then lay off an angle ACD = 19°, the angle subtended by A and B, and next lay off an angle CAD = 25°, the angle subtended by B and C; describe a circle to pass through the three points A, D, and C, draw out the line DB, and produce it, and it will cut the circle in P; join PA, PB, PC, then shall P be the position of that part of the shoal from which the angles were taken.

Let fall the perpendicular BH upon AC, then in the triangle ABC the sides are given; therefore find the segments AH, HC, and the angle BAC = 35° 34′ 38″.

In the triangle DAC the angle ACD = 19°, and DAC = 25°, having been by construction made equal to the observed angles, consequently the angle ADC, which is their supplement, is equal to 136°, and the side AC = 12. Thence find the side AD = 5·624, and the side DC = 7·3.

In the triangle DAB, the side AD = 5·624, AB = 8, and the angle BAC = 35° 34′ 38″ + DAC = 25°, gives the whole angle DAB = 60° 34′ 38″, with which find the angle ABD = 43° 5′ 14″; from this angle (being the exterior angle of the triangle PBC) subtract the observed angle APB = 19°, and there remains the angle PAB = 24° 5′ 14″.

Again, in the triangle APB the side AB = 8, the angle APB = 19°, PAB = 24° 5′ 14″, and ABP, their supplement, = 136° 54′ 46″. From these find the side AP = 16·7857, and the side PB = 10·0286.

And, lastly, in the triangle DCP, the angle DPC or BPC = 25°; then, because the opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles (Euc. iii. 22), if we subtract the angle PAB 24° 5′ 14″ *plus* the whole angle DAB = 60° 34′ 38″ from the sum of the angles DCP + DAP = 180°, it will leave the angle DCP = 95° 20′ 8″; consequently the supplement PDC = 59° 39′ 52″. With these angles and the side DC = 7·3 find the side PC = 14·9095.

The station-point will also be determined correctly, by describing on the sides AB and BC segments that shall contain the angles subtended by those sides. For example:—With G as a centre, describe a segment that will contain 19°, being the angle measured between the peaks A and B, and the circle will pass through the points A and B, and the station point P, whence the angle was observed, will also lie in that circle. Then, in like manner, with R as a centre, and radius RB or

RC, describe a segment that will contain 25°, being the angle subtended by the peaks B and C, and this circle will intersect the former circle in the station point P; then by joining the points PA, PB, PC, we obtain the same distances as before.

TO SURVEY A BAY OR HARBOUR.

WISHING to make a survey of a small bay, but not finding any conspicuous objects on shore by which I could carry on the survey and determine my position while sounding round it, I measured a base line, MN (fig. 83), 2140 fathoms, on a convenient part of the shore; after which I placed three buoys A, B, and C, forming a triangle, and in such a position as I supposed would enable me to see them from all parts of the bay. Then, from one end of the base line, N, I took with a sextant the angles ANM = 39°, BNM = 65°, CNM = 83° 10′, next, from M, the other end of the base line, I measured the angles C M N = 36°, B M N = 55° 30′, A M N = 76°. With a large circular protractor I protracted the measured angles respectively from each end of the base, then drawing out the lines MA, NA, MB, NB, MC, NC, they intersected in the points A, B, and C, which gave me the fixed position of the three buoys. Then to find the sides by calculation:—

In the triangle ANM, there are given the side MN = 2140, the angle AMN = 76°, and ANM = 39°, consequently the angle NAM = 65° being their supplement; therefore find the side MA = 1995·5, and the side NA = 2148·5.

Then, in the triangle CNM, in like manner, with the three angles C N M = 83° 10′, C M N = 36°, and their supplement NCM = 60° 50′, and the side MN = 2140, find the side NC = 1992·25, and the side MC = 2158·5.

Next, in the triangle NCA, there are given the two sides NA = 2148·5, and NC = 1992·25, and the included angle ANC = 44° 10′, being the difference between CNM = 83° 10′ and ANM = 39°; therefore find the side AC = 2106.

In the triangle BMN, there are given the angle BMN = 55° 30′, the angle BNM = 65°, consequently the angle MBN = 59° 30′, and the side MN = 2140; therefore find the side NB = 2131.

Next, in the triangle ANB, there are given the side AN = 2148·5, the side NB = 2131, and the included angle ANB = 26°; therefore find the side AB = 1065·5.

In the triangle BNC, there are given the side BN = 2131,

the side NC = 1992·25, and the included angle BNC = 18° 10′; hence find the side BC = 1053·25.

Having now determined, both by calculation and protraction, the positions of the three buoys, with their mutual distances from each other, I next proceeded to determine the position of a rock at G, which was just level with the water's edge. I landed on it, and with a sextant measured the angle between the buoys B and C, which was = 13° 30′, and between A and B, which was = 29° 50′. Then, according to Problem V. (fig. 75), we have a triangle formed by the buoys, with its vertex, B, towards the observer; consequently, to find the position of the rock from the measured angles, and their mutual distances, proceed as in the last part of the protraction of (fig. 82), by describing on the sides BC and AC segments, such as to contain the angles respectively subtended by those sides; and these two circles, when described, will intersect each other on the surface of the rock at the point whence the angles were taken, and, by calculation, the distance from the rock to the buoy at A = 2131, to B = 2107·4, and to C = 2151·4.

As I was anxious to make use of this newly fixed object in the course of my operations, I caused a signal-post with a flag on it to be erected on the spot from which I had measured the angles. Then, as a proof how far I had been correct in protracting and calculating the distance of the rock from the buoys, I proceeded to B and measured the angle between A and the signal-post which I had just put up, which was = 97°; and from A I measured the angle between B and the flag-staff, which was = 53° 10′. After a careful protraction of these two angles, the lines intersected each other exactly on my former station, and by adding these two angles to the angle which I first measured between A and B, namely 29° 50′, their sum will be found equal to 180° precisely, which is a clear demonstration of the truth of these observations.

I next proceeded to the extreme S.E. point of the bay, D, where I measured the angle between the flag-staff and B, which was 29° 30′, and between A and B, which was 26° 30′. Then by Problem V. (fig. 72), we have a triangle formed by three objects, with its vertex turned from the observer; therefore, by either of the given methods, protract and calculate the distances, and should any material difference exist, go over the protraction and calculation again, until you discover the error, and determine the true position of the point D on the paper, always placing greater confidence on the calculated distances than on the protraction of the figure.

The point D being now fixed, take the bearing of *e*, the bluff part of the point, from D, which happens to be on with A, and draw out the line on your rough sketch, which ought to be prepared for the purpose. If no conspicuous part of the point in that direction can be observed, so as to be known again, set up a pole and trace the outline of the beach between D and *e* as correctly as you can. From D take also a correct bearing of the N.E. extreme point of the bay, P, and draw it out; take also an angle between it and B, $= 47° 30'$; then go on to *e* and take a correct bearing of the flag-staff on the rock, draw out the line, and it will intersect the line D *e* in *e;* look back towards D, and see that the outline between these two points is properly laid down. As it would be an endless, and indeed unnecessary task, to protract and calculate the distances of every little point and curve along the coast, according to the methods given, it will be absolutely necessary to have a compass of the best description, with good sights and a tripod-stand for observing the bearings, as done from D to *e*, and to note them with the utmost caution. The prismatic compass will be found an admirable instrument for this purpose. It would also greatly facilitate the delineation of the portions of the coast from station to station, if the surveyor could guess at the different distances, within a slight deviation of the truth, by the eye alone, after the bearing has been taken and the line drawn out, as it affords a good guide to him in filling up the outlines, by at once commencing from the two extremes, which cannot be done if an indefinite line of bearing is merely drawn out. From *e* take the bearing of the small point *f*, and also mark on the sketch where that line of bearing cuts the shore at *h*. Go on to *f*, make a rough guess at the distance between *f* and *h*, and lay them down from your original scale of equal parts, and these two points will enable you to fill up the coast between them pretty correctly by the eye; and, because the bearing of the signal-post on the rock would fall too obliquely on the line from *e* to *f*, rather take the bearing of B, which will give you the true position of *f* on your paper. Correct the outline between the last two points, and go on to *g*, which is a small creek, correcting the curvature of the shore as you go on. From *g* it will be observed that the two buoys, B and C, are seen in a direct line with each other; consequently measure the angle between A and B $= 36° 10'$, and by Problem II., protract and calculate the distances, *g* A $= 1991{\cdot}25$, *g* B $= 2111{\cdot}10$. Take a bearing of the points *i* and *k* from *g*, lay their distances down by the eye as near as

you can from your scale, and trace upon your sketch the representation of the line of shore between them. Go on to *h*, and determine its position by taking the bearing of A, which line also intersects the line of bearing *gi*, in *i*; look towards *g* and *f*, and correct the curvature between the points *f*, *g*, *h*, and *i*. From *j*, with the two buoys A and C on with each other (as in Problem II.), take the bearing of *i*, which will fix the point *j*, then fill up the curve between *i* and *k*, which is a rocky point. The line *g k* is intersected at *k* by a line produced through BA, the two buoys forming that side of the triangle. It will also be observed that the same line, produced towards the N.E. point of the bay, exactly passes over the extreme at P.

At *l*, the beach being very low and sandy, cause a pole to be put up, without which the point of bearing could not be found out when you advanced to it. Take now its bearing, and also the bearing of the rocky points *m* and *n*, where poles ought also to be set up; fix the point *l* by a bearing of the buoy C, then take the direction of *m* from *l*, and having traced out the curve *k l m*, from *m* take a bearing of *n*. Then, to corroborate this last intersection, from *n* take the angles between B and C = 33° 10′, B and A = 42° 30′; then, by Problem V. (fig. 72), protract and calculate the distances *n* C = 1885, *n* B = 1995·10 and *n* A = 1885, which agrees with the position given to *n* by the magnetic bearings *m n*, and *k n*. The operations, so far, may therefore be considered correct, after the coast has been correctly traced in between *m* and *n*. Next, take the bearings of *o*, *p*, and *r* from *n*, then fix *o* by a bearing of the buoy B, intersect the line *n p* by the bearing *o p*, and trace the curve *n o p*. From *p* take the bearing of *q*, proceed to *q* and fix it by the bearing B *q*, which also fixes the point *r* at the intersection of B *r*, *n r*; draw the curve *p q r*, and from *r* take the bearingr of *s* and *t*, the two points at the mouth of the creek; judge the distances and sketch the curvature and entrance to the creek; fix the points *s* and *t* by bearings of the buoy C; set up poles at *u* and *v*, and take their bearings from *t*, the point *u* will be determined correctly by drawing out the side formed by the buoys B and C, which will intersect the line *t u*, in *u*. The point *v* will in like manner be found by producing the side AC, which will intersect *t v*, in *v*. These may also be verified by a protraction and calculation, according to the method in Problem II., by measuring the angle between C and the flag-staff on the rock to the left, to fix the point *v*, and either the angle between C and the flag-staff to the left, or between

C and the buoy A to the right, to determine the position of *u* on the sketch, after which trace in correctly the outline from *t* to *v*. Take the bearing of the small point *w*, and determine it by the direction of C. Next, take the bearings of *x* and *y* from *w*, the latter of which will also pass through the extreme point P; fix *x* by a bearing of B, and mark the direction of *y* from it, and trace in the curvature between *w* and *y*, then determine *y* by the bearing of B, and correct the outline from *w* to *y*, and complete it to P. Then, to obtain correctly the true position of the point P, take the angle subtended by B and C = 21° 11′, and, by Problem II., protract and calculate the distances BP = 2148.25 and CP = 2126·18; or the angle between the signal-staff and B, = 37° 30′, may be used, which will (by the same problem, A and B being on with each other) give the distance between the flag-staff and P = 2171·25 and PB as before = 2148·25; or, so far as a magnetic bearing can be relied upon, the bearing of B and C, or B and the flag-staff, as explained under the article (fig. 56), should give the position of P; but, as P is a material point in the survey, it ought (by all means) to be determined by trigonometrical calculation. From the station P measure the angle DPB = 40° 40′, and at B take the angle PBD = 91° 50′; their sum + PDB = 47° 30′, being equal to 180°, shews that the observations are correct, and by calculation the side PB = 2148·25, BD = 2129·75, and DP = 3203.

The outline of the bay being now correctly filled in between the various points, the next operation is to lay down the soundings and shoals, which are to be determined exactly in the same manner as the different points along the shore, by a protraction of the measured angles between the fixed objects; but as it will only be found necessary to have recourse to logarithmic calculation when a rock or the extremes of a shoal are to be laid down, a very expeditious method of laying down or protracting a line of soundings is by means of the *Station-Pointer* (fig. 86), of which useful instrument a description, with the method of using it, is as follows:—

The Station-Pointer is formed by three limbs, A, B, C, which move upon a common centre in such a manner that B and C can be set to form any angles with A. The middle limb, A, has two verniers, *a* and *a*, attached to it, which are adapted to two graduated arcs, *b* and *c*, connected with the limbs B and C, and moving with them. The use of the graduated arcs *b* and *c* is to set the limbs of the instrument to any given angles; and, that this may be done with greater accuracy than if the edges merely were used, a fine wire is stretched from the common

centre along each limb, the angles formed by the wires being the required angles. Through the centre of the instrument there is an opening sufficient to admit a steel pricker; and when the instrument is used, if the pricker be pressed into the paper, it will mark the required station. We shall illustrate the use of the instrument by the following example:—

Being at the entrance of a bay, and wishing to determine the position of a sunken rock, I anchored a boat over it, and with a sextant took the angles subtended by three objects on shore, D, E, F, whose distances were known by chart. These angles were DPE $= 40° \ 25'$, FPE $= 62° \ 12'$, P being the point of station of the boat. To mark the position of the boat in the chart, open the limbs of the station-pointer, and set them, by means of the circular arcs *b* and *c*, to the observed angles; then place the instrument on the paper, and move it till the wires on the limbs pass through the three given points, D, E, F. By means of the steel pricker mark the centre, which will give the position of the sunken rock.

From the above description, it will be seen that the *station-pointer* affords an easy method of protracting angles, and of laying down on paper any number of soundings whatever, without reference to the rough sketch when they were obtained, as it will only be necessary to note the direction of the ship's or boat's head, and the position of the objects, in a memorandum-book, or on the page of the log-book, when the soundings and angles were taken. Neither will it be found necessary to mark down (nor indeed would the sketch or chart contain) every separate cast of the lead that might be taken during the operation; but it will be of great advantage and assistance, in filling up the draught, to have an officer to note on the log-slate every cast that is taken; and at the instant any change takes place in the depth, either *increasing* or *decreasing*, let two persons immediately measure the angles between the three objects, and note them down, following the cast of the lead, which will give the vessel's position; and the regular or steady casts can be marked down on the line of sailing at proper distances from each other, between every *changing* cast so fixed by the protracted angles. It will be also of the utmost importance if a careful look-out be kept for the coincidence of any two of the objects *in any part of the bay*, which must be frequently occurring while cruizing about; in which case, by Problem II., a single angle between any other object, either left or right, will readily give the station of the ship. Before we proceed to protract the soundings on our present sketch of

the bay (fig. 84), it will be necessary to rub out the whole of the lines and intersections which were used for determining the points along the shore, to prevent a confusion of lines, the three buoys and signal-staffs being alone preserved for the procedure.

Suppose, then, the first course is from the extreme point of the bay D, directly towards P, the other extreme point, the soundings, angles, courses, and remarks, may be noted down on the log-slate, and afterwards entered into the log-book, in the manner shown in the formula hereafter given; or they may be noted as follows:—Supposing the base line MN to be in a due north and south direction, the ship's head N. 16° W. on the larboard tack, the wind W. by S. to W. ½ N., extreme point P right a-head, soundings 12, 12, 12, 12, 13, where the flag-staff on the rock was on with the buoy A, B being to the right of A at the same instant 27° 25′. By protracting this angle according to Problem ii., we obtain the position of the ship, when the soundings increased or deepened to 13 fathoms. Then, again, 13, 13, 13, 13, 13, 13½, A left of B 13° 45′, C right of B 28°; hence, by Problem v. (fig. 75), where the vertex of the triangle falls towards you, find the ship's position when this cast 13½ was taken. 13½, 13½, 13½, 13½, 13½, and 14, B nearly on with A, tacked ship, sent the boat in shore to sound, 14, 14. Boat deepened into 14½ close in shore, with the buoys A and B exactly on with each other; C right of B 21° 10′ 30″.

I have here at once introduced the practical method of transferring to paper the line of soundings, B P, as they were obtained, which, of course, could not be done whilst the survey was actually going on, without the aid of several assistants. The soundings and angles are generally put down as above, during the day, on the log-slate on board both vessels, from which they are copied off, and handed in to the head surveyor every evening.

The next course, PB, is S. 25° W., A and B on with each other, soundings 14½, 14½, 14½, &c., 14, C right of B 37° 10′. By Problem ii. find the station. Then 14, 14, 14, 14, &c., 13¼, C right of B 81°. Again, 13¼, 13¼, 13¼, 13¼, &c., 13. Tacked ship close to B and stood towards C N. 29° 30′ W. 13, 13, 13, 13, 13, &c., 12¾, A right of B 33° 20′; here we have the station in one side of the triangle formed by the objects: therefore, by Problem iii., find the position. 12¾, 12¾, 12¾, &c., passed close to C. 12¾, 12¾, &c., 12½, C on with B, and A to the right 22° 10′. Tacked ship close in shore and stood S. 25° W.; small point at *n* a little on the starboard bow, soundings 12½, 12½, 12½,

&c., 12, c left of B 43° 40′, A right of B 43° 50′; then, by Problem v. (fig. 72), when the vertex of the triangle is turned from the observer, find the station. Then 12, 12, 12, 11, c left of B 36° 20′, A right of B 47° 12′. 11, 11, 11, &c., 10, c left of B 33° 50′, A right of B 45°; bore away and stood over E. 34° 30′ S.; small creek at *g* a little open to the left of A. 10, 10, 10, &c., 9½, c left of B 37° 20′, A right of B 66°. 9½, 9½, 9½, 9½, &c.; passed close to A, 9, 9, 9, 9, 9, &c., 8¼, B right of C 11° 10′, A left of c 41° 50′. 8¼, 8¼, &c., 7½, B right of c 5° 50′, A left of c 34° 30′. Hauled round to N. 25° E.; when close in shore, between *g* and *h*, 7½, 7½, &c., 8; B and c on with each other; A left of B 50° 30′. 8, 8, 8, &c., 9; A left of B 38° 25′, c right of B 19°. 9, 10, A left of B 35° 30′; c right of B 22° 13′. 11, 12, 13; A left of B 20° 56′, c right of B 23° 45′. The last two angles place the vessel again in the line DP, at the point F. Another line, *n*F, would be marked down in the same manner; and all casts of the lead obtained within the triangle formed by the buoys would be laid down by Problem iv. Of course, it cannot be expected that the actual number of casts taken during a cruize round a bay of the description represented in our figure, should be entered individually in the above work. A few have been noted, which it is hoped will sufficiently illustrate the method of conducting the survey. Indeed it would be impossible to render the process more familiar to the understanding of the young surveyor, by protracting more soundings in the figure, as it would only create difficulty by setting before him a complicated labyrinth of intersections. The whole of the surveying problems have been already in the above process called into practice, and it will immediately occur to the young officer, that, so soon as he has protracted with care a single line of soundings, such as DP, and marked in the figures with ink, he may that instant remove from his chart with Indian rubber the pencil lines and arcs by which he obtained the direction of that line, and at once proceed to lay down a second line, for instance PB, and so on, from line to line, until a sufficient number have been obtained; but this cannot be done in our present *draught*, they must be preserved like the lines in the demonstration of a proposition, until the reasoning is complete and satisfactory:—and it is to be hoped that figs. 83 and 84 will not, from their formidable appearance, prevent the student from patiently unravelling the various links by which the whole process is bound. If the station pointer be used, there will be no necessity for lines. In the foregoing example in the practice of Marine Surveying,

it must be understood that the whole of the operations were carried on in fine weather, that the buoys were perfectly steady and upright, and, being in a bay, it is supposed that they were uninfluenced by either tide or current, and maintained an unchangeable position during the time the survey was going on. It would be satisfactory to employ two needles in going round the shore, as a check on each other, as it will frequently be found that the same rock, hill, or stone, does not contain the same magnetic attraction on all parts of its surface; but at the distance of a few feet sometimes from where the attraction was found most powerful, the needle will be observed to rest quite undisturbed by its effects.

To make a survey of the entrance of a river, and to fix several remarkable objects, by which the survey may be continued along the coast.

From the summit of the hills F, D, or A (fig. 85), first take a general view and outline of the coast, and mark down, as near as the eye can estimate, the most remarkable objects, such as the hills at A, B, C, D, E, F, G.

Then, supposing that no level plane can be found conveniently situated for a base line, determine the distance between A and C, or F and D, by the result of several experiments made by *sound*, choosing the most favourable opportunity to conduct the operations; or the latitudes of two remarkable objects, making a small angle with the meridian, whose distance from each other is about twenty miles, may be carefully determined with an artificial horizon, and from these latitudes the true distance between the objects in miles and parts of a mile may be found, (page 101).

Then, having obtained the distance between any of these objects, either by velocity of sound, or by difference of latitude, next proced to take the angles between the different hills and each object at the two extremes of this newly measured *base line;* then take the magnetic bearing of the two objects, and from a scale of equal parts, lay them down in a convenient part of a sheet of paper, not using less than one inch and a half to a mile; draw out the angles, and mark their intersections, also protract the magnetic meridian, and determine the angle contained between it and the true meridian, by an amplitude or azimuth, or by the methods given for finding a meridian line, page, 108; the true bearing of any one object being thus found, the true bearing and direction of the *base line* and other objects may be easily laid down. But should it so

fortunately happen that an extensive plane could be found, then measure the *base line* PR, and let it run in such a direction that all the objects may be fixed with precision by the angles taken from its extremes, agreeable to the instructions laid down in page 91, on the measurement of a base line; then in your field-book make your remarks, and mark the angles down as follows:—

Fixed on a point P on the north side of the river (fig. 85), and measured a base line in the direction PR, equal to 3080 fathoms, or 3½ miles, the magnetic bearing of the station R from P being E. 5° 30′ N.

ANGLES TAKEN AT STATION P, LEFT OF PR.

CPR	36° 50′
BPR	67 10
APR	75 12

ANGLES TAKEN AT STATION P, RIGHT OF PR.

DPR on with point *g*, . . .	44° 15′
*a*PR a sharp projecting point, north side,	50 40
EPR on with the mouth of creek *bc*, .	67 50
FPR on with rocky point *d*, . .	94 55
GPR	99 13
*k*PR southern point, entrance to river, .	105 30

ANGLES TAKEN AT STATION R, RIGHT OF RP.

CRP	95° 57′
BRP	76
ARP	44 11

ANGLES TAKEN AT STATION R, LEFT OF RP.

DRP	100° 30′
ERP	78
*c*RP eastern point of creek mouth, .	71 55
*b*RP western point of creek mouth, .	67 50
*a*RP projecting point on north side of river,	59 30
GRP	56 49
FRP	47 48
*d*RP rocky point, on with a small point *g*,	37 59
*k*RP southern point at the entrance to the river,	30 2

The angles between every remarkable object, and the extremes of the *base line* being now taken—First, Draw out the lines that are to fix the principal objects for carrying on the future survey; RC and PC, RB and PB, RA and PA, will, when protracted so as to contain the respective angles, intersect each other in the points A, B, C, being the summits of the three hills on the northern side of the *base line*. In like man-

ner RD and PD, RE and PE, RG and PG, RF and PF, when drawn out, will fix the objects D, E, G, F, on the southern side of the *base line;* R*k* and P*k*, will also determine the southern point *k*, at the entrance of the river. Next proceed to find the distances of the principal objects by logarithmic calculation.

In the triangle RPC, we have the angle CPR = 36° 50′, and the angle CRP = 95° 57′, consequently PCR = 47° 13′ and the side PR = 3½ miles, therefore find the side RC = 2½ miles and 352 fathoms, and PC = 4½ miles and 220 fathoms. In the triangle RPB, the angle BPR = 67° 10′, and BRP = 76°; consequently PBR = 36° 50′, and the side PR = 3½ miles. Therefore find the side RB = 5 miles and 242 fathoms, and PB = 5½ miles and 88 fathoms.

In the triangle BPC, the sides PB and PC are known, and the contained angle CPB = 30° 20′, being the difference between BPR and CPR; therefore find the distance between the objects B and C = 2½ miles and 264 fathoms. Next, in the triangle PAR, the angle APR = 75° 12′, and ARP = 44° 11′, and the side PR = 3½ miles; therefore find the side RA = 3½ miles and 264 fathoms, and the side PA = 2½ miles and 264 fathoms. In the triangle BRA, the two sides RB and RA are known, and the included angle BRA = 31° 59′, being the difference between BRP and ARP; consequently, find the distance between the objects B and A = 2½ miles and 308 fathoms. In the triangle RCA, the two sides RC and RA are known, and the included angle CRA = 51° 46′, being the difference between ARP and CRP; therefore find the distance between the objects A and C = 3 miles and 25 fathoms. In the triangle RDP the angle DRP = 100° 30′, DPR = 44° 15′, consequently RDP = 35° 15′, and the side RP = 3½ miles; therefore find the side RD = 4 miles and 154 fathoms, and the side PD = 5½ miles and 330 fathoms. In the triangle REP, the angle ERP = 78°, EPR = 67° 50′, consequently REP = 34° 10′, and the side RP = 3½ miles; consequently find the side RE = 5½ miles and 176 fathoms, and the side PE = 6 miles and 18 fathoms. In the triangle RDE, the sides RD and RE are known, and the contained angle DRE = 22° 30′, being the difference between DRP and ERP; hence the distance between the hills D and E will be found equal to 2 miles and 352 fathoms. Next in the triangle RGP, the angle GPR = 99° 13′, GRP = 56° 49′, consequently PGR = 23° 58′, and the side RP = 3½ miles; therefore find the side RG = 8½ miles, and the side PG = 7 miles and 194 fathoms. Then in the triangle RGE, the sides RE and RG are known, and the included angle ERG = 21° 11′, being

the difference between ERP and GRP; hence find the distance between the hills E and G = 3½ miles and 264 fathoms. Again, in the triangle RFP, the angle FRP = 47° 48′, and FPR = 94° 55′, consequently PFR = 37° 17′, and the side RP = 3½ miles; hence find the side RF = 5½ miles and 176 fathoms, and the side PF = 4 miles and 176 fathoms. Next, in the triangle RFG, the sides RG and RF are known, and the included angle GRF = 9° 1′, being the difference between GRP and FRP; therefore find the distance between the hills F and G equal to 3 miles. And last, in the triangle R*k*P, the angle *k*RP = 30° 2′, and *k*PR = 105° 30′, consequently P*k*R = 44° 28′, and the side RP 3½ miles; hence, find the side R*k* = 5½ miles and 132 fathoms, and the side P*k* 3 miles and 42 fathoms. The positions of the hills being thus determined by logarithmic calculation, make a *dot* at each point, and encircle it by a black line to render it more conspicuous, and write the name of the object it is intended to represent near it; then completely erase the intersecting lines RD, RG, RA, &c. &c., that the paper may be clear for the protraction of the other angles.

Next, the distances between the hills CD, CE, CG, GA, GB, &c. &c., may be found by calculation, which completes the scheme of points, for carrying on the survey along the coast and in the harbour. The hill at H lying too oblique to the *base line*, its position will be best determined by measuring the angles RAH and ARH, after the position of A has been found; by this means a more direct intersection is obtained.

The paper may again be cleared of its lines and pencil marks, and several lines parallel to the magnetic meridian may be drawn. These lines are of great service in protracting angles, &c., and afford great facility in laying down a magnetic bearing, when filling up the little points and curvatures along the coast. Then draw out the lines R*c*, R*b*, and they will intersect the line PE in the two points *c* and *b*, at the entrance of the creek; drawn out R*a*, P*a*, which will fix the sharp projecting point *a*. Next draw out the line R*d*, and it will intersect the line PD in the point *g*, and PF in the rocky point *d*. Then as *d* and *a* are two conspicuous points that ships must pass, calculate their distances and lay them down. Determine the point *e* by measuring the angles F*e*E and F*e*H; then trace the curvature roughly by the eye, and take the bearing of any conspicuous mark at *f*. Measure the same angles, and by the problem which determined the point *e*, find the true place of *f*, and correct the curvature from the point *k* to *e*; or the bearing of E would have been, for most purposes, sufficient for

finding *f*. The points *k*, *d*, *b*, *c* have already been fixed on the paper, by angles taken from both extremes of the base line; therefore determine by bearings the extent of the curve between these points, and trace it with a pencil; sketch the mouth of the creek, and take the bearing of *h*, and guess its distance as near as you can, and fix it by a bearing of D, which also cuts the point *j*. Trace in the line of shore between *c* and *h*; from *h* take bearings of *i*, and points above it, in the same manner, as far as you intend to extend the survey. Fix the point *i* by bearings of E and D; from *i* take a bearing of *j*, and sketch the curvature between them. After you have determined its extent by bearings, verify the position of *j* by bearings of *a* and F; go round the curve between *a* and *j*, and find its extent; sketch it, and find the sharp curve or deep *bight* at *l*, and trace it correctly in from *g* to *a*. From *a* take bearings of *g*, and the north point at the entrance to the river *p*, and go carefully round the winding coast between them, fixing by bearings the most remarkable curves and points; determine the point *p* by one of the problems, and calculate its position from *k*, the opposite point of the river mouth. Lay down the coast from *p* to *s* in the same manner, and determine *s* by the measured angles between A and C, and B and C. The point *t* may likewise be easily determined by Problem ii., for the hill at B is in a direct line with the hill at C, the angle between B and A to the right will give the position of *t*; then trace in the coast between *t* and *s*, and the land part will be completely laid down. But before commencing a survey of the coast and harbour, to lay down the soundings, islands, rocks, &c., it will be necessary to trace in the line of coast on your chart paper with Indian ink, marking at the same time the places of the different hills with ink also. The next operation is to lay down the soundings, which can be best conducted on a weather tide, by standing from side to side, with either a fair or a working wind. Suppose, by our present figure, that the operation is commenced between the points *i* and *j*, with an ebbing tide, and wind from the W.N.W, the first tack to the S.W. towards the point *h*; then note down carefully the casts 4, 8, &c. of the lead, and take angles between F and E, G and E being *on* with each other; then by Problem ii. protract this line of soundings, and proceed in the same manner towards the mouth of the river, watching particularly for the coming on of one hill with another, also making such observations and remarks as will enable you to give proper sailing directions, both for working or running in and out of the

harbour, and be particular in sketching the different appearances of the land, such as rocky or sandy coasts, hills, towns, churches, or trees, that may serve for leading marks for clearing rocks and shoals, &c. Buoys, if you have them, must also be placed on the different spits or shoals, after carefully sounding round each in the boat, where it would not be prudent to approach with the vessel. To determine and lay down the extreme of the reef of rocks near the entrance of the harbour, great care and labour must be bestowed from the dangerous position which they occupy. At low water let the boat be anchored on the very extreme of the reef; then with a sextant take the angles between G and F, and F and D, also between D and H, and H and B; then first find the station of the boat by the first angles, which is a case where the vertex of the triangle is towards you, verify the result by a protraction of the angles subtended by FD and DH, which is a case where the vertex is turned from the observer; and lastly, protract the angles subtended by DH and HB; then, if they all coincide, calculate the distance of at least *three* of the objects, suppose H, D, and F, and if the calculated sides agree with the protracted sides, the station has been correctly determined, and may be laid down on the chart. The soundings along the edge of the reef can be found with the assistance of the boat also. Next stand off and on the coast, and determine in the same way the soundings, and mark down the different appearances of the land, and best leading marks for sailing or working into the harbour. Land on the extreme points of the island, and fix your Theodolite, and measure the angles between all the objects and those points which are sufficiently conspicuous to be distinguished; fix the extremes of the island by protracting and calculating the angles subtended by GE, FE, ED, DF, &c., go round the island and fill up the intermediate spaces by bearings, then lay down the other angles which you took with the Theodolite, and see whether they will intersect their respective points from the position given to the island by protraction, &c. of the above angles: if not, go over the work until the error is detected. When on shore, should any of the fixed hills be hid by others that have not been determined, or should it be judged that the fixed object is not conveniently situated for determining any point along the shore, then proceed to the object so intervening, and by any of the problems find its position, and it will supply the place of those obstructed from view. Thus at all times a difficulty of this nature may readily be removed by pitching upon adjacent objects from

which the fixed objects can be more advantageously seen, to assist us in determining a station on the shore. It may sometimes happen that the angle between two of the fixed objects cannot be *directly* had, in consequence of some interposing object, in which case proceed as follows:—Suppose you required the angle CRA (fig. 85), but at N, in the direction RC, a long low eminence intervened, but of sufficient elevation to hide the object at C entirely; then proceed to N, and erect poles on each side of it, until, by shifting them, you can fix a pole at N exactly in the line RC; then the angle between this pole and A is evidently the angle required. It is a more tedious operation than would at first be imagined, to find a point such as N between two determined extremes, R and C, even on a plane surface, seeing that you have but the breadth of a *line* to hit upon. It of course becomes doubly so, where the required point is on an eminence, because you cannot so conveniently shift the poles. The survey may be continued along the coast, both north and south, until the angles subtended by the different fixed objects, A, B, C, &c. become *too acute*, which must evidently take place as we recede from them. It will be necessary, before this actually happens, to determine the position of other objects, either by measuring another *base line* on shore, if it can be done, as in the previous cases, or by the velocity of sound between two vessels at anchor off the coast; or by proceeding to the summit of other objects, their position may be determined by any of the problems, after the angles subtended by some of the former fixed objects shall have been measured; and so the series of triangles may be continued to whatever distance it may be required to extend the survey; observing that it would be advisable, in consequence of an error which must arise from the difficulty of measuring the angles to a sufficient degree of nicety, the imperfection of instruments, and still more the accumulating error in protracting, &c. to determine the position of a new set of objects at proper distances along the coast. Should a suitable plane present itself for a new *base line*, some of the former objects must be included in this new scheme, and by a judicious arrangement they may be easily connected together.

Directions for running into the Harbour. (Fig. 85.)

On sighting the land, run in for the harbour (which you will know from your latitude), keeping the hill at F on with a remarkable sharp peak at E; and on approaching the land,

the soundings will gradually decrease to 9 fathoms, distant about 2 miles from the shore at *k*. Should the flood-tide be running, which sets directly across the mouth of the river at a rapid rate to the northward, it will be necessary to make due allowance on that account, by keeping the peak at E well open to the right of F, which will lead the ship clear of the extreme point of the reef of rocks, which extend from the northern point of the river. When abreast of it, you will have a few casts of 8 fathoms, and then deepen again into 9 and 10 fathoms.

When the hill at A is open to the right of H, steer E. by N. until you bring the hill at D a little open to the left of the point at *d*; then, if the wind hangs to the southward, haul up east, and round the sand between *d* and *b* in 6 fathoms, observing not to shut the point *k* in with the point *d*, but keep them open until the hill at C is open to the right of the point at *g*, you will then be clear of and above the *spit* of the sand, and may run into the anchorage close in shore above the point at *a* in 8 fathoms. The bottom stiff clay, and good holding ground; high-water at $3^{h.}$ $30^{m.}$ P. M. full and change of the moon; rise of tide 14 feet.

At L is given a view of the land between *t* and *p* on approaching the harbour, bearing E. by N. distant about 4 miles; and at M will be seen a view of the town from the anchorage. The harbour is extensive, well sheltered, and capable of containing 200 vessels of heavy tonnage, under a strong fortification, with abundance of fresh water.

After finishing the survey, it was necessary to determine the true direction of the *base line* with regard to the meridian, also the variation of the compass. Consequently from R, at sunset, the angle between the sun's south limb and P (a conspicuous mark at the other extreme of the base line) was found to be 25° 46′ when the sun's centre was considered to be on the horizon, making due allowance for the effect of refraction; then subtracting the sun's semidiameter from the measured angle, we obtain the correct angle between P and the sun's centre, = 25° 30′, and the true amplitude by the methods given at page 105 was found to be W. 10° N., which, subtracted from 25° 30′, gives the true direction of the *base line*, W. 15° 30′ S. and the variation of the compass 10° west.

The soundings were taken between the first quarter ebb and low-water, on account of the irregularity of the fall of tide during the first quarter ebb, which is much greater than at any other part of the tide, especially in rivers, where the

tide runs at a rapid rate, and has a considerable rise. In most cases, however, it will be found sufficiently correct (where the water is not very shallow) to deduct $\frac{3}{4}$ of the full rise from all soundings taken during the first quarter ebb; and at half ebb deduct half of the whole rise from the soundings then taken; at the last quarter ebb deduct $\frac{1}{4}$ of the full rise: but from various causes, which operate on the tides, such as the positions of the sun and moon with respect to each other and the earth, also gales of wind, and other extraordinary causes, it would be difficult to give any fixed rule for reducing the soundings taken during a rising or ebbing tide to low-water, which ought to be the case when the water is very shallow. It must be the business, therefore, of him who undertakes the survey of a harbour or river to ascertain by every exertion and experiment the full rise during ordinary spring-tides, by which, and other circumstances, he must regulate his scale of soundings accordingly, being at all times careful to mark down on his chart less water, rather than give a greater depth than there actually is, by which serious accidents to shipping might occur. It is necessary also to note particularly the direction of the tide at the different quarters, and the number of hours of ebb and flow, respectively.

To survey a coast which lies in an east and west direction.

Let fig. 87 be part of a coast extending in an easterly and westerly direction; then, if no suitable plane can be found on which a *base line* can be measured, anchor the two surveying vessels as nearly east and west from each other as possible, making allowance for the variation of the compass when you are about to bring the ships to an anchor. Suppose A and B are the stations taken by the vessels, and so situated that the different islands outside of them may be correctly laid down.

Then let several guns be fired from each vessel alternately, whilst two or three observers with chronometers, or if possible with a stop watch, such as described at page 97, note the exact interval between the flash and report of the guns. Suppose the result of several experiments gave a distance of seven miles between the ships; the operation must of course be conducted in calm, clear weather, and to prevent the vessels from swinging about, it will be necessary to ride with as little scope of cable as possible; when the intervals have been all carefully noted on board each ship, next let the signal "prepare to take angles" be made on board the senior officer's vessel, and when answered by the assistant, the signal should

be hauled down, and another flag should be hoisted at the main royal mast-head, to signify that the operation of measuring the angles is to commence. The flag must be kept flying until the whole of the angles on the one side of the *base line* shall have been completed by both vessels at the same instant, and commencing with the same object from both extremes, if possible, which will have a tendency to counteract any errors in the angles that might arise from the motion of the vessels, while the measurement is going on. One assistant should mark down the angles and remarks, whilst the other measures and reads them off. The following angles in our present figure will be noted thus, commencing from A, which we will consider the station of the senior officer's ship:

Angles taken at A *to the left of* B, *the assistant vessel's main-mast.*

C	a hill to the left of B, ...	110°	50′
D	a bluff hill to the left of B, and on with a hill on island *a*,	85	50
E	a round peak to the left of B, ...	63	45
F	a high peak to the left of B, ...	54	10
G	a remarkable sharp-pointed peak, and on with two rocks *b* and *c*,	45	10

Angles taken at B *to the right of* A, *the senior officer's main-mast.*

C	to the right of A, and on with the hill on the island *a*,	43°	20′
D	to the right of A, and on with rock *b*, . . .	67	
E	to the right of A, and on with rock *c*, . . .	87	15
F	to the right of A, ...	104	
G	to the right of A, ...	108	16

The angles being finished on the north side of the *base line*, the flag may be hauled down, and the former preparatory signal may again be made until answered by the assistant vessel; and when the signal flag is again seen flying at the senior officer's main royal mast-head, commence taking the angles on the south side of the *base line* as follows:—

Angles taken at B *to the left of* A, *the senior officer's main-mast.*

H	a peak on long island to the left of senior officer's main-mast,	31°	40′
I	a hill on round island, on with a peak *d*, and two other small islands *e* and *f*, ...	47	47
K	a conical-shaped hill on a large island, ...	67	
g	and *h* two small islands on with each other, . . .	86	52
L	a peak on high island, left of A, ...	106	6

Angles taken at A *to the right of* B, *the assistant vessel's main-mast.*

f and *h* two islands on with each other to the right of B,	23° 4′
e and *g* on with high island L, ...	36 20
d on with conical hill at K, ...	68
I round island hill, ...	92 2
H peak on long island, ...	111

After all the angles have been taken, also a sketch of the coast and islands, next determine the true direction of the *base line* by an amplitude, as in last example. The vessels may then weigh and close, and it will be the duty of the assistant to furnish the senior officer with a copy of all the angles and observations he may have made, who will protract and calculate the distances of the different objects, and lay them down on a chart, as described at page 139 (fig. 83.) Then, when a rough sketch has been completed, land with the boat on some of the islands or objects most favourably situated, and with a Theodolite take angles between the most conspicuous of them, and protract them on the rough chart. If they agree with the positions given to them by the angles taken from the extremes of the *base line*, the work is so far correct; and we may look for such a result, providing that the distance between the two ships was, under favourable circumstances, correctly obtained at the commencement of the operations, which in a great measure depends on the nicety of the instruments with which the observations were made.

The next operation is to determine the points and outline of the coast, none of which were fixed by angles taken from the extremes of the *base line*, in consequence of the uncertainty of marking the identical spot on any part of the coast from both ships at anchor so far apart, without which the two angles would not intersect on the point required. Proceed, therefore, to the different points, and fix them by intersections of the needle, and sketch in the curvature of the coast between them, as shown at page 141 (fig. 83); but when any material point is to be determined, recourse must be had to logarithmic calculation by some of the methods given amongst the problems. The objects in our present (fig. 87) will be found most conveniently situated, lying, as many of them do, on the islands far from the coast, and therefore unobstructed by woods or intervening heights; but it may happen that the coast to be surveyed is not at all times so favourably circumstanced; there

may be no objects whatever on the sea side of the coast, and those inland (although perfectly visible at a small distance from shore) may, on approaching it, be kept from our view by high land or a thick intervening wood. In which case, determine the true positions of the inland objects from the extremes of a *base line*, either measured by the velocity of sound, as in fig. 87, or on a convenient plane beyond the wood or high land, and suppose *k* is a point to be laid down on the coast, but that the objects C, D, E, &c., cannot be seen from it. Set up a high pole at *k*, and station an officer by it, and send another officer in the boat with instructions to pull out until he can see the summit of C above the intervening land or wood; and when he has brought the pole at *k* exactly on with C, let him anchor the boat if possible; or, if there is no anchorage, let him hoist a pendant or flag at the instant the coincidence takes place; and he must contrive to keep the boat in that position, until the officer on shore makes some previously concerted signal, signifying that he has obtained a correct bearing of the boat, which we will suppose was at *m*, S. 13° E., two miles and a half from the point *k*, at the instant the bearing was taken. The officer must next pull or sail until he in like manner brings the pole at *k* in a direct line with the summit of the hill at D, which, being further inland, and higher than C, will be seen sooner and at a less distance from shore: the pendant must again be hoisted until the officer on shore takes a precise bearing of the boat at *n*, S. 29° 30′ W., two miles distant from *k*. It will readily be seen that these two bearings of the boat from the point *k*, with the hills C and D respectively on with *k*, are precisely what they would have been had the observations been made by an observer from the summits of the objects, with the point *k* on with the boat; consequently, if a line be drawn, S. 13° E. from C, and from D S. 29° 30′ W., these two lines will intersect each other at *k*, the true position of the point required, so far as a magnetic bearing may be depended upon; and any number of intersections may be used, always observing to begin either to the right or to the left, taking the objects in succession, by which arrangement the officers will have no difficulty in following the movements of each other, until the objects can be seen distinctly from all parts of the shore; the services of the boat will then be of no further utility. The point *p* would in like manner be fixed by lines drawn from E and F, in the direction of the two bearings of the boat, taken from the point *p*, and so on with all other points or rocks. The curvature of the coast between them

must be filled up as directed at page 139, fig. 83, and when the outline of the coast is correctly laid down, proceed to lay down the soundings, shoals, and rocks, as directed at page 145, fig. 84, and draw on one part of the chart the different appearances of the land as you sail along it; observe the sets of tide, and currents, with their peculiarities; also the *rise* and *fall* of tide, and the time of high-water, at the full, change, and new moon. Ascertain and describe particularly all leading land-marks, for avoiding rocks or shoals; point out good anchorage, by inserting an anchor with leading marks and directions for finding them; also watering-places, with every particular relative to the nature of the anchorage, and most expeditious mode of procuring the water. State particularly whether the coast affords places of shelter into which vessels might run in bad weather, under what circumstances, the capacity of the bay or river, and safest method of approaching it in a gale of wind. Determine with the utmost exactness the latitude and longitude of the most conspicuous hills, or points, near the entrance of the harbour, or adjacent to it on the coast; and take careful views of them at different distances from the land; insert these in the chart, with appropriate names, that they may be easily and readily recognised by commanders of ships, and those who may happen to make these points in first approaching the coast.

How to obtain a Base Line by Latitudes or Longitudes, and from thence to fix the objects for surveying the coast.

If the coast extends from east to west, as in last example, fig. 87, and, from circumstances, a *base line* by the velocity of sound cannot be measured, and no level plane can be found, then determine the latitudes of two hills in the direction of G and F, but select those as nearly north and south of each other as possible, and at the distance of at least twenty miles; then by their difference of latitude and bearings find the distance between them, which will be an arc of a great circle of the earth: but its chord, which we have already shown to be twice the sine of half the arc, must be used as the *base line;* the true bearing, as well as magnetic bearing of the objects, must be observed, to obtain their distance. From the extremes of this *base line* find the position of C, D, &c. and then proceed as before to delineate the coast, and fix the objects lying off the coast, such as H, I, K, L, &c., also rocks, shoals, and soundings.

Should there not be a fit object in the direction of F, the longitude and latitude of the hill G must be determined, and by the method given at page 101 find the latitude and longitude of the hill at R, about 34 miles from G; make the chord of that arc the *base line*, and from its extremes fix the objects lying off the coast, by which the different stations along the coast may easily be laid down, and the survey completed as before.

If it should so happen that the coast is flat, without any remarkable objects inland, or at a distance off shore, whose latitude or longitude might be determined for one extreme of a *base line;* in that case find a meridian line, making some conspicuous point on the coast, such as *k*, fig. 87, one extreme of it: then, at as great a distance in that line as the report of a gun can be heard and signals be made out, erect a conspicuous flag-staff, and, by the result of several trials made from each extreme simultaneously, determine its distance from *k* by the velocity of sound; then determine trigonometrically the distances of other flag-staffs, set up at convenient positions along the coast, and from these fix others; observing always, if possible, to lay down the soundings as far from the shore as the signal-posts can be seen, before they are removed to any other station. When the angles become too oblique, discontinue the operation, and again measure a *base line* as before, providing no plane can be found; observing to include some of the former flag-staffs in the new scheme. Few coasts, however, are so bare as not to offer some conspicuous objects by which the outline and soundings may be laid down: but it may occur that *one* high object only can be found for several miles along a coast at a short distance inland, suppose at D, fig. 87. Some of the principal points may, in that case, be found from it as follows:—

Determine the distance of D from the point *k*, by the velocity of sound, and observe well its magnetic bearing. Erect flag-staffs, if necessary, at *s* and *r*, and at *k* with a sextant measure the angles D *k s* = 67° 50′, and D *k r* = 22° 5′; also take magnetic bearings of any little projection between *k*, *s*, or *k*, *r*, such as the point *t*, after which proceed to *s* and take the angle D *s k* = 88° 10′. Here we have obtained two angles of the triangle D *k s*, consequently the third angle *k* D *s* is = 24°. Lay off the angle *k* D *s* and it will intersect the side *k s* at the point *s*. Next go to *r*, and take the angles D *r k* = 119° 30′, and D *r p* = 107° 10′: then in the triangle D *r k* two angles are known; therefore the third angle *r* D *k* is found = 35° 25′.

draw out the angle and it will intersect *k r* in the point *r*; from *r* take the bearing of *t*, and it will intersect the former bearing in that point. This easy method of fixing the points on the coast may be more extensively employed in surveying a bay or harbour that may have some conspicuous object or island, situated in such manner that angles may be taken right round it, when a *base line* cannot be measured on shore, or when other objects do not present themselves. Small islands may also be surveyed singly in this manner, by having conspicuous signal-posts erected near their centre, and then going round the island, commencing at a point whose distance and magnetic bearing from that signal-post had already been determined, in the manner in which the distance between D and the point *k* was found. Suppose it were required to determine the outline of the island (fig. 88), of three or four miles extent, on which a base line could not be measured, in consequence of woods or uneven ground. At P, near the centre of the island, or at any station near the centre, from which all the points along the coast can be distinctly seen, erect a conspicuous flag-staff, and determine its distance and bearing from the point G, either by measurement or velocity of sound, then, if the points at *a*, *b*, *c*, *d*, &c. be low and ill-defined, erect poles with flags on them, and then take, with a theodolite or sextant, the angles PG*b* PG*a* PG*c*, PG*d*, &c. and note them in your book, then go on to *b* and *a*, and take the angles P*b*G, P*a*G; hence we obtain the third angle GP*b* and GP*a*. Then to protract these angles, first draw on paper the line PG, and lay down the distance between these two points from a scale of equal parts, next protract the angles PG*b*, PG*a*, and draw out the lines G*b*, G*a*, &c.; then from 180° subtract the sum of the angles PG*b*, P*b*G, and we obtain the remaining angle of the triangle GP*b*; protract this angle, and it will intersect the line G*b* in *b*; in like manner find the angle GP*a*, and draw it out, and it will intersect the line G*a* in *a*; the curvature of the coast between G and *a*, or between any of the points thus determined, must necessarily be sketched in while on the spot. Go round the island in this way until the whole is filled in, making use of occasionally a magnetic bearing where it may be found convenient.

If several islands are together, their positions must be laid down in a similar manner, if the group is not very extensive, and even then the method will answer, providing the fixed object can be seen from the extremes of the cluster; but two objects in that case might be erected, if the islands did not happen to be of a hilly nature; and the distance between the

might be determined by the velocity of sound; or by their difference of latitude, if in a north and south direction, and at a sufficient distance apart; or by difference of longitude, determined by chronometer, if the islands run in an east and west direction; for instance, the islands H and L (fig. 87) would be conveniently situated for a base line in an east and west direction, or the distance, K*e* or K*f*, might be determined by two latitudes and a bearing. From the extremes of either base line, the position of other objects on the different islands may be fixed, after which a distinct survey of each island may be carried on, as in fig. 88; then sound between them, and at a distance around them, note the rise and fall of tide, and its direction, or the set of the current, and every other particular as before directed. If it were found impracticable to land on an extensive chain of islands, either from the unfavourable nature of the shore, or the savage state of the natives, a pretty correct survey may be made, by sailing until two of the most remarkable objects (at a great distance apart) are brought on with each other, at which instant let a correct bearing be taken, which must be afterwards corrected for variation, to obtain a true bearing; then if they lie in a north and south direction, anchor the vessel due east or west of each object, on following days, and determine their latitudes; or if there is no anchorage, heave the ship to in that position, until you have made the observation. For example, suppose it were required to make a survey of the island (fig. 88) on which, from some cause, it was not possible to land. From O, with the objects A and B on with each other, take with an azimuth compass their magnetic bearing, S.E. 14° 40′, which bearing, corrected for the variation previously found, gave the true bearing of the objects S. 5° E. At M and N, on different days, with the objects A and B due west, anchor the ship if possible, or heave to, until the latitude of each object has been correctly obtained; then, with the true bearing and difference of latitude find the distance AB; that is, in this problem we have given one side of a right-angled triangle, being equal to the difference of latitude laid off on the true meridian; also the angle included between the meridian and true bearing of the object, consequently the third angle of the triangle is known; with these data find the hypothenuse, which is the distance required between the objects. If the distance thus found be very great, it must be considered, as before observed, as an arc of a great circle of the earth, the cord of which must be used as the base line for carrying on the survey round the coast of the island; but if the distance

does not exceed 10 or 15 miles, the arc may be considered as differing in no sensible degree from the cord itself, and may consequently be taken at once from a scale of equal parts as the base line.

Send the boat in shore when the water is perfectly smooth, and when she has approached the point F as close as she prudently can, let go the anchor, and take with an azimuth compass the correct bearing of the objects A and B, which, according to the method at page 116, will give the position of the boat; guess the small distance between the boat and the point, and mark it down on the rough draft, which cannot be far wrong, supposing the whole distance to be only one or two hundred yards; also sketch in the curvature between *g* and F, observing that the bearing of B passes over the small point *g;* from the position of the boat near the point F, take a correct bearing of the point at D, and as many other points or projections as may be seen from this station, and draw them out on the chart, guessing their distances as near as possible, which will greatly facilitate the delineation of the coast between those points.

Next proceed towards the point D, noting the soundings and corresponding bearings of A and B as you go on; also correct the outline of the coast, and sketch its different appearances; at *s* take a bearing of A, and draw it out, and it will intersect the bearing F*g* in the true position of *g;* anchor the boat as close into the point D as possible, and fix its position by a bearing of one or both the objects A and B, and again correct the outline of the coast which you have sailed along; another bearing of the point *g* may also be taken to verify the position given to it by the first observations; at this station also note the correct bearing of the hill at R, and the headland at H, and proceed on in the same manner as from F to D, and reaching the point H, fix the position of the boat by bearings of the two objects, and sketch in the curvature from D; also take another bearing of the hill at R, which will intersect the former bearing taken at D in the true position of R; and in like manner any other remarkable object may be fixed, should it be on the island, and required. The object at R will be found useful in surveying that part of the coast from H to *a*.

For the correctness of this method of surveying, of course, much depends on the accuracy of the latitudes and bearing of the objects, and it may be applied to the survey of an extensive coast as well as islands, when two objects can conveniently be brought on with each other to obtain their bearing, such as by

sailing round a headland, or into a bay, &c.; and it would be advantageous, when the coast runs in a north and south direction, to choose those objects (if a choice can be made) which make as small an angle as possible with the meridian; but if the coast lies in an east and west direction, two objects which subtend an angle of about 45° from the meridian may be chosen.

A survey of a coast sufficiently correct to answer the purposes of navigation in a certain degree, may be made while sailing along it. For example, let it be required to survey that part of the coast between the headland at Y and the point *k*, (fig. 87). Stand as close in to Y as you can, and, if practicable, anchor the ship; but if there should be no anchorage, then heave the ship steadily to, until you make the necessary observations. First take the bearing of the extreme headland Y, and draw out the line on your draft, making some mark, such as V, or any other mark you choose, to denote the position of the ship; next, guess with great care the distance of Y from the ship, which we will estimate at ¼ of a mile. Take this distance from a large scale of equal parts, and lay it off on the bearing line. Sketch, in the form of the headland, also the land and coast between Y and the point *k*, the bearing of which must be most correctly observed and drawn out on the rough draft, with its estimated distance. Then take the bearing of the following objects and projections of the coast, namely, the hill at R, the small point *x* on with the hill at K, the points *w* and *z*; then draw out these bearing lines. This part of the work being completed, obtain the latitude of the ship by sun, moon, or stars, also the longitude by chronometer. After these observations have been made, and the whole carefully noted on the log-slate, or in a book, make sail, if the wind will permit, and stand directly towards the point *k*, noting precisely the minute the ship starts, and carefully measure, by a correct log-line, the rate of the ship's going. This must be repeatedly done, to obtain her mean rate of sailing. Massey's patent log may also be towed astern, as a check on the other, or the distances given by each can be added together, and a mean of the two be taken for the distance run; also let the soundings be noted as you proceed on; when you are abreast of the point *x* (as at No. 1), take its bearing and draw it out, and it will intersect the bearing line of that point taken at V, in the true position of *x*; but to know the distances of the point when each bearing was taken, we must ascertain the distance run between the station at V and No. 1.; this is to be found by a proportion, having the

time elapsed whilst running between the two stations; then say, if the ship sails a certain number of miles in one hour, what distance had she run during the elapsed time the distance thus found may be considered as a base line? and, providing the objects be not at a great distance, their positions will be correctly obtained by the bearings taken from its extremes, if the ship's run has been exactly measured.

From No. 2 take bearings of the point *w*, which happens to be on with the high object at K, noting the time elapsed, as before; then take a bearing of the hill at R, and that will fix its position by intersecting the former bearing taken at V; this line also passes over the point at *x*. From this station, No. 2, take a bearing of the island at *v*, and again sketch and correct your former rough outline of the coast from Y to *z*. At station 3 it would be advisable to heave the ship *to*, whilst bearings of all the points and remarkable objects are being observed; take a bearing of R, and fix the position of the hill at K by a bearing, which also passes over and determines the position of the island at *v*. From this station take the bearing of some remarkable object which extends beyond the point *k*, such as E or G, and which can afterwards be fixed by an intersecting bearing when opposite to it, having the distance run by log; if two or more objects can be selected, they will be of use in carrying on the survey on that side of *k*. When abreast of the point *z*, determine its position by a bearing; and when you have reached the point *k*, put the vessel into a similar position to that which you occupied when off the point Y; then take the bearing of the point and guess its distance; take also the bearing of R and K, and fix the ship's position by these objects, which ought to place her in the same station as the distance run; then sketch the point *k* and coast correctly in, and take a bearing of some point or headland further on towards G, for a second operation, and so continue the survey as far as necessary; observing, that when you relinquish the work, to do it as near to some remarkable headland or conspicuous object as possible, that you may resume the work with certainty, and before leaving each station, obtain its latitude and longitude if possible; afterwards cruise between the stations, and lay down correctly the soundings, rocks, and shoals, ascertain the rise and fall of tide, and its direction on both ebb and flood with the time of high-water; mark down particularly the entrance of any bay or river, and if opportunity offers, make a faithful sketch of it, and give proper directions for finding it on the coast.

THE SHIP ———, on Survey in the Straits of ———,

H.	K.	F.	Courses.	Winds.	Soundings.	REMARKS—*Wednesday, 23d January*
1	...	...	At anchor.	S.E.	10¾	A.M. Fine clear weather.
2	...	...		Eastw^{d}.	11	2 A.M. Tended head to the eastward.
3	...	...		N.E.	11	
4	...	...		Eastw^{d}.	11¼	4 A.M. Weighed and stood to join the senior officer.
5	2	...	S.W.	...	14	☉ rise. St John's hill right of ☉'s south limb 29° 8′, alt. ☉'s L. L. 1° 19′. St John's and south hill, 46° 40′. St John's hill and Peak Antonio, 78° 39′. St John's and Platform hill, 28° 53′. Angles left of St John's. Round Island, 48° 27′. Summer Island, 57° 12′. White Rock, 60° 10′. Northern extreme of Long Island, 62° 15′. Southern extreme of ditto, 98° 20′. Extreme of mainland, 110° 30′.
6	2	...	S. by W.	...	10	
7	2	...	...	...	19	
8	1	6	S. by E.	S.E.	19	
9	1	4	S.S.W.	Tacked.	18	
10	1		S.S.E.	...	19	
	1		...		8½	
11	1	4	E.	Tacked.	7¾	
12	4	4	S.W.	W.N.W.	6¼	Noon. Light variable airs and hot sultry weather. Part of the crew employed altering sails; carpenter making oars for the launch; other part of the crew variously employed. High peak on Summer Island and St John's hill, 71° 3′. St John's and Platform hill, 32° 18′. St John's and Peak Antonio, 89° 57′. St John's, S. 31° E. Senior officer's vessel S. 4° W. at anchor, distant about 1¼ miles. Latitude observed, 13° 12′ N.
1	1	...	S. by W.	Westw^{d}.	5¼	1 P.M.
2	1	4	S.S.W.	...	6	2 P.M. Light variable airs. Spoke the senior officer.
	1	4	...	...	5	
3	1	4	E.	Tacked.	4¾	3 P.M. Senior officer weighed and stood down in company, having made the signal "prepare to take up a position for measuring a base line."
4	2	6	...	...	9½	
	2	4				
5	2	...	W.S.W.	N.W.	9	
	1	6				
6	2	...	E. by N.	...	8¾	☉ set. Cloudy weather, with squally appearance to the S.E^{d}. 6^{h}. 15^{m}. P.M. a heavy squall from the S.E. with rain. Clewed up the topsails and courses, and furled them, and sent down top-gallant yards. Anchored with the larboard bower in 8¾ fathoms to measure a base line; senior officer E. by S. about 5 miles. St John's, S. 50° E. High peak on Summer Island and St John's, 95° 12′. St John's and Peak Antonio, 65° 33′. Extreme of the main land, S. 30° W.
7	...	...	At anchor.	S.E.	8¾	
8	...	...		Calm	8½	
9	...	...		Northw^{d}.	8¼	
10	...	...		Eastw^{d}.	8¼	
11	...	...		...	8¼	
12	...	...		...	8	Midnight. Fine clear weather.

———————————, *Commander.*

K.	F.	Courses.	Winds.	Soundings.	REMARKS—*Thursday, 24th January.*
...	...	At anchor.	N.W.	7½	A.M. Fine clear weather.
...	...		..	7	
...	...		...	6¾	3 A.M. Tended to the flood.
...	...		...	7¼	
...	...		...	7¾	5 A.M. Sent the launch on shore for water.
...	...		N.E.	8¼	6 A.M. Cut six fathoms of the starboard bower-cable, and rebent it again. Sent up the top-gallant yards.
...	...		...	8¾	
...	..		...	9	7 A.M. Tended to the ebb; hove in the slack chain preparatory to taking angles.
...	...		...	8½	8 A.M. Launch returned; cleared and sent her on shore again, with instructions to note the soundings on her way.
...	...		N.N.E.	8¼	
...	...		Calm.	8	11 A.M. Senior officer made signal, "prepare to measure a base line by sound." Answered the signal, and noted a mean interval of 27″ between the flash and report of four guns. Repeated the experiment, by firing a gun four times whilst the senior officer noted the interval on board his ship. Took a set of angles on both sides of the base line, as per book. Part of the crew employed filling water: sail-maker and six hands repairing sails.
...	...		...	8	
...	...	At anchor.	N.N.W.	7¾	P.M. Light airs and clear weather.
...	...		N.N.E.	7½	2 P.M. Senior officer made signal for the boat; sent her with an officer, and a copy of the angles taken.
...	...		...	7¼	
...	...		...	7	4 A.M. Tended to the flood.
...	...		S.W.	7¾	5 P.M. Launch returned with water; moored her astern.
...	...		Westw^{d}.	8½	⊙ set. High peak on the mainland, and ⊙'s S.L. 72° 12′. White Rock and ⊙'s N.L. 29° 10′. White Rock and Round Island, 58° 7′.
...	...		Calm.	8¾	
...	...		...	9	
...	...		...	9¼	9 P.M. Tended to the ebb. Boat returned with orders for us to weigh in the morning, if clear weather, and sound along the shore. Moored the boat astern; *the work being done*, gave the crew an extra dram.
...	...		...	8½	
...	...		...	8	11 P.M. Squally appearance to the eastward.
...	...		E.S.E.	7¾	Midnight. Moderate breeze, with light rain.

A TABLE

CONTAINING

THE LOGARITHMS OF NUMBERS,

FROM AN UNIT TO 10,000.

M

EXPLANATION OF THE TABLE OF THE LOGARITHMS OF NUMBERS.

LOGARITHMS are a system of numbers designed to facilitate the arithmetical operations of multiplication, division, involution, evolution, &c., and are arranged in tables so that each number is placed opposite to its logarithm. The product of any two or more numbers in the table is found by adding together their logarithms, and the quotient of any two numbers by subtracting from the logarithm of the dividend that of the divisor.

In the common system of logarithms, the logarithm of 1 is 0, the logarithm of 10 is 1, the logarithm of 100 is 2, the logarithm of 1000 is 3, and so on. Hence, it is evident that the logarithm of any number between 1 and 10 must be a decimal fraction; that of any number between 10 and 100 must be 1 with some decimal annexed; that of any number between 100 and 1000 must be 2 with a decimal annexed; and in short, *the integral part of any whole or mixed number is always a unit less than the number of figures in the integral part of the corresponding natural number.* The integral part of a logarithm is called the *index* or *characteristic.*

Since the logarithm of 1 is 0, the logarithm of any number less than 1, that is, of any fraction, must be *negative.* When the logarithm of a fraction is required, it is always convenient first to convert it into a decimal, and then to find the logarithm corresponding to the integral number represented by the same digits, prefixing a negative index, *that index being always a unit more than the number of cyphers between the decimal point and the first significant figure.* The negative sign is generally written over the index instead of before it.

To shew at one view the method of prefixing the index of a logarithm, we give the following examples of numbers, con-

sisting of the same digits, with their corresponding logarithms:—

NUMBERS.	LOGARITHMS.
3598	3.556061
359.8	2.556061
35.98	1.556061
3.598	0.556061
.3598	$\bar{1}$.556061
.03598	$\bar{2}$.556061
.003598	$\bar{3}$.556061

To find the logarithm of any whole number between 1 *and* 100.

Look for the number in the column marked N in the first page of the table of logarithms, and immediately on the right of it will be found the logarithm sought, with its proper index: thus, log. 72 is 1.857332.

To find the logarithm of any number between 100 *and* 1000.

Look for the given number in the column marked N in any of the pages of the table except the first, and immediately on the right of it, in the column marked 0 at the top will be found the decimal part of the logarithm sought, to which the proper index must be prefixed: thus, log. 698 is 2.843855.

To find the logarithm of any number between 1000 *and* 10,000.

Look for the three first figures in the column marked N, as before, and the fourth figure at the top or bottom of the page, then the number in a straight line with the three figures in the column N, and directly under the fourth figure, is the decimal part of the logarithm sought, to which the proper index must be prefixed: thus, log. 4968 is 3.696182.

To find the logarithm of any number above 10,000.

Find the logarithm of the first four figures in the manner already described, to which prefix the proper index; then take the number opposite to it in the column of differences marked

D, and multiply this number by the remaining figures of the given number; from the right of the product cut off as many figures as the multiplier contains; the remaining figures added to the logarithm already found will be the logarithm required: thus, let it be required to find the logarithm of 465·976.

Log. of 465·900 is	2·668293	Diff.	93
Add	70		× 76
Log. of 465·976	2·668363		70|68

It may be remarked, that when the first figure of the product of the tabular difference by the remaining figures of the given number which is cut off is above 5, it is usual to carry 1 to the next number: thus, in the above example 71 should be added to the logarithm found in the table instead of 70, since the first figure cut off is 6, and the true logarithm will be 2·668364.

To find the natural number answering to any given logarithm.

Look for the decimal part of the logarithm in the different columns, and in the column marked N in a straight line with it the three first figures of the natural number will be found; the figure at the top of the column in which the logarithm stands will be the fourth figure, which is to be annexed to the other three.

But if the given logarithm be not found exactly in any of the columns of logarithms, take that which is next less, and subtract it from the given logarithm; divide the remainder by the tabular difference, annexing as many cyphers as there are figures required, and annex the quotient to the figures already found; place the decimal point according to the index of the logarithm, which will give the number required: thus, let it be required to find the number whose logarithm is 3·496548.

Given log.	3·496548	Tab. diff. 139)3300(24
The next less is log. of 3137	3·496515	
Remainder	33	

Hence the number sought is 3137·24.

EXPLANATION OF THE TABLES OF LOGARITHMIC SINES, TANGENTS, AND SECANTS.

To find the logarithmic sine, tangent, and secant of any number of degrees and minutes.

Find the number of degrees, if less than 45°, at the top of the page, and the minutes in the left-hand column marked M, then in a straight line with the number of minutes, under the words Sine, Tangent, or Secant, will be found the logarithm of the given number of degrees and minutes.

If the number of degrees be above 45° and less than 90°, find the degrees at the bottom of the page, and the minutes in the right-hand column marked M.

When the degrees are above 90°, take their supplement and find its sine, tangent, or secant, which is also the sine, tangent, or secant of the given number of degrees and minutes. (See page 5.)

To find the Logarithmic sines, tangents, and secants, of any number of degrees, minutes, and seconds.

Find the sine, tangent, or secant of the number of degrees and minutes in the manner already described. From the column marked D take the tabular difference, and multiply it by the number of seconds; from the right of the product cut off two figures, and add the remaining figures to the logarithm already found.

Find the logarithmic sine of 36° 25′.

Sine 36° 25′. Log. 9·773533.

Find the logarithmic tangent of 47° 23′ 36″.

Tangent 47° 23′	Log. 10·036173	Tab. diff.	423
	Add . 152		36
Tangent 47° 23′ 36″	Log. 10·036325		152|28

Find the logarithmic secant of 125° 22′ 56″.

The number of degrees being above 90°, subtract 125° 22′ 56″ from 180°, which gives the supplement 54° 37′ 4″.

Secant 54° 37′	Log. 10·237288	Tab. diff.	296
	Add . 11		4
Secant 54° 37′ 4″ or 125° 22′ 56″	10·237299		11|84

To find the degrees, minutes, and seconds, corresponding to any given logarithmic sine, tangent, or secant.

Look for the given logarithmic sine, tangent, or secant, in the respective columns, and if it be found exactly, there will be no seconds. If the given logarithm be found in the left-hand columns of sines, tangents, or secants, the degrees stand on the top of the page, and the minutes on the left, counting downwards; but if it be found in the right-hand columns of sines, tangents, and secants, the degrees are at the bottom of the page, and the minutes on the right, counting upwards.

If the given logarithm be not found exactly in the tables, take that which is next less, and subtract it from the given logarithm; to the remainder annex two cyphers, and divide by the tabular difference, the quotient will be the number of seconds.

Required the degrees, minutes, and seconds, corresponding to the logarithmic sine 9·844238?

Given logarithm, . 9·844238
Log. next less in tables, 9·844114 = sine 44° 18′

Tab. diff. 216)12400(57″
Hence 9·844238 is the logarithmic sine of 44° 18′ 57″.

Required the degrees, minutes, and seconds, corresponding to the logarithmic tangent, 10·385426?

Given logarithm, . . 10·385426
Log. next less found in table, 10·385282 = 67° 37′

Tab. diff. 598)14400(24″
Hence 10·385426 is the logarithmic tangent of 67° 37′ 24″.

Required the degrees, minutes, and seconds, corresponding to the logarithmic secant, 10·458379?

Given logarithm, . . 10·458379
Log. next less found in table, 10·458047 = 69° 37′

Tab. diff. 566)33200(59″
Hence 10·458379 is the logarithmic secant of 69° 37′ 59″.

The cosine, cotangent, and cosecant of any number of degrees, minutes, and seconds, are found in the same manner as the sine, tangent, and secant, except that, when a correction is made for seconds, the seconds must be subtracted instead of added.

A TABLE

CONTAINING

THE LOGARITHMS OF ALL NUMBERS,

FROM AN UNIT TO 10,000.

Numbers from 1 *to* 100, *and their Logarithms, with Indices.*

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	26	1.414973	51	1.707570	76	1.880814
2	0.301030	27	1.431364	52	1.716003	77	1.886491
3	0.477121	28	1.447158	53	1.724276	78	1.892095
4	0.602060	29	1.462398	54	1.732394	79	1.897627
5	0.698970	30	1.477121	55	1.740363	80	1.903090
6	0.778151	31	1.491362	56	1.748188	81	1.908485
7	0.845098	32	1.505150	57	1.755875	82	1.913814
8	0.903090	33	1.518514	58	1.763428	83	1.919078
9	0.954243	34	1.531479	59	1.770852	84	1.924279
10	1.000000	35	1.544068	60	1.778151	85	1.929419
11	1.041393	36	1.556303	61	1.785330	86	1.934498
12	1.079181	37	1.568202	62	1.792392	87	1.939519
13	1.113943	38	1.579784	63	1.799341	88	1.944483
14	1.146128	39	1.591065	64	1.806180	89	1.949390
15	1.176091	40	1.602060	65	1.812913	90	1.954243
16	1.204120	41	1.612784	66	1.819544	91	1.959041
17	1.230449	42	1.623249	67	1.826075	92	1.963788
18	1.255273	43	1.633468	68	1.832509	93	1.968483
19	1.278754	44	1.643453	69	1.838849	94	1.973128
20	1.301030	45	1.653213	70	1.845098	95	1.977724
21	1.322219	46	1.662758	71	1.851258	96	1.982271
22	1.342423	47	1.672098	72	1.857332	97	1.986772
23	1.361728	48	1.681241	73	1.863323	98	1.991226
24	1.380211	49	1.690196	74	1.869232	99	1.995635
25	1.397940	50	1.698970	75	1.875061	100	2.000000
N.	Log.	N.	Log.	N.	Log.	N.	Log.

N. B.—In the following part of the Table the Indices are omitted, as they are easily supplied, being always, each of them, in the case of whole or mixed numbers, an unit less than the number of figures in the integral part of the corresponding natural number. If the number is a decimal, the index is negative, and is always an unit greater than the number of cyphers between the decimal point and the first significant figure of the decimal.

A Table of Logarithms from 1 to 10,000.

N.	0	1	2	3	4	5	6	7	8	9	D.
340	531479	531607	531734	531862	531990	532117	532245	532372	532500	532627	128
341	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
343	5294	5421	5547	5674	5800	5927	6053	6179	6306	6432	126
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	9076	9202	9327	9452	9578	9703	9829	9954	540079	540204	125
347	540329	540455	540580	540705	540830	540955	541080	541205	1330	1454	125
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183	124
351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	550106	123
355	550228	550351	550473	550595	550717	550840	550962	551084	551206	1328	122
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	9907	560026	560146	560265	560385	560504	560624	560743	560863	560982	119
364	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
371	9374	9491	9608	9725	9842	9959	570076	570193	570309	570426	117
372	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	579784	579898	580012	580126	580241	580355	580469	580583	580697	580811	114
381	580925	581039	1153	1267	1381	1495	1608	1722	1836	1950	114
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	9950	590061	590173	590284	590396	590507	590619	590730	590842	590953	112
390	591065	591176	591287	591399	591510	591621	591732	591843	591955	592066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
296	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
399	600973	601082	1191	1299	1408	1517	1625	1734	1843	1951	109
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
400	602060	602169	602277	602386	602494	602603	602711	602819	602928	603036	108
401	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
407	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
408	610660	610767	610873	610979	1086	1192	1298	1405	1511	1617	106
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	9093	9198	9302	9406	9511	9615	9719	9824	9928	620032	104
417	620136	620240	620344	620448	620552	620656	620760	620864	620968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	623249	623353	623456	623559	623663	623766	623869	623973	624076	624179	103
421	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
426	9410	9512	9613	9715	9817	9919	630021	630123	630224	630326	102
427	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	100
431	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	100
432	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
434	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	99
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	99
436	9486	9586	9686	9785	9885	9984	640084	640183	640283	640382	99
437	640481	640581	640680	640779	640879	640978	1077	1177	1276	1375	99
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
439	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	98
441	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	9335	9432	9530	9627	9724	9821	9919	650016	650113	650210	97
447	650308	650405	650502	650599	650696	650793	650890	0987	1084	1181	97
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080	96
451	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
457	9916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
458	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
459	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
460	662758	662852	662947	663041	663135	663230	663324	663418	663512	663607	94
461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
468	670246	670339	670431	670524	670617	670710	670802	670895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
479	680336	680426	680517	680607	680698	680789	680879	0970	1060	1151	91
480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
481	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	8100	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
501	9838	9924	700011	700098	700184	700271	700358	700444	700531	700617	87
502	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	85
513	710117	710202	710287	710371	710456	710540	710625	710710	710794	0879	85
514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3659	3742	3826	3910	3994	4078	4163	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
519	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
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520	716003	716087	716170	716254	716337	716421	716504	716588	716671	716754	83
521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
525	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
538	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
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544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
563	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
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568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
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578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
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354	9003	9126	9249	9371	9494	9616	9739	9861	9984	550106	123
355	550228	550351	550473	550595	550717	550840	550962	551084	551206	1328	122
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
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367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
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371	9374	9491	9608	9725	9842	9959	570076	570193	570309	570426	117
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373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
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375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
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398	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
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407	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
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409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	9093	9198	9302	9406	9511	9615	9719	9824	9928	620032	104
417	620136	620240	620344	620448	620552	620656	620760	620864	620968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
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427	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	100
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443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	9335	9432	9530	9627	9724	9821	9919	650016	650113	650210	97
447	650308	650405	650502	650599	650696	650793	650890	0987	1084	1181	97
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449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080	96
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454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
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457	9916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
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461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
468	670246	670339	670431	670524	670617	670710	670802	670895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
479	680336	680426	680517	680607	680698	680789	680879	0970	1060	1151	91
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481	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
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484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
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486	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
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494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
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503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
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518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
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521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
525	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
538	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
543	4800	4880	4960	5040	5120	5199	5279	5359	5439	5519	80
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
563	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
576	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
577	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
579	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
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A Table of Logarithms from 1 *to* 10,000.

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580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
581	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
589	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8179	72
600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
602	9596	9669	9741	9813	9885	9957	780029	780101	780173	780245	72
603	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	9581	9651	9722	9792	9863	9933	790004	790074	790144	790215	70
617	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
619	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961	69
631	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
632	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
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640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790	68
641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
646	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
661	820201	820267	820333	820399	820464	820530	820595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
677	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
681	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415	63
691	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
692	840106	840169	840232	840294	840357	840420	840482	840545	840608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
N.	0	1	2	3	4	5	6	7	8	9	D.

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700	845098	845160	845222	845284	845346	845408	845470	845532	845594	845656	62
701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	9739	9799	9859	9918	9978	860038	860098	860158	860218	860278	60
725	860338	860398	860458	860518	860578	0637	0697	0757	0817	0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
741	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	59
742	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
759	880242	880299	880356	880413	880471	880528	0585	0642	0699	0756	57
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761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1890	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998	56
771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
776	9862	9918	9974	890030	890086	890141	890197	890253	890309	890365	56
777	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
795	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
801	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
806	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	7949	8002	8056	8109	8163	8217	8270	8324	8378	8431	54
810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	9556	9609	9663	9716	9770	9823	9877	9930	9984	910037	53
813	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
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821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
831	9601	9653	9706	9758	9810	9862	9914	9967	920019	920071	52
832	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6239	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
851	9930	9981	930032	930083	930134	930185	930236	930287	930338	930389	51
852	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1203	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1864	1915	51
855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	7016	7066	7116	7167	7217	7267	7317	7367	7418	7468	50
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8319	8370	8420	8470	50
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	9020	9070	9120	9170	9220	9270	9319	9369	9419	9469	50
870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
871	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
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A Table of Logarithms from 1 *to* 10,000.

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880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7139	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8608	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
891	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
892	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2259	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3479	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	960042	960090	960138	960185	960233	960281	960328	960376	960423	48
913	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5060	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8389	8436	47
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
934	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
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940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5933	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
978	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9130	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

A TABLE

OF

LOGARITHMIC SINES, TANGENTS, AND SECANTS.

FOR EVERY DEGREE AND MINUTE OF THE QUADRANT.

N. B.—The minutes in the left-hand column of each page, increasing downwards, belong to the degrees at the top; and those increasing upwards, in the right-hand column, belong to the degrees below.

M.	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	0.000000	- - -	Infinite	0.000000	- - -	Infinite.	10.000000		10.000000	60
1	6.463726	501717	13.536274	6.463726	501717	13.536274	000000	00	000600	59
2	764756	293485	235244	764756	293483	235244	000000	00	000000	58
3	940847	208231	059153	940847	208231	059153	000000	00	000000	57
4	7.065786	161517	12.934214	7.065786	161517	12.934214	000000	00	000000	56
5	162696	131968	837304	162696	131969	837304	000000	00	000000	55
6	241877	111575	758123	241878	111578	758122	000001	01	9.999999	54
7	308824	96653	691176	308825	99653	691175	000001	01	999999	53
8	366816	85254	633184	366817	85254	633183	000001	01	999999	52
9	417968	76263	582032	417970	76263	582030	000001	01	999999	51
10	463725	68988	536275	463727	68988	536273	000002	01	999998	50
11	7.505118	62981	12.494882	7.505120	62981	12.494880	10.000002	01	9.999998	49
12	542906	57936	457094	542909	57933	457091	000003	01	999997	48
13	577668	53641	422332	577672	53642	422328	000003	01	999997	47
14	609853	49938	390147	609857	49939	390143	000004	01	999996	46
15	639816	46714	360184	639820	46715	360180	000004	01	999996	45
16	667845	43881	332156	667849	43882	332151	000005	01	999995	44
17	694173	41372	305827	694179	41373	305821	000005	01	999995	43
18	718997	39135	281003	719003	39136	280997	000006	01	999994	42
19	742477	37127	257523	742484	37128	257516	000007	01	999993	41
20	764754	35315	235246	764761	35136	235239	000007	01	999993	40
21	7.785943	33672	12.214057	7.785951	33673	12.214049	10.000008	01	9.999992	39
22	806146	32175	193854	806155	32176	193845	000009	01	999991	38
23	825451	30805	174549	825460	30806	174540	000010	01	999990	37
24	843934	29547	156066	843944	29549	156056	000011	02	999989	36
25	861662	28388	138338	861674	28390	138326	000011	02	999988	35
26	878695	27317	121305	878708	27318	121292	000012	02	999988	34
27	895085	26323	104915	895099	26325	104901	000013	02	999987	33
28	910879	25399	089121	910894	25401	089106	000014	02	999986	32
29	926119	24538	073881	926134	24540	073866	000015	02	999985	31
30	940842	23733	059158	940858	23735	059142	000017	02	999983	30
31	7.955082	22980	12.044918	7.955100	22981	12.044900	10.000018	02	9.999982	29
32	968870	22273	031130	968889	22275	031111	000019	02	999981	28
33	982233	21608	017767	982253	21610	017747	000020	02	999980	27
34	995198	20981	004802	995219	20983	004781	000021	02	999979	26
35	8.007787	20390	11.992213	8.007809	20392	11.992191	000023	02	999977	25
36	020021	19831	979979	020045	19833	979955	000024	02	999976	24
37	031919	19302	968081	031945	19305	968055	000025	02	999975	23
38	043501	18801	956499	043527	18803	956473	000027	02	999973	22
39	054781	18325	945219	054809	18327	945191	000028	02	999972	21
40	065776	17872	934224	065806	17874	934194	000029	02	999971	20
41	8.076500	17441	11.923500	8.076531	17444	11.923469	10.000031	02	9.999969	19
42	086965	17031	913035	086997	17034	913003	000032	02	999968	18
43	097183	16639	902817	097217	16642	902783	000034	02	999966	17
44	107167	16265	892833	107202	16268	892797	000036	03	999964	16
45	116926	15908	883074	116963	15910	883037	000037	03	999963	15
46	126471	15566	873529	126510	15568	873490	000039	03	999961	14
47	135810	15238	864190	135851	15241	864149	000041	03	999959	13
48	144953	14924	855047	144996	14927	855004	000042	03	999958	12
49	153907	14622	846093	153952	14625	846048	000044	03	999956	11
50	162681	14333	837319	162727	14336	837273	000046	03	999954	10
51	8.171280	14054	11.828720	8.171328	14057	11.828672	10.000048	03	9.999952	9
52	179713	13786	820287	179763	13790	820237	000050	03	999950	8
53	187985	13529	812015	188036	13532	811964	000052	03	999948	7
54	196102	13280	803898	196156	13284	803844	000054	03	999946	6
55	204070	13041	795930	204126	13044	795874	000056	03	999944	5
56	211895	12810	788105	211953	12814	788047	000058	04	990942	4
57	219581	12587	780419	219641	12590	780359	000060	04	999940	8
58	227134	12372	772866	227195	12376	772805	000062	04	999938	3
59	234557	12164	765443	234621	12168	765379	000064	04	999936	1
60	241855	11963	758145	241921	11967	758079	000066	04	999934	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	8.241855	11963	11.758145	8.241921	11967	11.758079	10.000066	04	9.999934	60
1	249033	11768	750967	249102	11772	750898	000068	04	999932	59
2	256094	11580	743906	256165	11584	743835	000071	04	999929	58
3	263042	11398	736958	263115	11402	736885	000073	04	999927	57
4	269881	11221	730119	269956	11225	730044	000075	04	999925	56
5	276614	11050	723386	276691	11054	723309	000078	04	999922	55
6	283243	10883	716757	283323	10887	716677	000080	04	999920	54
7	289773	10721	710227	289856	10726	710144	000082	04	999918	53
8	296207	10565	703793	296292	10570	703708	000085	04	999915	52
9	302546	10413	697454	302634	10418	697366	000087	04	999913	51
10	308794	10266	691206	308884	10270	691116	000090	04	9.:9910	50
11	8.314954	10122	11.685046	8 315046	10126	11.684954	10.000093	04	9.999907	49
12	321027	9982	678973	321122	9987	678878	000095	04	999905	48
13	327016	9847	972984	327114	9851	672886	000098	04	999902	47
14	332924	9714	667076	333025	9719	666975	000101	05	999899	46
15	338753	9586	661247	338856	9590	661144	000103	05	999897	45
16	344504	9460	655496	344610	9465	655390	000106	05	999894	44
17	350181	9338	649819	350289	9343	649711	000109	05	999891	43
18	355783	9219	644217	355895	9224	644105	000112	05	999888	42
19	361315	9103	638685	361430	9108	638570	000115	05	999885	41
20	366777	8990	633223	366895	8995	633105	000118	05	999882	40
21	8.372171	8880	11.627829	8.372292	8885	11 627708	10.000121	05	9.999879	39
22	377499	8772	622501	377622	8777	622378	000124	05	999876	38
23	382762	8667	617238	382889	8672	617111	000127	05	999873	37
24	387962	8564	612038	388092	8570	611908	000130	05	999870	36
25	393101	8464	606899	393234	8470	606766	000133	05	999867	35
26	398179	8366	601821	398315	8371	601685	000136	05	999864	34
27	403199	8271	596801	403338	8276	596662	000139	05	999861	33
28	408161	8177	591839	408304	8182	591696	000142	05	999858	32
29	413068	8086	586932	413213	8091	586787	000146	05	999854	31
30	417919	7996	582081	418068	8002	581932	000149	06	999851	30
31	8.422717	7909	11.577283	8.422869	7914	11.577131	10.000152	06	9.999848	29
32	427462	7823	572538	427618	7830	572382	000156	06	999344	28
33	432156	7740	567844	432315	7745	567685	000159	06	999841	27
34	436800	7657	563200	326962	7663	563038	000162	06	999838	26
35	441394	7577	558606	441560	7583	558440	000166	06	999834	25
36	445941	7499	554059	446110	7505	553890	000169	06	999831	24
37	450440	7422	549560	450613	7428	549387	000173	06	999827	23
38	454893	7346	545107	455070	7352	544930	000177	06	999823	22
39	459301	7273	540699	459481	7279	540519	000180	06	999820	21
40	463665	7200	536335	463849	7206	536151	000184	06	999816	20
41	8.467985	7129	11.532015	8.468172	7135	11.531828	10.000188	06	9.999812	19
42	472263	7060	527737	472454	7066	527546	000191	06	999809	18
43	476498	6991	523502	476693	6998	523307	000195	06	999805	17
44	480693	6924	519307	480892	6931	519108	000199	06	999801	16
45	484848	6859	515152	485050	6865	514950	000203	07	999797	15
46	488963	6794	511037	489170	6801	510830	000207	07	999793	14
47	493040	6731	506960	493250	6738	506750	000210	07	999790	13
48	497078	6669	502922	497293	6676	502707	000214	07	999786	12
49	501080	6608	498920	501298	6615	498702	000218	07	999782	11
50	505045	6548	494955	505267	6555	494733	000222	07	999778	10
51	8.508974	6489	11.491026	8.509200	6496	11.490800	10.000226	07	9.999774	9
52	512867	6431	487133	513098	6439	486902	000231	07	999769	8
53	516726	6375	483274	516961	6382	483039	000235	07	999765	7
54	520551	6319	479449	520790	6326	479210	000239	07	999761	6
55	524343	6264	475657	524586	6272	475414	000243	07	999757	5
56	528102	6211	471898	528349	6218	471651	000247	07	999753	4
57	531828	6158	468172	532080	6165	467920	000252	07	999748	3
58	535523	6106	464477	535779	6113	464221	000256	07	999744	2
59	539186	6055	460814	539447	6062	460553	000260	07	999740	1
60	542819	6004	457181	543084	6012	456916	000265	07	999735	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	8.542819	6004	11.457181	8.543084	6012	11.456916	10.000265	07	9.999735	60
1	546422	5955	453578	546691	5962	453309	000269	07	999731	59
2	549995	5906	450005	550268	5914	449732	000274	07	999726	58
3	553539	5858	446461	553817	5866	446183	000278	08	999722	57
4	557054	5811	442946	557336	5819	442664	000283	08	999717	56
5	560540	5765	439460	560828	5773	439172	000287	08	999713	55
6	563999	5719	436001	564291	5727	435709	000292	08	999708	54
7	567431	5674	432569	567727	5682	432273	000296	08	999704	53
8	570836	5630	429164	571137	5638	428863	000301	08	999699	52
9	574214	5587	425786	574520	5595	425480	000306	08	999694	51
10	577566	5544	422434	577877	5552	422123	000311	08	999689	50
11	8.580892	5502	11.419108	8.581208	5510	11.418792	11.000315	08	9.999685	49
12	58·193	5460	415807	584514	5468	415486	000320	08	999680	48
13	587469	5419	412531	587795	5427	412205	000325	08	999675	47
14	590721	5379	409279	591051	5387	408949	000330	08	999670	46
15	593948	5339	406052	594283	5347	405717	000335	08	999665	45
16	597152	5300	402848	597492	5308	402508	000340	08	999660	44
17	600332	5261	399668	600677	5270	399323	000345	08	999655	43
.18	603489	5223	396511	603839	5232	396161	000350	08	999650	42
19	606623	5186	393377	606978	5194	393022	000355	09	999645	41
20	609734	5149	390266	610094	5158	389906	000360	09	999640	40
21	8.612823	5112	11.387177	8.613189	5121	11.386811	10.000365	09	9.999635	39
22	615891	5076	384109	616262	5085	383738	000371	09	999629	38
23	618937	5041	381063	619313	5050	380687	000376	09	999624	37
24	621962	5006	378038	622343	5015	377657	000381	09	999619	36
25	624965	4972	375035	625352	4981	374648	000386	09	999614	35
26	627948	4938	372052	628340	4947	371660	000392	09	999608	34
27	630911	4904	369089	631308	4913	368692	000397	09	999603	33
28	633854	4871	366146	634256	4880	365744	000403	09	999597	32
29	636776	4839	363224	637184	4848	362816	000408	09	995592	31
30	639680	4806	360320	640093	4816	359907	000414	09	999686	30
31	8 642563	4775	11.357437	8.642982	4784	11.357018	10.000419	09	9.999581	29
32	645428	4743	354572	645853	4753	354147	000425	09	999575	28
33	648274	4712	351726	648704	4722	351296	000430	09	999570	27
34	651102	4682	348898	651537	4691	348463	000436	09	999564	26
35	653911	4652	346089	654352	4661	345648	000442	10	999558	25
36	656702	4622	343298	657149	4631	342851	000447	10	999553	24
37	659475	4592	340525	659928	4602	340072	000453	10	999547	23
38	662230	4563	337770	662689	4573	337311	000459	10	999541	22
39	664968	4535	335032	665433	4544	334567	000465	10	999535	21
40	667689	4506	332311	668160	4516	331840	000471	10	999529	20
41	8.670393	4479	11.329607	8.670870	4488	11.329130	10.000476	10	9.999524	19
42	673080	4451	326920	673563	4461	326437	000482	10	999518	18
43	675751	4424	324249	676239	4434	323761	000488	10	999512	17
44	678405	4397	321595	678900	4417	321100	000494	10	999506	16
45	681043	4370	318957	681544	4380	318456	000500	10	999500	15
46	683665	4344	316335	684172	4354	315828	000507	10	999493	14
47	686272	4318	313728	686784	4328	313216	000513	10	999487	13
48	688863	4292	311137	689381	4303	310619	000519	10	999481	12
49	691438	4267	308562	691963	4277	308037	000525	10	999475	11
50	693998	4242	306002	694529	4252	305471	000531	10	999469	10
51	8.696543	4217	11.303457	8.697081	4228	11.302919	10.000537	11	9.999463	9
52	699073	4192	300927	699617	4203	300383	000544	11	999456	8
53	701589	4168	298411	702139	4179	297861	000550	11	999450	7
54	704090	4144	295910	704646	4155	295354	000557	11	999443	6
55	706577	4121	293423	707140	4132	292860	000563	11	999437	5
56	709049	4097	290951	709618	4108	290382	000569	11	999431	4
57	711507	4074	288493	712083	4085	287917	000576	11	999424	3
58	713952	4051	286048	714534	4062	285465	000582	11	999418	2
59	716383	4029	283617	716972	4040	283028	000589	11	999411	1
60	718800	4006	281200	719396	4017	280604	000596	11	999404	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	8.718800	4006	11.281200	8.719396	4017	11.280604	10 000596	11	9.999404	60
1	721204	3984	278796	721806	3995	278194	000602	11	999398	59
2	723595	3962	276405	724204	3974	275796	000609	11	999391	58
3	725972	3941	274028	726588	3952	273412	000616	11	999384	57
4	728337	3919	271663	728959	3931	271041	000622	11	999378	56
5	730688	3898	269312	731317	3909	268683	000629	11	999371	55
6	733027	3877	266973	733663	3889	266337	000636	12	999364	54
7	735354	3857	264646	735996	3868	264004	000643	12	999357	53
8	737667	3836	262333	738317	3848	261683	000650	12	999350	52
9	739969	3816	260031	740626	3827	259374	000657	12	999343	51
10	742259	3796	257741	742922	3807	257078	000664	12	999336	50
11	8.744536	3776	11.255464	8.745207	3787	11.254793	10.000671	12	9.999329	49
12	746802	3756	253198	747479	3768	252521	000678	12	999322	48
13	749055	3737	250945	749740	3749	250260	000685	12	999315	47
14	751297	3717	248703	751989	3729	248011	000692	12	999308	46
15	753528	3698	246472	754227	3710	245773	000699	12	999301	45
16	755747	3679	244253	756453	3692	243547	000706	12	999294	44
17	757955	3661	242045	758668	3673	241332	000714	12	999286	43
18	760151	3642	239849	760872	3655	239128	000721	12	999279	42
19	762337	3624	237663	763065	3636	236935	000728	12	999272	41
20	764511	3606	235489	765246	3618	234754	000735	12	999265	40
21	8.766675	3588	11.233325	8.767417	3600	11.232583	10.000743	12	9.999257	39
22	768828	3570	231172	769578	3583	230422	000750	13	999250	38
23	770970	3553	229030	771727	3565	228273	000758	13	999242	37
24	773101	3535	226899	773866	3548	226134	000765	13	999235	36
25	775223	3518	224777	775995	3531	224005	000773	13	999227	35
26	777333	3501	222667	778114	3514	221886	000780	13	999220	34
27	779434	3484	220566	780222	3497	219778	000788	13	999212	33
28	781524	3467	218476	782320	3480	217680	000795	13	999205	32
29	783605	3451	216395	784408	3464	215592	000803	13	999197	31
30	785675	3434	214325	786486	3447	213514	000811	13	999189	30
31	8.787736	3418	11.212264	8.788554	3431	11.211446	10.000819	13	9.999181	29
32	789787	3402	210213	790613	3415	209387	000826	13	999174	28
33	791828	3386	208172	792662	3399	207338	000834	13	999166	27
34	793859	3370	206141	794701	3383	205299	000842	13	999158	26
35	795881	3354	204119	796731	3368	203269	000850	13	999150	25
36	797894	3339	202106	798752	3352	201248	000858	13	999142	24
37	799897	3323	200103	800763	3337	199237	000866	13	999134	23
38	801892	3308	198108	802765	3322	197235	000874	13	999126	22
39	803876	3293	196124	804758	3307	195242	000882	13	999118	21
40	805852	3278	194148	806742	3292	193258	000890	13	999110	20
41	8.807819	3263	11.192181	8.808717	3278	11.191283	10.000898	13	9.999102	19
42	809777	3249	190223	810683	3262	189317	000906	14	999094	18
43	811726	3234	188274	812641	3248	187359	000914	14	999086	17
44	813667	3219	186333	814589	3233	185411	000923	14	999077	16
45	815599	3205	184401	816529	3219	183471	000931	14	999069	15
46	817522	3191	182478	818461	3205	181539	000939	14	999061	14
47	819436	3177	180564	820384	3191	179616	000947	14	999053	13
48	821343	3163	178657	822298	3177	177702	000956	14	999044	12
49	823240	3149	176760	824205	3163	175795	000964	14	999036	11
50	825130	3135	174870	826103	3150	173897	000973	14	999027	10
51	8.827011	3122	11.172989	8.827992	3136	11.172008	10.000981	14	9.999019	9
52	828884	3108	171116	829874	3123	170126	000990	14	999010	8
53	830749	3095	169251	831748	3109	168252	000998	14	999002	7
54	832607	3082	167393	833613	3096	166387	001007	14	998993	6
55	834456	3069	165544	835471	3083	164529	001016	14	998984	5
56	836297	3056	163703	837321	3070	162679	001024	14	998976	4
57	838130	3043	161870	839163	3057	160837	001033	15	998967	3
58	839956	3030	160044	840998	3045	159002	001042	15	998958	2
59	841774	3017	158226	842825	3032	157175	001050	15	998950	1
60	843585	3005	156415	844644	3019	155356	001059	15	998941	0
	Cosine		Secant	Cotang.		Tang.	Cosec.	.	Sine	M.

86 Degrees.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	8.843585	3005	11.156415	8.844644	3019	11.155356	10.001059	15	9.998941	60
1	845387	2992	154613	846455	3007	153545	001068	15	998932	59
2	847183	2980	152817	848260	2995	151740	001077	15	998923	58
3	848971	2967	151029	850057	2982	149943	001086	15	998914	57
4	850751	2955	149249	851846	2970	148154	001095	15	998905	56
5	852525	2943	147475	853628	2958	146372	001104	15	998896	55
6	854291	2931	145709	855403	2946	144597	001113	15	998887	54
7	856049	2919	143951	857171	2935	142829	001122	15	998878	53
8	857801	2908	142199	858932	2923	141068	001131	15	998869	52
9	859546	2896	140454	860686	2911	139314	001140	15	998860	51
10	861283	2884	138717	862433	2900	137567	001149	15	998851	50
11	8.863014	2873	11.136986	8.864173	2888	11.135827	10.001159	15	9.998841	49
12	864738	2861	135262	865906	2877	134094	001168	16	998832	48
13	866455	2850	133545	867632	2866	132368	001177	16	998823	47
14	868165	2839	131835	869351	2854	130649	001187	16	998813	46
15	869868	2828	130132	871064	2843	128936	001196	16	998804	45
16	871565	2817	128435	872770	2832	127230	001205	16	998795	44
17	873255	2806	126745	874469	2821	125531	001215	16	998785	43
18	874938	2795	125062	876162	2811	123838	001224	16	998776	42
19	876615	2786	123385	877849	2800	122151	001234	16	998766	41
20	878285	2773	121715	879529	2789	120471	001243	16	998757	40
21	8.879949	2763	11.120051	8.881202	2779	11.118798	10.001253	16	9.998747	39
22	881607	2752	118393	882869	2768	117131	001262	16	998738	38
23	883258	2742	116742	884530	2758	115470	001272	16	998728	37
24	884903	2731	115097	886185	2747	113815	001282	16	998718	36
25	886542	2721	113458	887833	2737	112167	001292	16	998708	35
26	888174	2711	111826	889476	2727	110524	001301	16	998699	34
27	889801	2700	110199	891112	2717	108888	001311	16	998689	33
28	891421	2690	108579	892742	2707	107258	001321	17	998679	32
29	893035	2680	106965	894366	2697	105634	001331	17	998669	31
30	894643	2670	105357	895984	2687	104016	001341	17	998659	30
31	8.896246	2660	11.103754	8.897596	2677	11.102404	10.001351	17	9.998649	29
32	897842	2651	102158	899203	2667	100797	001361	17	998639	28
33	899432	2641	100568	900803	2658	099197	001371	17	998629	27
34	901017	2631	098983	902398	2648	097602	001381	17	998619	26
35	902596	2622	097404	903987	2638	096013	001391	17	998609	25
36	904169	2612	095831	905570	2629	094430	001401	17	998599	24
37	905736	2603	094264	907147	2620	092853	001411	17	998589	23
38	907297	2593	092703	908719	2610	091281	001422	17	998578	22
39	908853	2584	091147	910285	2601	089715	001432	17	998568	21
40	910404	2575	089596	911846	2592	088154	001442	17	998558	20
41	8.911949	2566	11.088051	8.913401	2583	11.086599	10.001452	17	9.998548	19
42	913488	2556	086512	914951	2574	085049	001463	17	998537	18
43	915022	2547	084978	916495	2565	083505	001473	18	998527	17
44	916550	2538	083450	918034	2556	081966	001484	18	998516	16
45	918073	2529	081927	919568	2547	080432	001494	18	998506	15
46	919591	2520	080409	921096	2538	078904	001505	18	998495	14
47	921103	2512	078897	922619	2530	077381	001515	18	998485	13
48	922610	2503	077390	924136	2521	075864	001526	18	998474	12
49	924112	2494	075888	925649	2512	074351	001536	18	998464	11
50	925609	2486	074391	927156	2503	072844	001547	18	998453	10
51	8.927100	2477	11.072900	8.928658	2495	11.071342	10.001558	18	9.998442	9
52	928587	2469	071413	930155	2486	069845	001569	18	998431	8
53	930068	2460	069932	931647	2478	068353	001579	18	998421	7
54	931544	2452	068456	933134	2470	066866	001590	18	998410	6
55	933015	2443	066985	934616	2461	065384	001601	18	998399	5
56	934481	2435	065519	936093	2453	063907	001612	18	998388	4
57	935942	2427	064058	937565	2445	062435	001623	18	998377	3
58	937398	2419	062602	939032	2437	060968	001634	19	998366	2
59	938850	2411	061150	940494	2430	059506	001645	18	998355	1
60	940296	2403	059704	941952	2421	058048	001656	18	998344	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	8.940296	2403	11.059704	8.941952	2421	11.058048	10 001658	18	9.998344	60
1	941738	2394	058262	943404	2413	056596	001667	19	998333	59
2	943174	2387	056826	944852	2405	055148	001678	19	998322	58
3	944606	2379	055394	946295	2397	053705	001689	19	998311	57
4	946034	2371	053966	947734	2390	052266	001700	19	998300	56
5	947456	2363	052544	949168	2382	050832	001711	19	998289	55
6	948874	2355	051126	950597	2374	049403	001723	19	998277	54
7	950287	2348	049713	952021	2366	047979	001734	19	998266	53
8	951696	2340	048304	953441	2359	046559	001745	19	998255	52
9	953100	2332	046900	954856	2351	045144	001757	19	998243	51
10	954499	2325	045501	956267	2344	043733	001768	19	998232	50
11	8.955894	2317	11.044106	8.957674	2337	11.042326	10.001780	19	9.998220	49
12	957284	2310	042716	959075	2329	040925	001791	19	998209	48
13	958670	2302	041330	960473	2322	039527	001803	19	998197	47
14	960052	2295	039948	961866	2314	038134	001814	19	998186	46
15	961429	2288	038571	963255	2307	036745	001826	10	998174	45
16	962801	2280	037199	964639	2300	035361	001837	19	998163	44
17	964170	2273	035830	966019	2293	033981	001849	19	998151	43
18	965534	2266	034466	967394	2286	032606	001861	20	998139	42
19	966893	2259	033107	968766	2279	031234	001872	20	998128	41
20	968249	2252	031751	970133	2271	029867	001884	20	998116	40
21	8.969600	2245	11.030400	8.971496	2265	11.028504	10.001896	20	9.998104	39
22	970917	2238	029053	972855	2257	027145	001908	20	998092	38
23	972289	2231	027711	974209	2251	025791	001920	20	998080	37
24	973628	2224	026372	975560	2244	024440	001932	20	998068	36
25	974962	2217	025038	976906	2237	023094	001944	20	998056	35
26	976293	2210	023707	978248	2230	021752	001956	20	998044	34
27	977619	2203	022381	979586	2223	020414	001968	20	998032	33
28	978941	2197	021059	980921	2217	019079	001980	20	998020	32
29	980259	2190	019741	982251	2210	017749	001992	20	998008	31
30	981573	2183	018427	983577	2204	016423	002004	20	997996	30
31	8.982883	2177	11.017117	8.984899	2197	11.015101	10.002016	20	9.997984	29
32	984189	2170	015811	986217	2191	013783	002028	20	997972	28
33	985491	2163	014509	987532	2184	012468	002041	20	997959	27
34	986789	2157	013211	988842	2178	011158	002053	21	997947	26
35	988083	2150	011917	990149	2171	009851	002065	21	997935	25
36	989374	2144	010626	991451	2165	008549	002078	21	997922	24
37	990660	2138	009340	992750	2158	007250	002090	21	997910	23
38	991943	2131	008057	994045	2152	005955	002103	21	997897	22
39	993222	2125	006778	995337	2146	004663	002115	21	997885	21
40	994497	2119	005503	996624	2140	003376	002128	21	997872	20
41	8.995768	2112	11.004232	8.997908	2134	11 002092	10.002140	21	9.997860	19
42	997036	2106	002964	999188	2127	000812	002153	21	997847	18
43	998299	2100	001701	9.000465	2121	10.999535	002165	21	997835	17
44	999560	2094	000441	001738	2115	998262	002178	21	997822	16
45	9.000816	2088	10.999184	003007	2109	996993	002191	21	997809	15
46	002069	2082	997931	004272	2103	995728	002203	21	997797	14
47	003318	2076	996682	005534	2097	994466	002216	21	997784	13
48	004563	2070	995437	006792	2091	993208	002229	21	997771	12
49	005805	2064	994195	008047	2085	991953	002242	22	997758	11
50	007044	2058	992956	009298	2080	990702	002255	22	997745	10
51	9.008278	2052	10.991722	9.010546	2074	10.989454	10.002268	22	9.997732	9
52	009510	2046	990490	011790	2068	988210	002281	22	997719	8
53	010737	2040	989263	013031	2062	986969	002294	22	997706	7
54	011962	2034	988038	014268	2056	985732	002307	22	997693	6
55	013182	2029	986818	015502	2051	984498	002320	22	997680	5
56	014400	2023	985600	016732	2045	983268	002333	22	997667	4
57	015613	2017	984387	017959	2040	982041	002346	22	997654	3
58	016824	2012	983176	019183	2034	980817	002359	22	997641	2
59	018031	2006	981969	020403	2028	979597	002372	22	997628	1
60	019235	2000	980765	021620	2023	978380	002386	22	997614	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine	
0	9.019235	2000	10.980765	9.021620	2023	10.978380	10.002386	22	9.997614	60
1	020435	1995	979565	022834	2017	977166	002399	22	997601	59
2	021632	1989	978368	024044	2011	975956	002412	22	997588	58
3	022825	1984	977175	025251	2006	974749	002426	22	997574	57
4	024016	1978	975984	026455	2001	973545	002439	22	997561	56
5	025203	1973	974797	027655	1995	972345	002453	22	997547	55
6	026386	1967	973614	028852	1990	971148	002466	23	997534	54
7	027567	1962	972433	030046	1985	969954	002480	23	997520	53
8	028744	1957	971256	031237	1979	968763	002493	23	997507	52
9	029918	1951	970082	032425	1974	967575	002507	23	997493	51
10	031089	1946	968911	033609	1969	966391	002520	23	997480	50
11	9.032257	1941	10.967743	9.034791	1964	10.965209	10.002534	23	9.997466	49
12	033421	1936	966579	035969	1958	964031	002548	23	997452	48
13	034582	1930	965418	037144	1953	962856	002561	23	997439	47
14	035741	1925	964259	038316	1948	961684	002575	23	997425	46
15	036896	1920	963104	039485	1943	960515	002589	23	997411	45
16	038048	1915	961952	040651	1938	959349	002603	23	997397	44
17	039197	1910	960803	041813	1933	958187	002617	23	997383	43
18	040342	1905	959658	042973	1928	957027	002631	23	997369	42
19	041485	1899	958515	044130	1923	955870	002645	23	997355	41
20	042625	1895	957375	045284	1918	954716	002659	23	997341	40
21	9.043762	1889	10.956238	9.046434	1913	10.953566	10.002673	24	9.997327	39
22	044895	1884	955105	047582	1908	952418	002687	24	997313	38
23	046026	1879	953974	048727	1903	951273	002701	24	997299	37
24	047154	1875	952846	049869	1898	950131	002715	24	997285	36
25	048279	1870	951721	051008	1893	948992	002729	24	997271	35
26	049400	1865	950600	052144	1889	947856	002743	24	997257	34
27	050519	1860	949481	053277	1884	946723	002758	24	997242	33
28	051635	1855	948365	054407	1879	945593	002772	24	997228	32
29	052749	1850	947251	055535	1874	944465	002786	24	997214	31
30	053859	1845	946141	056659	1870	943341	002801	24	997199	30
31	9.054966	1841	10.945034	9.057781	1865	10.942219	10.002815	24	9.997185	29
32	056071	1836	943929	058900	1860	941100	002830	24	997170	28
33	057172	1831	942828	060016	1855	939984	002844	24	997156	27
34	058271	1827	941729	061130	1851	938870	002859	24	997141	26
35	059367	1822	940633	062240	1846	937760	002873	24	997127	25
36	060460	1817	939540	063348	1842	936652	002888	24	997112	24
37	061551	1813	938449	064453	1837	935547	002902	24	997098	23
38	062639	1808	937361	065556	1833	934444	002917	25	997083	22
39	063724	1804	936276	066655	1828	933345	002932	25	997068	21
40	064806	1799	935194	067752	1824	932248	092947	25	997053	20
41	9.065885	1794	10.934115	9.068846	1819	10.931154	10.002961	25	9.997039	19
42	066962	1790	933038	069938	1815	930062	002976	25	997024	18
43	068036	1786	931964	071027	1810	928973	002991	25	997009	17
44	069107	1781	930893	072113	1806	927887	003006	25	996994	16
45	070176	1777	929824	073197	1802	926803	003021	25	996979	15
46	071242	1772	928758	074278	1797	925722	003036	25	996964	14
47	072306	1768	927694	075356	1793	924644	003051	25	996949	13
48	073366	1763	926634	076432	1789	923568	003066	25	996934	12
49	074424	1759	925576	077505	1784	922495	003081	25	996919	11
50	075480	1755	924520	078576	1780	921424	003096	25	996904	10
51	9.076533	1750	10.923467	9.079644	1776	10.920356	10.003111	25	9.996889	9
52	077583	1746	922417	080710	1772	919290	003126	25	996874	8
53	078631	1742	921369	081773	1767	918227	003142	25	996858	7
54	079676	1738	920324	082833	1763	917167	003157	25	996843	6
55	080719	1733	919281	083891	1759	916109	003172	25	996828	5
56	081759	1729	918241	084947	1755	915053	003188	26	996812	4
57	082797	1725	917203	086000	1751	914000	003203	26	996797	3
58	083832	1721	916168	087050	1747	912950	003218	26	996782	2
59	084864	1717	915136	088098	1743	911902	003234	26	996766	1
60	085894	1713	914106	089144	1738	910856	003249	26	996751	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.085894	1713	10.914106	9.089144	1738	10.910856	10.003249	26	9.996751	60
1	086922	1709	913078	090187	1735	909813	003265	26	996735	59
2	087947	1704	912053	091228	1730	908772	003280	26	996720	58
3	088970	1700	911030	092266	1727	907734	003296	26	996704	57
4	089990	1696	910010	093302	1722	906698	003312	26	996688	56
5	091008	1692	908992	094336	1719	905664	003327	26	996673	55
6	092024	1688	907976	095367	1715	904633	003343	26	996657	54
7	093037	1684	906963	096395	1711	903605	003359	26	996641	53
8	094047	1680	905953	097422	1707	902578	003375	26	996625	52
9	095056	1676	904944	098446	1703	901554	003390	26	996610	51
10	096062	1673	903938	099468	1699	900532	003406	26	996594	50
11	9.097065	1668	10.902935	9.100487	1695	10.899513	10.003422	27	9.996578	49
12	098066	1665	901934	101504	1691	898496	003438	27	996562	48
13	099065	1661	900935	102519	1687	897481	003454	27	996546	47
14	100062	1657	899938	103532	1684	896468	003470	27	996530	46
15	101056	1653	898944	104542	1680	895458	003486	27	996514	45
16	102048	1649	897952	105550	1676	894450	003502	27	996498	44
17	103037	1645	896963	106556	1672	893444	003518	27	996482	43
18	104025	1642	895975	107559	1669	892441	003535	27	996465	42
19	105010	1638	894990	108560	1665	891440	003551	27	996449	41
20	105992	1634	894008	109559	1661	890441	003567	27	996433	40
21	9.106973	1630	10.893027	9.110556	1658	10.889444	10.003583	27	9.996417	39
22	107951	1627	892049	111551	1654	888449	003600	27	996400	38
23	108927	1623	891073	112543	1650	887457	003616	27	996384	37
24	109901	1619	890099	113533	1647	886467	003632	27	996368	36
25	110873	1616	889127	114521	1643	885479	003649	27	996351	35
26	111842	1612	888158	115507	1639	884493	003665	27	996335	34
27	112809	1608	887191	116491	1636	883509	003682	27	996318	33
28	113774	1605	886226	117472	1632	882528	003698	28	996302	32
29	114737	1601	885263	118452	1629	881548	003715	28	996285	31
30	115698	1597	884302	119429	1625	880571	003731	28	996269	30
31	9.116656	1594	10.883344	9.120404	1622	10.879596	10.003748	28	9.996252	29
32	117613	1590	882387	121377	1618	878623	003765	28	996235	28
33	118567	1587	881433	122348	1615	877652	003781	28	996219	27
34	119519	1583	880481	123317	1611	876683	003798	28	996202	26
35	120469	1580	879531	124284	1608	875716	003815	28	996185	25
36	121417	1576	878583	125249	1604	874751	003832	28	996168	24
37	122362	1573	877638	126211	1601	873789	003849	28	996151	23
38	123306	1569	876694	127172	1597	872828	003866	28	996134	22
39	124248	1566	875752	128130	1594	871870	003883	28	996117	21
40	125187	1562	874813	129087	1591	870913	003900	28	996100	20
41	9.126125	1559	10.873875	9.130041	1587	10.869959	10.003917	29	9.996083	19
42	127060	1556	872940	130994	1584	869006	003934	29	996066	18
43	127993	1552	872007	131944	1581	868056	003951	29	996049	17
44	128925	1549	871075	132893	1577	867107	003968	29	996032	16
45	129854	1545	870146	133839	1574	866161	003985	29	996015	15
46	130781	1542	869219	134784	1571	865216	004002	29	995998	14
47	131706	1539	868294	135726	1567	864274	004020	29	995980	13
48	132630	1535	867370	136667	1564	863333	004037	29	995963	12
49	133551	1532	866449	137605	1561	862395	004054	29	995946	11
50	134470	1529	865530	138542	1558	861458	004072	29	995928	10
51	9.135387	1525	10.864613	9.139476	1555	10.860524	10.004089	29	9.995911	9
52	136303	1522	863697	140409	1551	859591	004106	29	995894	8
53	137216	1519	862784	141340	1548	858660	004124	29	995876	7
54	138128	1516	861872	142269	1545	857731	004141	29	995859	6
55	139037	1512	860963	143196	1543	856804	004159	29	995841	5
56	139944	1509	860056	144121	1539	855879	004177	29	995823	4
57	140850	1506	859150	145044	1535	854956	004194	29	995806	3
58	141754	1503	858246	145966	1532	854034	004212	29	995788	2
59	142655	1500	857345	146885	1529	853115	004229	29	995771	1
60	143555	1496	856445	147803	1526	852197	004247	30	995753	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9 143555	1496	10.856445	9 147803	1526	10 852197	10.004247	30	9.995753	60
1	144453	1493	855547	148718	1523	851282	004265	30	995735	59
2	145349	1490	854651	149632	1520	850368	004283	30	995717	58
3	146243	1487	853757	150544	1517	849456	004301	30	995699	57
4	147136	1484	852864	151454	1514	848546	004319	30	995681	56
5	148026	1481	851974	152363	1511	847637	004336	30	995664	55
6	148915	1478	851085	153269	1508	846731	004354	30	995646	54
7	149802	1475	850198	154174	1505	845826	004372	30	995628	53
8	150686	1472	849314	155077	1502	844923	004390	30	995610	52
9	151569	1469	848431	155978	1499	844022	004409	30	995591	51
10	152451	1466	847549	156877	1496	843123	004427	30	995573	50
11	9.153330	1462	10.846670	9.157775	1493	10 842225	10 004445	30	9.995555	49
12	154208	1460	845792	158671	1490	841329	004463	30	995537	48
13	155083	1457	844917	159565	1487	840435	004481	30	995519	47
14	155957	1454	844043	160457	1484	839543	004499	31	995501	46
15	156830	1451	843170	161347	1481	838653	004518	31	995482	45
16	157700	1448	842300	162236	1478	837764	004536	31	995464	44
17	158569	1445	841431	163123	1476	836877	004554	31	995446	43
18	159435	1442	840565	164008	1473	835992	004573	31	995427	42
19	160301	1439	839699	164892	1470	835108	004591	31	995409	41
20	161164	1436	838836	165774	1467	834226	004610	31	995390	40
21	9.162025	1433	10 837975	9.166654	1464	10.833346	10.004628	31	9.995372	39
22	162885	1430	837115	167532	1461	832468	004647	31	995353	38
23	163743	1427	836257	168409	1458	831591	004656	31	995334	37
24	164600	1424	835400	169284	1455	830716	004684	31	995316	36
25	165454	1422	834546	170157	1453	829843	004703	31	995297	35
26	166307	1419	833693	171029	1450	828971	004722	31	995278	34
27	167159	1416	832841	171899	1447	828101	004740	31	995260	33
28	168008	1413	831992	172767	1444	827233	004759	32	995241	32
29	168856	1410	831144	173634	1442	826366	004778	32	995222	31
30	169702	1407	830298	174499	1439	825501	004797	32	995203	30
31	9.170547	1405	10 829453	9 175362	1436	10.824638	10.004816	32	9.995184	29
32	171389	1402	828611	176224	1433	823776	004835	32	995165	28
33	172230	1399	827770	177084	1431	822916	004854	32	995146	27
34	173070	1306	826930	177942	1428	822058	004873	32	995127	26
35	173908	1394	826092	178799	1425	821201	004892	32	995108	25
36	174744	1391	825256	179655	1423	820345	004911	32	995089	24
37	175578	1388	824422	180508	1420	819492	004930	32	995070	23
38	176411	1386	823589	181360	1417	818640	004949	32	995051	22
39	177242	1383	822758	182211	1415	817789	004968	32	995032	21
40	178072	1380	821928	183059	1412	816941	004987	32	995013	20
41	9 178900	1377	10.821100	9.183907	1409	10 816093	10.005007	32	9.994993	19
42	179726	1374	820274	184752	1407	815248	005026	32	994974	18
43	180551	1372	819449	185597	1404	814403	005045	32	994955	17
44	181374	1369	818626	186439	1402	813561	005065	32	994935	16
45	182196	1367	817804	187280	1399	812720	005084	33	994916	15
46	183016	1364	816984	188120	1396	811880	005104	33	994896	14
47	183834	1361	816166	188958	1393	811042	005123	33	994877	13
48	184651	1359	815349	189794	1391	810206	005143	33	994857	12
49	185466	1356	814534	190629	1389	809371	005162	33	994838	11
50	186280	1353	813720	191462	1386	808538	005182	33	994819	10
51	9.187092	1351	10.812908	9.192294	1384	10.807706	10.005202	33	9.994798	9
52	187903	1348	812097	193124	1381	806876	005221	33	994779	8
53	188712	1346	811288	193953	1379	806047	005241	33	994759	7
54	189519	1343	810481	194780	1376	805220	005261	33	994739	6
55	190325	1341	809675	195606	1374	804394	005281	33	994719	5
56	191130	1338	808870	196430	1371	803570	005300	33	994700	4
57	191933	1336	808067	197253	1369	802747	005320	33	994680	3
58	192734	1333	807266	198074	1366	801926	005340	33	994660	2
59	193534	1330	806466	198894	1364	801106	005360	33	994640	1
60	194332	1328	805668	199713	1361	800287	005380	33	994620	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

Tangents and Secants. (9 Degrees.)

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.194332	1328	10.805568	9.199713	1361	10.800287	10.005380	33	9.994620	60
1	195129	1326	804871	200529	1359	799471	005400	33	994600	59
2	195925	1324	804075	201345	1356	798655	005420	33	994580	58
3	196719	1321	803281	202159	1354	797841	005440	34	994560	57
4	197511	1318	802489	202971	1352	797029	005460	34	994540	56
5	198302	1316	801698	203782	1349	796218	005481	34	994519	55
6	199091	1313	800909	204592	1347	795408	005501	34	994499	54
7	199879	1311	800121	205400	1345	794600	005521	34	994479	53
8	200666	1308	799334	206207	1342	793793	005541	34	994459	52
9	201451	1306	798549	207013	1340	792987	005562	34	994438	51
10	202234	1304	797766	207817	1338	792183	005582	34	994418	50
11	9.203017	1301	10.796983	9.208619	1335	10.791381	10.005603	34	9.994397	49
12	203797	1299	796203	209420	1333	790580	005623	34	994377	48
13	204577	1296	795423	210220	1331	789780	005643	34	994357	47
14	205354	1294	794646	211018	1328	788982	005664	34	994336	46
15	206131	1292	793869	211815	1326	788185	005684	34	994316	45
16	206906	1289	793094	212611	1324	787389	005705	34	994295	44
17	207679	1287	792321	213405	1321	786595	005726	35	994274	43
18	208452	1285	791548	214198	1319	785802	005746	35	994254	42
19	209222	1282	790778	214989	1317	785011	005767	35	994233	41
20	209992	1280	790008	215780	1315	784220	005788	35	994212	40
21	9.210760	1278	10.789240	9.216568	1312	10.783432	10.005809	35	9.994191	39
22	211526	1275	788474	217356	1310	782644	005829	35	994171	38
23	212291	1273	787709	218142	1308	781858	005850	35	994150	37
24	213055	1271	786945	218926	1305	781074	005871	35	994129	36
25	213818	1268	786182	219710	1303	780290	005892	35	994108	35
26	214579	1266	785421	220492	1301	779508	005913	35	994087	34
27	215338	1264	784662	221272	1299	778728	005934	35	994066	33
28	216097	1261	783903	222052	1297	777948	005955	35	994045	32
29	216854	1259	783146	222830	1294	777170	005976	35	994024	31
30	217609	1257	782391	223606	1292	776394	005997	35	994003	30
31	9.218363	1255	10.781637	9.224382	1290	10.775618	10.006019	35	9.993981	29
32	219116	1253	780884	225156	1288	774844	006040	35	993960	28
33	219868	1250	780132	225929	1286	774071	006061	35	993939	27
34	220618	1248	789382	226700	1284	773300	006082	35	993918	26
35	221367	1246	778633	227471	1281	772529	006104	36	993896	25
36	222115	1244	777885	228239	1279	771761	006125	36	993875	24
37	222861	1242	777139	229007	1277	770993	006146	36	993854	23
38	223606	1239	776394	229773	1275	770227	006168	36	993832	22
39	224349	1237	775651	230530	1273	769461	006189	36	993811	21
40	225092	1235	774908	231302	1271	768698	006211	36	993789	20
41	9.225833	1233	10.774167	9.232065	1269	10.767935	10.006232	36	9.993768	19
42	226573	1231	773427	232826	1267	767174	006254	36	993746	18
43	227311	1228	772689	233586	1265	766414	006275	36	993725	17
44	228048	1226	771952	234345	1262	765655	006297	36	993703	16
45	228784	1224	771216	235103	1260	764897	006319	36	993681	15
46	229518	1222	770482	235859	1258	764141	006340	36	993660	14
47	230252	1220	769748	236614	1256	763386	006362	36	993638	13
48	230984	1218	769016	237368	1254	762632	006384	36	993616	12
49	231714	1216	768286	238120	1252	761880	006406	37	993594	11
50	232444	1214	767556	238872	1250	761128	006428	37	993572	10
51	9.233172	1212	10.766828	9.239622	1248	10.760378	10.006450	37	9.993550	9
52	233899	1209	766101	240371	1246	759629	006472	37	993528	8
53	234625	1207	765375	241118	1244	758882	006494	37	993506	7
54	235349	1205	764651	241865	1242	758135	006516	37	993484	6
55	236073	1203	763927	242610	1240	757390	006538	37	993462	5
56	236795	1201	763205	243354	1238	756646	006560	37	993440	4
57	237515	1199	762485	244097	1236	755903	006582	37	993418	3
58	238235	1197	761765	244839	1234	755161	006604	37	993396	2
59	238953	1195	761047	245579	1232	754421	006626	37	993374	1
60	239670	1193	760330	246319	1230	753681	006649	37	993351	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

80 Degrees.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.239670	1193	10.760330	9.246319	1230	10.753681	10.006649	37	9.993351	60
1	240386	1191	759614	247057	1228	752943	006671	37	993329	59
2	241101	1189	758899	247794	1226	752206	006693	37	993307	58
3	241814	1187	758186	248530	1224	751470	006715	37	993285	57
4	242526	1185	757474	249264	1222	750736	006738	37	993262	56
5	243237	1183	756763	249998	1220	750002	006760	37	993240	55
6	243947	1181	756053	250730	1218	749270	006783	38	993217	54
7	244656	1179	755344	251461	1217	748539	006805	38	993195	53
8	245363	1177	754637	252191	1215	747809	006828	38	993172	52
9	246069	1175	753931	252920	1213	747080	006851	38	993149	51
10	246775	1173	753225	253648	1211	746352	006873	38	993127	50
11	9.247478	1171	10.752522	9.254374	1209	10.745626	10.006896	38	9.993104	49
12	248181	1169	751819	255100	1207	744900	006919	38	993081	48
13	248883	1167	751117	255824	1205	744176	006941	38	993059	47
14	249583	1165	750417	256547	1203	743453	006964	38	993036	46
15	250282	1163	749718	257269	1201	742731	006987	38	993013	45
16	250980	1161	749020	257990	1200	742010	007010	38	992990	44
17	251677	1159	748323	258710	1198	741290	007033	38	992967	43
18	252373	1158	747627	259429	1196	740571	007056	38	992944	42
19	253067	1156	746933	260146	1194	739854	007079	38	992921	41
20	253761	1154	746239	260863	1192	739137	007102	38	992898	40
21	9.254453	1152	10.745547	9.261578	1190	10.738422	10.007125	38	9.992875	39
22	255144	1150	744856	262292	1189	737708	007148	38	992852	38
23	255834	1148	744166	263005	1187	736995	007171	39	992829	37
24	256523	1146	743477	263717	1185	736283	007194	39	992806	36
25	257211	1144	742789	264428	1183	735572	007217	39	992783	35
26	257898	1142	742102	265138	1181	734862	007241	39	992759	34
27	258583	1141	741417	265847	1179	734153	007264	39	992736	33
28	259268	1139	740732	266555	1178	733445	007287	39	992713	32
29	259951	1137	740049	267261	1176	732739	007310	39	992690	31
30	260633	1135	739367	267967	1174	732033	007334	39	992666	30
31	9.261314	1133	10.738686	9.268671	1172	10.731329	10.007357	39	9.992643	29
32	261994	1131	738006	269375	1170	730625	007381	39	992619	28
33	262673	1130	737327	270077	1169	729923	007404	39	992596	27
34	263351	1128	736649	270779	1167	729221	007428	39	992572	26
35	264027	1126	735973	271479	1165	728521	007451	39	992549	25
36	264703	1124	735297	272178	1164	727822	007475	39	992525	24
37	265377	1122	734623	272876	1162	727124	007499	39	992501	23
38	266051	1120	733949	273573	1160	726427	007522	40	992478	22
39	266723	1119	733277	274269	1158	725731	007546	40	992454	21
40	267395	1117	732605	274964	1157	725036	007570	40	992430	20
41	9.268065	1115	10.731935	9.275658	1155	10.724342	10.007594	40	9.992406	19
42	268734	1113	731266	276351	1153	723649	007618	40	992382	18
43	269402	1111	730598	277043	1151	722957	007641	40	992359	17
44	270069	1110	729931	277734	1150	722266	007665	40	992335	16
45	270735	1108	729265	278424	1148	721576	007689	40	992311	15
46	271400	1106	728600	279113	1147	720887	007713	40	992287	14
47	272064	1105	727936	279801	1145	720199	007737	40	992263	13
48	272726	1103	727274	280488	1143	719512	007761	40	992239	12
49	273388	1101	726612	281174	1141	718826	007786	40	992214	11
50	274049	1099	725951	281858	1140	718142	007810	40	992190	10
51	9.274708	1098	10.725292	9.282542	1138	10.717458	10 007834	40	9.992166	9
52	275367	1096	724633	283225	1136	716775	007858	40	992142	8
53	276024	1094	723976	283907	1135	716093	007883	41	992117	7
54	276681	1092	723319	284588	1133	715412	007907	41	992093	6
55	277337	1091	722663	285268	1131	714732	007931	41	992069	5
56	277991	1089	722009	285947	1130	714053	007956	41	992044	4
57	278644	1087	721356	286624	1128	713376	007980	41	992020	3
58	279297	1086	720703	287301	1126	712699	008004	41	991996	2
59	279948	1084	720052	287977	1125	712023	808029	41	991971	1
60	280599	1082	719401	288652	1123	711348	008053	41	991947	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.280599	1082	10.719401	9.288652	1123	10.711348	10.008053	41	9.991947	60
1	281248	1081	718752	289326	1122	710674	008078	41	991922	59
2	281897	1079	718103	289999	1120	710001	008103	41	991897	58
3	282544	1077	717456	290671	1118	709329	008127	41	991873	57
4	283190	1076	716810	291342	1117	708658	008152	41	991848	56
5	283836	1074	716164	292013	1115	707987	008177	41	991823	55
6	284480	1072	715520	292682	1114	707318	008201	41	991799	54
7	285124	1071	714876	293350	1112	706650	008226	42	991774	53
8	285766	1069	714234	294017	1111	705983	008251	42	991749	52
9	286408	1067	713592	294684	1109	705316	008276	42	991724	51
10	287048	1066	712952	295349	1107	704651	008301	42	991699	50
11	9.287687	1064	10.712313	9.296013	1106	10.703987	10.008326	42	9.991674	49
12	288326	1063	711674	296677	1104	703323	008351	42	991649	48
13	288964	1061	711036	297339	1103	702661	008376	42	991624	47
14	289600	1059	710400	298001	1101	701999	008401	42	991599	46
15	290236	1058	709764	298662	1100	701338	008426	42	991574	45
16	290870	1056	709130	299322	1098	700678	008451	42	991549	44
17	291504	1054	708496	299980	1096	700020	008476	42	991524	43
18	292137	1053	707863	300638	1095	699362	008502	42	991498	42
19	292768	1051	707232	301295	1093	698705	008527	42	991473	41
20	293399	1050	706601	301951	1092	698049	008552	42	991448	40
21	9.294029	1048	10.705971	9.302607	1090	10.697393	10.008578	42	9.991422	39
22	294658	1046	705342	303261	1089	696739	008603	42	991397	38
23	295286	1045	704714	303914	1087	696086	008628	43	991372	37
24	295913	1043	704087	304567	1086	695433	008654	43	991346	36
25	296539	1042	703461	305218	1084	694782	008679	43	991321	35
26	297164	1040	702836	305869	1083	694131	008705	43	991295	34
27	297788	1039	702212	306519	1081	693481	008730	43	991270	33
28	298412	1037	701588	307168	1080	692832	008756	43	991244	32
29	299034	1036	700966	307815	1078	692185	008782	43	991218	31
30	299655	1034	700345	308463	1077	691537	008807	43	991193	30
31	9.300276	1032	10.699724	9.309109	1075	10.690891	10.008833	43	9.991167	29
32	300895	1031	699105	309754	1074	690246	008859	43	991141	28
33	301514	1029	698486	310398	1073	689602	008885	43	991115	27
34	302132	1028	697868	311042	1071	688958	008910	43	991090	26
35	302748	1026	697252	311685	1070	688315	008936	43	991064	25
36	303364	1025	696636	312327	1068	687673	008962	43	991038	24
37	303979	1023	696021	312967	1067	687033	008988	43	991012	23
38	304593	1022	695407	313608	1065	686392	009014	43	990986	22
39	305207	1020	694793	314247	1064	685753	009040	43	990960	21
40	305819	1019	694181	314885	1062	685115	009066	44	990934	20
41	9.306430	1017	10.693570	9.315523	1061	10.684477	10.009092	44	9.990908	19
42	307041	1016	692959	316159	1060	683841	009118	44	990882	18
43	307650	1014	692350	316795	1058	683205	009145	44	990855	17
44	308259	1013	691741	317430	1057	682570	009171	44	990829	16
45	308867	1011	691133	318064	1055	681936	009197	44	990803	15
46	309474	1010	690526	318697	1054	681303	009223	44	990777	14
47	310080	1008	689920	319329	1053	680671	009250	44	990750	13
48	310685	1007	689315	319961	1051	680039	009276	44	990724	12
49	311289	1005	688711	320592	1050	679408	009303	44	990697	11
50	311893	1004	688107	321222	1048	678778	009329	44	990671	10
51	9.312495	1003	10.687505	9.321851	1047	10.678149	10.009356	44	9.990644	9
52	313097	1001	686903	322479	1045	677521	009382	44	990618	8
53	313698	1000	686302	323106	1044	676894	009409	44	990591	7
54	314297	998	685703	323733	1043	676267	009435	44	990565	6
55	314897	997	685104	324358	1041	675642	009462	44	990538	5
56	315495	996	684505	324983	1040	675017	009489	45	990511	4
57	316092	994	683908	325607	1039	674393	009515	45	990485	3
58	316689	993	683311	326231	1037	673769	009542	45	990458	2
59	317284	991	682716	326853	1036	673147	009569	45	990431	1
60	317879	990	682121	327475	1035	672525	009596	45	990404	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.317879	990	10.682121	9.327474	1035	10.672526	10.009596	45	9.990404	60
1	318473	988	681527	328095	1033	671905	009622	45	990378	59
2	319066	987	680934	328715	1032	671285	009649	45	990351	58
3	319658	986	680342	329334	1030	670666	009676	45	990324	57
4	320249	984	679751	329953	1029	670047	009703	45	990297	56
5	320840	983	679160	330570	1028	669430	009730	45	990270	55
6	321430	982	678570	331187	1026	668813	009757	45	990243	54
7	322019	980	677981	331803	1025	668197	009785	45	990215	53
8	322607	979	677393	332418	1024	667582	009812	45	990188	52
9	323194	977	676806	333033	1023	666967	009839	45	990161	51
10	323780	976	676220	333646	1021	666354	009866	45	990134	50
11	9.324366	975	10.675634	9.334259	1020	10.665741	10.009893	46	9.990107	49
12	324950	973	675050	334871	1019	665129	009921	46	990079	48
13	325534	972	674466	335482	1017	664518	009948	46	990052	47
14	326117	970	673883	336093	1016	663907	009975	46	990025	46
15	326700	969	673300	336702	1015	663298	010003	46	989997	45
16	327281	968	672719	337311	1013	662689	010030	46	989970	44
17	327862	966	672138	337919	1012	662081	010058	46	989942	43
18	328442	965	671558	338527	1011	661473	010085	46	989915	42
19	329021	964	670979	339133	1010	660867	010113	46	989887	41
20	329599	962	670401	339739	1008	660261	010140	46	989860	40
21	9.330176	961	10.669824	9.340344	1007	10.659656	10.010168	46	9.989832	39
22	330753	960	669247	340948	1006	659052	010196	46	989804	38
23	331329	958	668671	341552	1004	658448	010223	46	989777	37
24	331903	957	668097	342155	1003	657845	010251	47	989749	36
25	332478	956	667522	342757	1002	657243	010279	47	989721	35
26	333051	954	666949	343358	1000	656642	010307	47	989693	34
27	333624	953	666376	343958	999	656042	010335	47	989665	33
28	334195	952	665805	344558	998	655442	010363	47	989637	32
29	334766	950	665234	345157	997	654843	010391	47	989609	31
30	335337	949	664663	345755	996	654245	010418	47	988582	30
31	9 335906	948	10.664094	9.346353	994	10.653647	10.010447	47	9.989553	29
32	336475	946	663525	346949	993	653051	010475	47	989525	28
33	337043	945	662957	347545	992	652455	010503	47	989497	27
34	337610	944	662390	348141	991	651859	010531	47	989469	26
35	338176	943	661824	348735	990	651265	010559	47	989441	25
36	333742	941	661258	349329	988	650671	010587	47	989413	24
37	339306	940	660694	349922	987	650078	010616	47	989384	23
38	339871	939	660129	350514	986	649486	010644	47	989356	22
39	340434	937	659566	351106	985	648894	010672	47	989328	21
40	340996	936	659004	351697	983	648303	010700	47	989300	20
41	9.341558	935	10.658442	9 352287	982	10.647713	10.010729	47	9.989271	19
42	342119	934	557881	352876	981	647124	010757	47	989243	18
43	342679	932	657321	353465	980	646535	010786	47	989214	17
44	343239	931	656761	354053	979	645947	010814	47	989186	16
45	343797	930	656203	354640	977	645360	010843	47	989157	15
46	344355	929	655645	355227	976	644773	010872	48	989128	14
47	344912	927	655088	355813	975	644187	010900	48	989100	13
48	345469	926	654531	356398	974	643602	010929	48	989071	12
49	346024	925	653976	356982	973	643018	010958	48	989042	11
50	346579	924	653421	357566	971	642434	010986	48	989014	10
51	9.347134	922	10.652866	9.358149	970	10.641851	10.011015	48	9.988985	9
52	347687	921	652313	358731	969	641269	011044	48	988956	8
53	348240	920	651760	359313	968	640687	011073	48	988927	7
54	348792	919	651208	359893	967	640107	011102	48	988898	6
55	349343	917	650657	360474	966	639526	011131	48	988869	5
56	349893	916	650107	361053	965	638947	011160	48	988840	4
57	350443	915	649557	361632	963	638368	011189	49	988811	3
58	350992	914	649008	362210	962	637790	011218	49	988782	2
59	351540	913	648460	362787	961	637213	011247	49	988753	1
60	352088	911	647912	363364	960	636636	011276	49	988724	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.352088	911	10.647912	9.363364	960	10.636636	10.011276	49	9.988724	60
1	352635	910	647365	363940	959	636060	011305	49	988695	59
2	353181	909	646819	364515	958	635485	011334	49	988666	58
3	353726	908	646274	365090	957	634910	011364	49	988636	57
4	354271	907	645729	365664	955	634336	011393	49	988607	56
5	354815	905	645185	366237	954	633763	011422	49	988578	55
6	355358	904	644642	366810	953	633190	011452	49	988548	54
7	355901	903	644099	367382	952	632618	011481	49	988519	53
8	356443	902	643557	367953	951	632047	011511	49	988489	52
9	356984	901	643016	368524	950	631476	011540	49	988460	51
10	357524	899	642476	369094	949	630906	011570	49	988430	50
11	9.358064	898	10.641936	9.369663	948	10.630337	10.011599	49	9.988401	49
12	358603	897	641397	370232	946	629768	011629	49	988371	48
13	359141	896	640859	370799	945	629201	011658	49	988342	47
14	359678	895	640322	371367	944	628633	011688	50	988312	46
15	360215	893	639785	371933	943	628067	011718	50	988282	45
16	360752	892	639249	372499	942	627501	011748	50	988252	44
17	361287	891	638713	373064	941	626936	011777	50	988223	43
18	361822	890	638178	373629	940	626371	011807	50	988193	42
19	362356	889	637644	374193	939	625807	011837	50	988163	41
20	362889	888	637111	374756	938	625244	011867	50	988133	40
21	9.363422	887	10.636578	9.375319	937	10.624681	10.011897	50	9.988103	39
22	363954	885	636046	375881	935	624119	011927	50	988073	38
23	364485	884	635515	376442	934	623558	011957	50	988043	37
24	365016	883	634984	377003	933	622997	011987	50	988013	36
25	365546	882	634454	377563	932	622437	012017	50	987983	35
26	366075	881	633925	378122	931	621878	012048	50	987953	34
27	366604	880	633396	378681	930	621319	012078	50	987922	33
28	367131	879	632869	379239	929	620761	012108	50	987892	32
29	367659	877	632341	379797	928	620203	012138	50	987862	31
30	368185	876	631815	380354	927	619646	012168	51	987832	30
31	9.368711	875	10.631289	9.380910	926	10.619090	10.012199	51	9.987801	29
32	369236	874	630764	381466	925	618534	012229	51	987771	28
33	369761	878	630239	382020	924	617980	012260	51	987740	27
34	370285	872	629715	382575	923	617425	012290	51	987710	26
35	370808	871	629192	383129	922	616871	012321	51	987679	25
36	371330	870	628670	383682	921	616318	012351	51	987049	24
37	371852	869	628148	384234	920	615766	012382	51	987618	23
38	372373	867	627627	384786	919	615214	012412	51	987588	22
39	372894	866	627106	385337	918	614663	012443	51	987557	21
40	373414	865	626586	385888	917	614112	012474	51	987526	20
41	9.373933	864	10.626067	9.386438	915	10.613562	10.012505	51	9.987496	19
42	374452	863	625548	386987	914	613013	012535	51	987465	18
43	374970	862	625030	387536	913	612464	012566	51	987434	17
44	375487	861	624513	388084	912	611916	012597	52	987403	16
45	376003	860	623997	388631	911	611369	012628	52	987372	15
46	376519	859	623481	389178	910	610822	012659	52	987341	14
47	377035	858	622965	389724	909	610276	012690	52	987310	13
48	377549	857	622451	390270	908	609730	012721	52	987279	12
49	378063	856	621937	390815	907	609185	012752	52	987248	11
50	378577	854	621423	391360	906	608640	012783	52	987217	10
51	9.379089	853	10.620911	9.391903	905	10.608097	10.012814	52	9.987186	9
52	379601	852	620399	392447	904	607553	012845	52	987155	8
53	380113	851	619887	392989	903	607011	012876	52	987124	7
54	380624	850	619376	393531	902	606469	012908	52	987092	6
55	381134	849	618866	394073	901	605927	012039	52	987061	5
56	381643	848	618357	394614	900	605386	012970	52	987030	4
57	382152	847	617848	395154	899	604846	013002	52	986998	3
58	382661	846	617339	395694	898	604306	013033	52	986967	2
59	383168	845	616832	396233	897	603767	013064	52	986936	1
60	383675	844	616325	396771	896	603229	013096	52	986904	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.383675	844	10.616325	9.396771	896	10.603229	10.013096	52	9.986904	60
1	384182	843	615818	397309	896	602691	013127	53	986873	59
2	384687	842	615313	397846	895	602154	013159	53	986841	58
3	385192	841	614808	398383	894	601617	013191	53	986809	57
4	385697	840	614303	398919	893	601081	013222	53	986778	56
5	386201	839	613799	399455	892	600545	013254	53	986746	55
6	386704	838	613296	399990	891	600010	013286	53	986714	54
7	387207	837	612793	400524	890	599476	013317	53	986683	53
8	387709	836	612291	401058	889	598942	013349	53	986651	52
9	388210	835	611790	401591	888	598409	013381	53	986619	51
10	388711	834	611289	402124	887	597876	013413	53	986587	50
11	9.389211	833	10.610789	9.402656	886	10.597344	10.013445	53	9.986555	49
12	389711	832	610289	403187	885	596813	013477	53	986523	48
13	390210	831	609790	403718	884	596282	013509	53	986491	47
14	390708	830	609292	404249	883	595751	013541	53	986459	46
15	391206	828	608794	404778	882	595222	013573	53	986427	45
16	391703	827	608297	405308	881	594692	013605	53	986395	44
17	392199	826	607801	405836	880	594164	013637	54	986363	43
18	392695	825	607305	406364	879	593636	013669	54	986331	42
19	393191	824	606809	406892	878	593108	013701	54	986299	41
20	393685	823	606315	407419	877	592581	013734	54	986266	40
21	9.394179	822	10.605821	9.407945	876	10.592055	10.013766	54	9.986234	39
22	394673	821	605327	408471	875	591529	013798	54	986202	38
23	395166	820	604834	408997	874	591003	013831	54	986169	37
24	395658	819	604342	409521	874	590479	013863	54	986137	36
25	396150	818	603850	410045	873	589955	013896	54	986104	35
26	396641	817	603359	410569	872	589431	013928	54	986072	34
27	397132	816	602868	411092	871	588908	013961	54	986039	33
28	397621	816	602379	411615	870	588385	013993	54	986007	32
29	398111	815	601889	412137	869	587863	014026	54	985974	31
30	398600	814	601400	412658	868	587342	014058	54	985942	30
31	9.399088	813	10.600912	9.413179	867	10.586821	10.014091	55	9.985909	29
32	399575	812	600425	413699	866	586301	014124	55	985876	28
33	400062	811	599938	414219	865	585781	014157	55	985843	27
34	400549	810	599451	414738	864	585262	014189	55	985811	26
35	401035	809	598965	415257	864	584743	014222	55	985778	25
36	401520	808	598480	415775	863	584225	014255	55	985745	24
37	402005	807	597995	416293	862	583707	014288	55	985712	23
38	402489	806	597511	416810	861	583190	014321	55	985679	22
39	402972	805	597028	417326	860	582674	014354	55	985646	21
40	403455	804	596545	417842	859	582158	014387	55	985613	20
41	9.403938	803	10.596062	9.418358	858	10.581642	10.014420	55	9.985580	19
42	404420	802	595580	418873	857	581127	014453	55	985547	18
43	404901	801	595099	419387	856	580613	[illegible]486	55	985514	17
44	405382	800	594618	419901	855	580099	014520	55	985480	16
45	405862	799	594138	420415	855	579585	014553	55	985447	15
46	406341	798	593659	420927	854	579073	014586	55	985414	14
47	406820	797	593180	421440	853	578560	014620	56	985380	13
48	407299	796	592701	421952	852	578048	014653	56	985347	12
49	407777	795	592223	422463	851	577537	014686	56	985314	11
50	408254	794	591746	422974	850	577026	014720	56	985280	10
51	9.408731	794	10.591269	9.423484	849	10.576516	10.014753	56	9.985247	9
52	409207	793	590793	423993	848	576007	014787	56	985213	8
53	409682	792	590318	424503	848	575497	014820	56	985180	7
54	410157	791	589843	425011	847	574989	014854	56	985146	6
55	410632	790	589368	425519	846	574481	014887	56	985113	5
56	411106	789	588894	426027	845	573973	014921	56	985079	4
57	411579	788	588421	426534	844	573466	014955	56	985045	3
58	412052	787	587948	427041	843	572959	014989	56	985011	2
59	412524	786	587476	427547	843	572453	015022	56	984978	1
60	412996	785	587004	428052	842	571948	015056	56	984944	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

75 Degrees.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.412996	785	10.587004	9.428052	842	10.571948	10.015056	57	9.984944	60
1	413467	784	586533	428557	841	571443	015090	57	984910	59
2	413938	783	586062	429062	840	570938	015124	57	984876	58
3	414408	783	585592	429566	839	570434	015158	57	984842	57
4	414878	782	585122	430070	838	569930	015192	57	984808	56
5	415347	781	584653	430573	838	569427	015226	57	984774	55
6	415815	780	584185	431075	837	568925	015260	57	984740	54
7	416283	779	583717	431577	836	568423	015294	57	984706	53
8	416751	778	583249	432079	835	567921	015328	57	984672	52
9	417217	777	582783	432580	834	567420	015363	57	984638	51
10	417684	776	582316	433080	833	566920	015397	57	984603	50
11	9.418150	775	10.581850	9.433580	832	10.566420	10 015431	57	9.984569	49
12	418615	774	581385	434080	832	565920	015465	57	984535	48
13	419079	773	580921	434579	831	565421	015500	57	984500	47
14	419544	773	580456	435078	830	564922	015534	57	984466	46
15	420007	772	579993	435576	829	564424	015568	58	984432	45
16	420470	771	579530	436073	828	563927	015603	58	984397	44
17	420933	770	579067	436570	828	563430	015637	58	984363	43
18	421395	769	578605	437067	827	562933	015672	58	984328	42
19	421857	768	578143	437563	826	562437	015706	58	984294	41
20	422318	767	577682	438059	825	561941	015741	58	984259	40
21	9.422778	767	10.577222	9.438554	824	10.561446	10 015776	58	9 984224	39
22	423238	766	576762	439048	823	560952	015810	58	984190	38
23	423697	765	576303	439543	823	560457	015845	58	984155	37
24	424156	764	575844	440036	822	559964	015880	58	984120	36
25	424615	763	575385	440529	821	559471	015915	58	984085	35
26	425073	762	574927	441022	820	558978	015950	58	984050	34
27	425530	761	574470	441514	819	558486	015985	58	984015	33
28	425987	760	574013	442006	819	557994	016019	58	983981	32
29	426443	760	573557	442497	818	557503	016054	58	983946	31
30	426899	759	573101	442988	817	557012	016089	58	983911	30
31	9.427354	758	10.572646	9.443479	816	10.556521	10 016125	58	9.983875	29
32	427809	757	572191	443968	816	556032	016160	59	983840	28
33	428263	756	571737	444458	815	555542	016195	59	983805	27
34	428717	755	571283	444947	814	555053	016230	59	983770	26
35	429170	754	570830	445435	813	554565	016265	59	983735	25
36	429623	753	570377	445923	812	554077	016300	59	983700	24
37	430075	752	569925	446411	812	553589	016336	59	983664	23
38	430527	752	569473	446898	811	553102	016371	59	983629	22
39	430978	751	569022	447384	810	552616	016406	59	983594	21
40	431429	750	568571	447870	809	552130	016442	59	983558	20
41	9 431879	749	10.568121	9.448356	809	10.551644	10.016477	59	9.983523	19
42	432329	749	567671	448841	808	551159	016513	59	983487	18
43	432778	748	567222	449326	807	550674	016548	59	983452	17
44	433226	747	566774	449810	806	550190	016584	59	983416	16
45	433675	746	566325	450294	806	549706	016619	59	983381	15
46	434122	745	565878	450777	805	549223	016655	59	983345	14
47	434569	744	565431	451260	804	548740	016691	59	983309	13
48	435016	744	564984	451743	803	548257	016727	60	983273	12
49	435462	743	564538	452225	802	547775	016762	60	983238	11
50	435908	742	564092	452706	802	547294	016798	60	983202	10
51	9.436353	741	10.563647	9.453187	801	10.546813	10 016834	60	9.983166	9
52	436798	740	563202	453668	800	546332	016870	60	983130	8
53	437242	740	562758	454148	799	545852	016906	60	983094	7
54	437686	739	562314	454628	799	545372	016942	60	983058	6
55	438129	738	561871	455107	798	544893	016978	60	983022	5
56	438572	737	561428	455586	797	544414	017014	60	982986	4
57	439014	736	560986	456064	796	543936	017050	60	982950	3
58	439456	736	560544	456542	796	543458	017086	60	982914	2
59	439897	735	560103	457019	795	542981	017122	60	982878	1
60	440338	734	559662	457496	794	542504	017158	60	982842	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.440338	734	10 559662	9 457496	794	10 542504	10.017158	60	9.982842	60
1	440778	733	559222	457973	793	542027	017195	60	982805	59
2	441218	732	558782	458449	793	541551	017231	61	982769	58
3	441658	731	558342	458925	792	541075	017267	61	982733	57
4	442096	731	557904	459400	791	540600	017304	61	982696	56
5	442535	730	557465	459875	790	540125	017340	61	982660	55
6	442973	729	557027	460349	790	539651	017376	61	982624	54
7	443410	728	556590	460823	789	539177	017413	61	982587	53
8	443847	727	556153	461297	788	538703	017449	61	982551	52
9	444284	727	555716	461770	788	538230	017486	61	982514	51
10	444720	726	555280	442242	787	537758	017523	61	982477	50
11	9.445155	725	10 554845	9 462714	786	10 537286	10 017559	61	9 982441	49
12	445590	724	554410	463186	785	536814	017596	61	982404	48
13	446025	723	553975	463658	785	536342	017633	61	982367	47
14	446459	723	553541	464129	784	535871	017669	61	982331	46
15	446893	722	553107	464599	783	535401	017706	61	982294	45
16	447326	721	552674	465069	783	534931	017743	61	982257	44
17	447759	720	552241	465539	782	534461	017780	62	982220	43
18	448191	720	551809	466008	781	533992	017817	62	982183	42
19	448623	719	551377	466476	780	533524	017854	62	982146	41
20	449054	718	550946	466945	780	533055	017891	62	982109	40
21	9 449485	717	10 550515	9 467413	779	10 532587	10 017928	62	9.982072	39
22	449915	716	550085	467880	778	532120	017965	62	982035	38
23	450345	716	549655	468347	778	531653	018002	62	981998	37
24	450775	715	549225	468814	777	531186	018039	62	981961	36
25	451204	714	548796	469280	776	530720	018076	62	981924	35
26	451632	713	548368	469746	775	530254	018114	62	981886	34
27	452060	713	547940	470211	775	529789	018151	62	981849	33
28	452488	712	547512	470676	774	529324	018188	62	981812	32
29	452915	711	547085	471141	773	528859	018226	62	981774	31
30	453342	710	546658	471605	773	528395	018263	62	981737	30
31	9.453768	710	10 546232	9 472068	772	10.527932	10.018301	63	9 981699	29
32	454194	709	545806	472532	771	527468	018338	63	981662	28
33	454619	708	545381	472995	771	527005	018375	63	981625	27
34	455044	707	544956	473457	770	526543	018413	63	981587	26
35	455469	707	544531	473919	769	526081	018451	63	981549	25
36	455893	706	544107	474381	769	525619	018488	63	981512	24
37	456316	705	543684	474842	768	525158	018526	63	981474	23
38	456739	704	543261	475303	767	524697	018564	63	981436	22
39	457162	704	542838	475763	767	524237	018601	63	981399	21
40	457584	703	542416	476223	766	523777	018639	63	981361	20
41	9 458006	702	10 541994	9.476683	765	10.523317	10 018677	63	9 981323	19
42	458427	701	541573	477142	765	522858	018715	63	981285	18
43	458848	701	541152	477601	764	522399	018753	63	981247	17
44	459268	700	540732	478059	763	521941	018791	63	981209	16
45	459688	699	540312	478517	763	521483	018829	63	981171	15
46	460108	698	539892	478975	762	521025	018867	64	981133	14
47	460527	698	539473	479432	761	520568	018905	64	981095	13
48	460946	697	539054	479889	761	520111	018943	64	981057	12
49	461364	696	538636	480345	760	519655	018981	64	981019	11
50	461782	695	538218	480801	759	519199	019019	64	980981	10
51	9.462199	695	10 537801	9.481257	759	10.518743	10 019058	64	9 980942	9
52	462616	694	537384	481712	758	518288	019096	64	980904	8
53	463032	693	536968	482167	757	517833	019134	64	980866	7
54	463448	693	536552	482621	757	517379	019173	64	980827	6
55	463864	692	536136	483075	756	516925	019211	64	980789	5
56	464279	691	535721	483529	755	516471	019250	64	980750	4
57	464694	690	535306	483982	755	516018	019288	64	980712	3
58	465108	690	534892	484435	754	515565	019327	64	980673	2
59	465522	689	534478	484887	753	515113	019365	64	980635	1
60	465935	688	534065	485339	753	514661	019404	64	980596	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.465935	688	10.534065	9.485339	753	10.514661	10.019404	64	9.980596	60
1	466348	688	533652	485791	752	514209	019442	64	980558	59
2	466761	687	533239	486242	751	513758	019481	65	980519	58
3	467173	686	532827	486693	751	513307	019520	65	980480	57
4	467585	685	532415	487143	750	512857	019558	65	980442	56
5	467996	685	532004	487593	749	512407	019597	65	980403	55
6	468407	684	531593	488043	749	511957	019636	65	980364	54
7	468817	683	531183	488492	748	511508	019675	65	980325	53
8	469227	683	530773	488941	747	511059	019714	65	980286	52
9	469637	682	530363	489390	747	510610	019753	65	980247	51
10	470046	681	529954	489838	746	510162	019792	65	980208	50
11	9.470455	680	10.529545	9 490286	746	10.509714	10.019831	65	9.980169	49
12	470863	680	529137	490733	745	509267	019870	65	980130	48
13	471271	679	528729	491180	744	508820	019909	65	980091	47
14	471679	678	528321	491627	744	508373	019948	65	980052	46
15	472086	678	527914	492073	743	507927	019988	65	980012	45
16	472492	677	527508	492519	743	507481	020027	65	979973	44
17	472898	676	527102	492965	742	507035	020066	66	979934	43
18	473304	676	526696	493410	741	506590	020105	66	979895	42
19	473710	675	526290	493854	740	506146	020145	66	979855	41
20	474115	674	525885	494299	740	505701	020184	66	979816	40
21	9.474519	674	10.525481	9.494743	739	10.505257	10.020224	66	9.979776	39
22	474923	673	525077	495186	739	504814	020263	66	979737	38
23	475327	872	524673	495630	738	504370	020303	66	979697	37
24	475730	672	524270	496073	737	503927	020342	66	979658	36
25	476133	671	523867	496515	737	503485	020382	66	979618	35
26	476536	670	523464	496957	736	503043	020421	66	979579	34
27	476938	669	523062	497399	736	502601	020461	66	979539	33
28	477340	669	522660	497841	735	502159	020501	66	979499	32
29	477741	668	522259	498282	734	501718	020541	66	979459	31
30	478142	667	521858	498722	734	501278	020580	66	979420	30
31	9.478542	667	10.521458	9.499163	733	10.500837	10.020620	66	9.979380	29
32	478942	666	521058	499603	733	500397	020660	66	979340	28
33	479342	665	520658	500042	732	499958	020700	67	979300	27
34	479741	665	520259	500481	731	499519	020740	67	979260	26
35	480140	664	519860	500920	731	499080	020780	67	979220	25
36	480539	663	519461	501359	730	498641	020820	67	979180	24
37	480937	663	519063	501797	730	498203	020860	67	979140	23
38	481334	662	518666	502235	729	497765	020900	67	979100	22
39	481731	661	518269	502672	728	497328	020941	67	979059	21
40	482128	661	517872	503109	728	496891	020981	67	979019	20
41	9.482525	660	10.517475	9.503546	727	10.496454	10.021021	67	9.978979	19
42	482921	659	517079	503982	727	496018	021061	67	978939	18
43	483316	659	516684	504418	726	495582	021102	67	978898	17
44	483712	658	516288	504854	725	495146	021142	67	978858	16
45	484107	657	515893	505289	725	494711	021183	67	978817	15
46	484501	657	515499	505724	724	494276	021223	67	978777	14
47	484895	656	515105	506159	724	493841	021264	67	978736	13
48	485289	655	514711	506593	723	493407	021304	67	978696	12
49	485682	655	514318	507027	722	492973	021345	68	978655	11
50	486075	654	513925	507460	722	492540	021385	68	978615	10
51	9.486467	653	10.513533	9.507893	721	10.492107	10.021426	68	9.978574	9
52	486860	653	513140	508326	721	491674	021467	68	978533	8
53	487251	652	512749	508759	720	491241	021507	68	978493	7
54	487643	651	512357	509191	719	490809	021548	68	978452	6
55	488034	651	511966	509622	719	490378	021589	68	978411	5
56	488424	650	511576	510054	718	489946	021630	68	978370	4
57	488814	650	511186	510485	718	489515	021671	68	978329	3
58	489204	649	510796	510916	717	489084	021712	68	978288	2
59	489593	648	510407	511346	717	488654	021753	68	978247	1
60	489982	648	510018	511776	716	488224	021794	68	978206	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.489982	648	10.510018	9.511776	716	10 488224	10.021794	68	9.978206	60
1	490371	648	509629	512206	716	487794	021835	68	978165	59
2	490759	647	509241	512635	715	487365	021876	68	978124	58
3	491147	646	508853	513064	714	486936	021917	69	978083	57
4	491535	646	508465	513493	714	486507	021958	69	978042	56
5	491922	645	508078	513921	713	486079	021999	69	978001	55
6	492308	644	507692	514349	713	485651	022041	69	977959	54
7	492695	644	507305	514777	712	485223	022082	69	977918	53
8	493081	643	506919	515204	712	484796	022123	69	977877	52
9	493466	642	506534	515631	711	484369	022165	69	977835	51
10	493851	642	506149	516057	710	483943	022206	69	977794	50
11	9.494236	641	10.505764	9.516484	710	10.483516	10.022248	69	9.977752	49
12	494621	641	505379	516910	709	483090	022289	69	977711	48
13	495005	640	504995	517335	709	482665	022331	69	977669	47
14	495388	639	504612	517761	708	482239	022372	69	977628	46
15	495772	639	504228	518185	708	481815	022414	69	977586	45
16	496154	638	503846	518610	707	481390	022456	70	977544	44
17	496537	637	503463	519034	706	480966	022497	70	977503	43
18	496919	637	503081	519458	706	480542	022539	70	977461	42
19	497301	636	502699	519882	705	480118	022581	70	977419	41
20	497682	636	502318	520305	705	479695	022623	70	977377	40
21	9.498064	635	10.501936	9.520728	704	10.479272	10.022665	70	9.977335	39
22	498444	634	501556	521151	703	478849	022707	70	977293	38
23	498825	634	501175	521573	703	478427	022749	70	977251	37
24	499204	633	500796	521995	703	478005	022791	70	977209	36
25	499584	632	500416	522417	702	477583	022833	70	977167	35
26	499963	632	500037	522838	702	477162	022875	70	977125	34
27	500342	631	499658	523259	701	476741	022917	70	977083	33
28	500721	631	499279	523680	701	476320	022959	70	977041	32
29	501099	630	498901	524100	700	475900	023001	70	976999	31
30	501476	629	498524	524520	699	475480	023043	70	976957	30
31	9.501854	629	10.498146	9.524939	699	10.475061	10 023086	70	9.976914	29
32	502231	628	497769	525359	698	474641	023128	71	976872	28
33	502607	628	497393	525778	698	474222	023170	71	976830	27
34	502984	627	497016	526197	697	473803	023213	71	976787	26
35	503360	626	496640	526615	697	473385	023255	71	976745	25
36	503735	626	496265	527033	696	472967	023298	71	976702	24
37	504110	625	495890	527451	696	472549	023340	71	976660	23
38	504485	625	495515	527868	695	472132	023383	71	976617	22
39	504860	624	425140	538285	695	471715	023426	71	976574	21
40	505234	623	494766	528702	694	471298	023468	71	976532	20
41	9.505608	623	10.494392	9.529119	693	10.470881	10.023511	71	9.976489	19
42	505981	623	494019	529535	693	470465	023554	71	976446	18
43	506354	622	493646	529950	693	470050	023596	71	976404	17
44	506727	621	493273	530366	692	469634	023639	71	976361	16
45	507099	620	492901	530781	691	469219	023682	71	976318	15
46	507471	620	492529	531196	691	468804	023725	71	976275	14
47	507843	619	492157	531611	690	468389	023768	72	974232	13
48	508214	619	491786	532025	690	467975	023811	72	976189	12
49	508585	618	491415	532439	689	467561	023854	72	976146	11
50	508956	618	491044	532853	689	467147	023897	72	976103	10
51	9.509326	617	10.490674	9.533266	688	10.466734	10.023940	72	9.976060	9
52	509696	616	490304	533679	688	466321	023983	72	976017	8
53	510065	616	489935	534092	687	465908	024026	72	975974	7
54	510434	615	489566	534504	687	465496	024070	72	975930	6
55	510803	615	489197	534916	686	465084	024113	72	975887	5
56	511172	614	488828	535328	686	464672	024156	72	975844	4
57	511540	613	488460	535739	685	464261	024200	72	975800	3
58	511907	613	488093	536150	685	463850	024243	72	975757	2
59	512275	612	487725	536561	684	463439	024286	72	975714	1
60	512642	612	487358	536972	684	463028	024330	72	975670	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

Degrees.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.512642	612	10.487358	9.536972	684	10.463028	10.024330	73	9.975670	60
1	513009	611	486991	537382	683	462618	024373	73	975627	59
2	513375	611	486625	537792	683	462208	024417	73	975583	58
3	513741	610	486259	538202	682	461798	024461	73	975539	57
4	514107	609	485893	538611	682	461389	024504	73	975496	56
5	514472	609	485528	539020	681	460980	024548	73	975452	55
6	514837	608	485163	539429	681	460571	024592	73	975408	54
7	515202	608	484798	539837	680	460163	024635	73	975365	53
8	515566	607	484434	540245	680	459755	024679	73	975321	52
9	515930	607	484070	540653	679	459347	024723	73	975277	51
10	516294	606	483706	541061	679	458939	024767	73	975233	50
11	9.516657	605	10.483343	9.541468	678	10.458532	10.024811	73	9.975189	49
12	517020	605	482980	541875	678	458125	024855	73	975145	48
13	517382	604	482618	542281	677	457719	024899	73	975101	47
14	517745	604	482255	542688	677	457312	024943	73	975057	46
15	518107	603	481893	543094	676	456906	024987	73	975013	45
16	518468	603	481532	543499	676	456501	025031	74	974969	44
17	518829	602	481171	543905	675	456095	025075	74	974925	43
18	519190	601	480810	544310	675	455690	025120	74	974880	42
19	519551	601	480449	544715	674	455285	025164	74	974836	41
20	519911	600	480089	545119	674	454881	025208	74	974792	40
21	9.520271	600	10.479729	9.545524	673	10.454476	10.025252	74	9.974748	39
22	520631	599	479369	545928	673	454072	025297	74	974703	38
23	520990	599	479010	546331	672	453669	025341	74	974659	37
24	521349	598	478651	546735	672	453265	025386	74	974614	36
25	521707	598	478293	547138	671	452862	025430	74	974570	35
26	522066	597	477934	547540	671	452460	025475	74	974525	34
27	522424	596	477576	547943	670	452057	025519	74	974481	33
28	522781	596	477219	548345	670	451655	025564	74	974436	32
29	523138	595	476862	548747	669	451253	025609	74	974391	31
30	523495	595	476505	549149	669	450851	025653	75	974347	30
31	9.523852	594	10.476148	9.549550	668	10.450450	10.025698	75	9.974302	29
32	524208	594	475792	549951	668	450049	025743	75	974257	28
33	524564	593	475436	550352	667	449648	025788	75	974212	27
34	524920	593	475080	550752	667	449248	025833	75	974167	26
35	525275	592	474725	551152	666	448848	025878	75	974122	25
36	525630	591	474370	551552	666	448448	025923	75	974077	24
37	525984	591	474016	551952	665	448048	025968	75	974032	23
38	526339	590	473661	552351	665	447649	026013	75	973987	22
39	526693	590	473307	552750	665	447250	026058	75	973942	21
40	527046	589	472954	553149	664	446851	026103	75	973897	20
41	9.527400	589	10.472600	9.553548	664	10.446452	10.026148	75	9.973852	19
42	527753	588	472247	553946	663	446054	026193	75	973807	18
43	528105	588	471895	554343	663	445656	026239	75	973761	17
44	528458	587	471542	554741	662	445259	026284	76	973716	16
45	528810	587	471190	555139	662	444861	026329	76	973671	15
46	529161	586	470839	555536	661	444464	026375	76	973625	14
47	529513	586	470487	555933	661	444067	026420	76	973580	13
48	529864	585	470136	556329	660	443671	026465	76	973535	12
49	530215	585	469785	556725	660	443275	026511	76	973489	11
50	530565	584	469435	557121	659	442879	026556	76	973444	10
51	9.530915	584	10.469085	9.557517	659	10.442483	10.026602	76	9.973398	9
52	531265	583	468735	557913	659	442087	026648	76	973352	8
53	531614	582	468386	558308	658	441692	026693	76	973307	7
54	531963	582	468037	558702	658	441298	026739	76	973261	6
55	532312	581	467688	559097	657	440903	026785	76	973215	5
56	532661	581	467339	559491	657	440509	026831	76	973169	4
57	533009	580	466991	559885	656	440115	026876	76	973124	3
58	533357	580	466643	560279	656	439721	026922	76	973078	2
59	533704	579	466296	560673	655	439327	026968	77	973032	1
60	534052	578	465948	561066	655	438934	027014	77	972986	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

70 Degrees.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.534052	578	10.465948	9.561066	655	10.438934	10.027014	77	9.972986	60
1	534399	577	465601	561459	654	438541	027060	77	972940	59
2	534745	577	465255	561851	654	438149	027106	77	972894	58
3	535092	577	464908	562244	653	437756	027152	77	972848	57
4	535438	576	464562	562636	653	437364	027198	77	972802	56
5	535783	576	464217	563028	653	436972	027245	77	972755	55
6	536129	575	463871	563419	652	436581	027291	77	972709	54
7	536474	574	463526	563811	652	436189	027337	77	972663	53
8	536818	574	463182	564202	651	435798	027383	77	972617	52
9	537163	573	462837	564592	651	435408	027430	77	972570	51
10	537507	573	462493	564983	650	435017	027476	77	972524	50
11	9.537851	572	10.462149	9.565373	650	10.434627	10.027522	77	9.972478	49
12	538194	572	461806	565763	649	434237	027569	78	972431	48
13	538538	571	461462	566153	649	453847	027615	78	972385	47
14	538880	571	461120	566542	649	433458	027662	78	972338	46
15	539223	570	460777	566932	648	433068	027709	78	972291	45
16	539565	570	460435	567320	648	432680	027755	78	972245	44
17	539907	569	460093	567709	647	432291	027802	78	972198	43
18	540249	569	459751	568098	647	431902	027849	78	972151	42
19	540590	568	459410	568486	646	431514	027895	78	972105	41
20	540931	568	459069	568873	646	431127	027942	78	972058	40
21	9.541272	567	10.458728	9.569261	645	10.430739	10.027989	78	9.972011	39
22	541613	567	458387	569648	645	430352	028036	78	971964	38
23	541953	566	458047	570035	645	429965	028083	78	971917	37
24	542293	566	457707	570422	644	429578	028130	78	971870	36
25	542632	565	457368	570809	644	429191	028177	78	971823	35
26	542971	565	457029	571195	643	428805	028224	78	971776	34
27	543310	564	456690	571581	643	428419	028271	79	971729	33
28	543649	564	456351	571967	642	428033	028318	79	971682	32
29	543987	563	456013	572352	642	427648	028665	79	971635	31
30	544325	563	455675	572738	642	427262	028112	79	971588	30
31	9.544663	562	10.455337	9.573123	641	10.426877	10 028460	79	9.971540	29
32	545000	562	455000	573507	641	426493	028507	79	971493	28
33	545338	561	454662	573892	640	426108	028554	79	971446	27
34	545674	561	454326	574276	640	425724	028602	79	971398	26
35	546011	560	453989	574660	639	425340	028649	79	971351	25
36	546347	560	453653	575044	639	424956	028697	79	971303	24
37	546683	559	453317	575427	639	424573	028744	79	971256	23
38	547019	559	452981	575810	638	424190	028792	79	971208	22
39	547354	558	452646	576193	638	423807	028839	79	971161	21
40	547689	558	452311	576576	637	423424	028887	79	971113	20
41	9.548024	557	10.451976	9.576959	637	10.423041	10.028934	80	9.971066	19
42	548359	557	451641	577341	636	422659	028982	80	971018	18
43	548693	556	451307	577723	636	422277	029030	80	970970	17
44	549027	556	450973	578104	636	421896	029078	80	970922	16
45	549360	555	450640	578486	635	421514	029126	80	970874	15
46	549693	555	450307	578867	635	421133	029173	80	970827	14
47	550026	554	449974	579248	634	420752	029221	80	970779	13
48	550359	554	449641	579629	634	420371	029269	80	970731	12
49	550692	553	449308	580009	634	419991	029317	80	970683	11
50	551024	553	448976	580389	033	419611	029365	80	970635	10
51	9.551356	552	10.448644	9.580769	633	10.419231	10.029414	80	9.970586	9
52	551687	552	448313	581149	632	418851	029462	80	970538	8
53	552018	552	447982	581528	632	418472	029510	80	970490	7
54	552349	551	447651	581907	632	418093	029558	80	970442	6
55	552680	551	447320	582286	631	417714	029606	80	970394	5
56	553010	550	446990	582665	631	417335	029655	81	970345	4
57	553341	550	446659	583043	630	416957	029703	81	970297	3
58	553670	549	446330	583422	630	416578	029751	81	970249	2
59	554000	549	446000	583800	629	416200	029800	81	970200	1
60	554329	548	445671	584177	629	415823	029848	81	970152	0
	Cosine.		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.554329	548	10.445671	9.584177	629	10.415823	10.029848	81	9.970152	60
1	554658	548	445342	584555	629	415445	029897	81	970103	59
2	554987	547	445013	584932	628	415068	029945	81	970055	58
3	555315	547	444685	585309	628	414691	029994	81	970006	57
4	555643	546	444357	585686	627	414314	030043	81	969957	56
5	555971	546	444029	586062	627	413938	030091	81	969909	55
6	556299	545	443701	586439	627	413561	030140	81	969860	54
7	556626	545	443374	586815	626	413185	030189	81	969811	53
8	556953	544	443047	587190	626	412810	030238	81	969762	52
9	557280	544	442720	587566	625	412434	030286	81	969714	51
10	557606	543	442394	587941	625	412059	030335	81	969665	50
11	9.557932	543	10.442068	9.588316	625	10.411684	10.030384	82	9.969616	49
12	558258	543	441742	588691	624	411309	030433	82	969567	48
13	558583	542	441417	589066	624	410934	030482	82	969518	47
14	558909	542	441091	589440	623	410560	030531	82	969469	46
15	559234	541	440766	589814	623	410186	030580	82	969420	45
16	559558	541	440442	590188	623	409312	030630	82	969370	44
17	559883	540	440117	590562	622	409438	030679	82	909321	43
18	560207	540	439793	590935	622	409065	030728	82	969272	42
19	560531	539	439469	591308	622	408692	030777	82	969223	41
20	560855	539	429145	591681	621	408319	030827	82	969173	40
21	9.561178	538	10.438822	9.592054	621	10.407946	10.030876	82	9.969124	39
22	561501	538	438499	592426	620	407574	030925	82	969075	38
23	561824	537	438176	592798	620	407202	030975	82	969025	37
24	562146	537	437854	593171	620	406829	031024	82	968976	36
25	562468	536	437532	593542	619	406458	031074	83	968926	35
26	562790	536	437210	593914	619	406086	031123	83	968877	34
27	563112	536	436888	594285	618	405715	031173	83	968827	33
28	563433	535	436567	594656	618	405344	031223	83	968777	32
29	563755	535	436245	595022	618	404973	031272	83	968728	31
30	564075	534	435925	595398	617	404602	031322	83	968678	30
31	9.564396	534	10.435604	9.595768	617	10.404232	10.031372	83	9.968628	29
32	564716	533	435284	596138	616	403862	031422	83	968578	28
33	565036	533	434964	596508	616	403492	031472	83	968528	27
34	565356	532	434644	596878	616	403122	031521	83	968479	26
35	565676	532	434324	597247	615	402753	031571	83	968429	25
36	565995	531	434005	597616	615	402384	031621	83	968379	24
37	566314	531	433686	597985	615	402015	031671	83	968329	23
38	566632	531	433368	598354	614	401646	031722	83	968278	22
39	566951	530	433049	598722	614	401278	031772	84	968228	21
40	567269	530	332731	599091	613	400909	031822	84	968178	20
41	9.567587	529	10.432413	9.599459	613	10.400541	10.031872	84	9.968128	19
42	567904	529	432096	599827	613	400173	031922	84	968078	18
43	568222	528	431778	600194	612	399806	031973	84	968027	17
44	568539	528	431461	600562	612	399438	032023	84	967977	16
45	568856	528	431144	600929	611	399071	032073	84	967927	15
46	569172	527	430828	601296	611	398704	032124	84	967876	14
47	569488	527	430512	601662	611	398338	032174	84	967826	13
48	569804	526	430196	602029	610	397971	032225	84	967775	12
49	570120	526	429880	602395	610	397605	032275	84	967725	11
50	570435	525	429565	602761	610	397239	032326	84	967674	10
51	9.570751	525	10.429249	9.603127	609	10.396873	10.032376	84	9.967624	9
52	571066	524	428934	603493	609	396507	032427	84	967573	8
53	571380	524	428620	603858	609	396142	032478	85	967522	7
54	571695	523	428305	604223	608	395777	032529	85	967471	6
55	572009	523	427991	604588	608	395412	032579	85	967421	5
56	572323	523	427677	604953	607	395047	032630	85	967370	4
57	572636	522	427364	605317	607	394683	032681	85	967319	3
58	572950	522	427050	604682	607	394318	032732	85	967268	2
59	573263	521	426737	606046	606	393954	032783	85	997217	1
60	573575	521	426425	606410	606	393590	032834	85	967166	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.573575	521	10.426425	9.606410	606	10.393590	10.032834	85	9.967166	60
1	573888	520	426112	606773	606	393227	032885	85	967115	59
2	574200	520	425800	607137	605	392863	032936	85	967064	58
3	574512	519	425488	607500	605	392500	032987	85	967013	57
4	574824	519	425176	607863	604	392137	033039	85	966961	56
5	575136	519	424864	608225	604	391775	033090	85	966910	55
6	575447	518	424553	608588	604	391412	033141	85	966859	54
7	575758	518	424242	608950	603	391050	033192	85	966808	53
8	576069	517	423931	609312	603	390688	033244	86	966756	52
9	576379	517	423621	609674	603	390326	033295	86	966705	51
10	576689	516	423311	610036	602	389964	033347	86	966653	50
11	9.576999	516	10.423001	9.610397	602	10.389603	10.033398	86	9.966602	49
12	577309	516	422691	610759	602	389241	033450	86	966550	48
13	577618	515	422382	611120	601	388880	033501	86	966499	47
14	577927	515	422073	611480	601	388520	033553	86	966447	46
15	578236	514	421764	611841	601	388159	033605	86	966395	45
16	578545	514	421455	612201	600	387799	033656	86	966344	44
17	578853	513	421147	612561	600	387439	033708	86	966292	43
18	579162	513	420838	612921	600	387079	033760	86	966240	42
19	579470	513	420530	613281	599	386719	033812	86	966188	41
20	579777	512	420223	613641	599	386359	033864	86	966136	40
21	9.580085	512	10.419915	9.614000	598	10 386000	10.033915	87	9.966085	39
22	580392	511	419608	614359	598	385641	033967	87	966033	38
23	580699	511	419301	614718	598	385282	034019	87	965981	37
24	581005	511	418995	615077	597	384923	034072	87	965928	36
25	581312	510	418688	615435	597	384565	034124	87	965876	35
26	581618	510	418382	615793	597	384207	034176	87	965824	34
27	581924	509	418076	616151	596	383849	034228	87	965772	33
28	582229	509	417771	616509	596	383491	034280	87	965720	32
29	582535	509	417465	616867	596	383133	034332	87	965668	31
30	582840	508	417160	617224	595	382776	034385	87	965615	30
31	9.583145	508	10.416855	9.617582	595	10.382418	10.034437	87	9.965563	29
32	583449	507	416551	617939	595	382061	034489	87	965511	28
33	583754	507	416246	618295	594	381705	034542	87	965458	27
34	584058	506	415942	618652	594	381348	034594	87	965406	26
35	584361	506	415639	619008	594	380992	034647	88	965353	25
36	584665	506	415335	619364	593	380636	034699	88	965301	24
37	584968	505	415032	619721	593	380279	034752	88	965248	23
38	585272	505	414728	620076	593	379924	034805	88	966195	22
39	585574	504	414426	620432	592	379568	034857	88	965143	21
40	585877	504	414123	620787	592	379213	034910	88	965090	20
41	9.586179	503	10.413821	9.621142	592	10.378858	10.034963	88	9.965037	19
42	586482	503	413518	621497	591	378503	035016	88	964984	18
43	586783	503	413217	621852	591	378148	035069	88	964931	17
44	587085	502	412915	622207	590	377793	035121	88	964879	16
45	587386	502	412614	622561	590	377439	035174	88	964826	15
46	587688	501	412312	622915	590	377085	035227	88	964773	14
47	587989	501	412011	623269	589	376731	035281	88	964719	13
48	588289	501	411711	623623	589	376377	035334	89	964666	12
49	588590	500	411410	623976	589	376024	035386	89	964613	11
50	588890	500	411110	624330	588	375670	035440	89	964560	10
51	9.589190	499	10.410810	9.624683	588	10.375317	10.035493	89	9.964507	9
52	589489	499	410511	625036	588	374964	035546	89	964454	8
53	589789	499	410211	625388	587	374612	035600	89	964400	7
54	590088	498	409912	625741	587	374259	035653	89	964347	6
55	590387	498	409613	626093	587	373907	035706	89	964294	5
56	590686	497	409314	626445	586	373555	035760	89	964240	4
57	590984	497	409016	626797	586	373203	035813	89	964187	3
58	591282	497	408718	627149	586	372851	035867	89	964133	2
59	591580	496	408420	627501	585	372499	035920	89	964080	1
60	591878	496	408122	627852	585	372148	035974	89	964026	0
	Cosine		Secant	Cotang		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.591878	496	10.408122	9.627852	585	10.372148	10.035974	89	9.964026	60
1	592176	495	407824	628203	585	371797	036028	89	963972	59
2	592473	495	407527	628554	585	371446	036081	89	963919	58
3	592770	495	407230	628905	584	371095	036135	90	963865	57
4	593067	494	406933	629255	584	370745	036189	90	963811	56
5	593363	494	406637	629606	583	370394	036243	90	963757	55
6	593659	493	406341	629956	583	370044	036296	90	963704	54
7	593955	493	406045	630306	583	369694	036350	90	963650	53
8	594251	493	405749	630656	583	369344	036404	90	963596	52
9	594547	492	405453	631005	582	368995	036458	90	963542	51
10	594842	492	405158	631355	582	368645	036512	90	963488	50
11	9.595137	491	10.404863	9.631704	582	10.368296	10.036566	90	9.963434	49
12	595432	491	404568	632053	581	367947	036621	90	963379	48
13	595727	491	404273	632401	581	367599	036675	90	963325	47
14	·596021	490	403979	632750	581	367250	036729	90	963271	46
15	596315	490	403685	633098	580	366902	036783	90	963217	45
16	596609	489	403391	633447	580	366553	036837	90	963163	44
17	596903	489	403097	633795	580	366205	036892	91	963108	43
18	597196	489	402804	634143	579	365857	036946	91	963054	42
19	597490	488	402510	634490	579	365510	037001	91	962999	41
20	597783	488	402217	634838	579	365162	037055	91	962945	40
21	9.598075	487	10.401925	9.635185	578	10.364815	10.037110	91	9 962890	39
22	598368	487	401632	635532	578	364468	037164	91	962836	38
23	598660	487	401340	635879	578	364121	037219	91	962781	37
24	598952	486	401048	636226	577	363774	037273	91	962727	36
25	599244	486	400756	636572	577	363428	037328	91	962672	35
26	599536	485	400464	636919	577	363081	037383	91	932617	34
27	599827	485	400173	637265	577	362735	037438	91	962562	33
28	600118	485	399882	637611	676	362389	037492	91	962508	32
29	600409	484	399591	637956	576	362044	037547	91	962453	31
30	600700	484	399300	638302	576	361698	037602	92	962398	30
31	9.600990	484	10.399010	9.638647	575	10.361353	10.037657	92	9 962343	29
32	601280	483	398720	638992	575	361008	037712	92	962288	28
33	601570	483	398430	639337	575	360663	037767	92	962233	27
34	601860	482	398140	639682	574	360318	037822	92	962178	26
35	602150	482	397850	640027	574	359973	037877	92	962123	25
36	602439	482	397561	640371	574	359629	037933	92	962067	24
37	602728	481	397272	640716	573	359284	037988	92	962012	23
38	603017	481	396983	641060	573	358940	038043	92	961957	22
39	603305	481	396695	641404	573	358596	038098	92	961902	21
40	603594	480	396406	641747	572	358253	038154	92	961846	20
41	9.603882	480	10.396118	9.642091	572	10.357909	10 038209	92	9.961791	19
42	604170	479	395830	642434	572	357566	038265	92	961735	18
43	604457	479	395543	642777	572	357223	038320	92	961680	17
44	604745	479	395255	643120	571	356880	038376	93	961624	16
45	605032	478	394968	643463	571	356537	038431	93	961569	15
46	605319	478	394681	643806	571	356194	038487	93	961513	14
47	605606	478	394394	644148	570	355852	038542	93	961458	13
48	605892	477	394108	644490	570	355510	038598	93	961402	12
49	606179	477	393821	644832	570	355168	038654	93	961346	11
50	606465	476	393535	645174	569	354826	038710	93	961290	10
51	9.606751	476	10.393249	9.645516	569	10.354484	10 038765	93	9.961235	9
52	607036	476	392964	645857	569	354143	038821	93	961179	8
53	607322	475	392678	646199	569	353801	038877	93	961123	7
54	607607	475	392393	646540	568	353460	038933	93	961067	6
55	607892	474	392108	646881	568	353119	038989	93	961011	5
56	608177	474	391823	647222	568	352778	039045	93	960955	4
57	608461	474	391539	647562	567	352438	039101	93	960899	3
58	608745	473	391255	647903	567	352097	039157	94	960843	2
59	609029	473	390971	648243	567	351757	039214	94	960786	1
60	609313	473	390687	648583	566	351417	039270	94	960730	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.609313	473	10.390687	9.648583	566	10.351417	10.039270	94	9.960730	60
1	609597	472	390403	648923	566	351077	039326	94	960674	59
2	609880	472	390120	649263	566	350737	039382	94	960618	58
3	610164	472	389836	649602	566	350398	039439	94	960561	57
4	610447	471	389553	649942	565	350058	039495	94	960505	56
5	610729	471	389271	650281	565	349719	039552	94	960448	55
6	611012	470	388988	650620	565	349380	039608	94	960392	54
7	611294	470	388706	650959	564	349041	039665	94	960335	53
8	611576	470	388424	651297	564	348703	039721	94	960279	52
9	611858	469	388142	651636	564	348364	039778	94	960222	51
10	612140	469	387860	651974	563	348026	039835	94	960165	50
11	9.612421	469	10.387579	9.652312	563	10.347688	10.039891	95	9.960109	49
12	612702	468	387298	652650	563	347350	039948	95	960052	48
13	612983	468	387017	652988	563	347012	040005	95	959995	47
14	613264	467	386736	653326	562	346674	040062	95	959938	46
15	613545	467	386455	653663	562	346337	040118	95	959882	45
16	613825	467	386175	654000	562	346000	040175	95	959825	44
17	614105	466	385895	654337	561	345663	040232	95	959768	43
18	614385	466	385615	654674	561	345326	040289	95	959711	42
19	614665	466	385335	655011	561	344989	040346	95	959654	41
20	614944	465	385056	655348	561	344652	040404	95	959596	40
21	9.615223	465	10.384777	9.655684	560	10.344316	10.040461	95	9.959539	39
22	615502	465	384498	656020	560	343980	040518	95	959482	38
23	615781	464	384219	656356	560	343644	040575	95	959425	37
24	616060	464	383940	656692	559	343308	040632	95	959368	36
25	616338	464	383662	657028	559	342972	040690	96	959310	35
26	616616	463	383384	657364	559	342636	040747	96	959253	34
27	616894	463	383106	657699	559	342301	040805	96	959195	33
28	617172	462	382828	658034	558	341966	040862	96	959138	32
29	617450	462	382550	658369	558	341631	040919	96	959081	31
30	617727	462	382273	658704	558	341296	040977	96	959023	30
31	9.618004	461	10.381996	9.659039	558	10.340961	10.041035	96	9.958965	29
32	618281	461	381719	659373	557	340627	041092	96	958908	28
33	618558	461	381442	659708	557	340292	041150	96	958850	27
34	618834	460	381166	660042	557	339958	041208	96	958792	26
35	619110	460	380890	660376	557	339624	041266	96	958734	25
36	619386	460	380614	660710	556	339290	041323	96	958677	24
37	619662	459	380338	661043	556	338957	041381	96	958619	23
38	619938	459	380062	661377	556	338623	041439	96	958561	22
39	620213	459	379787	661710	555	338290	041497	97	958503	21
40	620488	458	379512	662043	555	337957	041555	97	958445	20
41	9.620763	458	10.379237	9.662376	555	10.337624	10.041613	97	9.958387	19
42	621038	457	378962	662709	554	337291	041671	97	958329	18
43	621313	457	378687	663042	554	336958	041729	97	958271	17
44	621587	457	378413	663375	554	336625	041787	97	958213	16
45	621861	456	378139	663707	554	336293	041846	97	958154	15
46	622135	456	377865	664039	553	335961	041904	97	958096	14
47	622409	456	377591	664371	553	335629	041962	97	958038	13
48	622682	455	377318	664703	553	335297	042021	97	957979	12
49	622956	455	377044	665035	553	334965	042079	97	957921	11
50	623229	455	376771	665366	552	334634	042137	97	957863	10
51	9.623502	454	10.376498	9.665697	552	10.334303	10.042196	97	9.957804	9
52	623774	454	376226	666029	552	333971	042254	98	957746	8
53	624047	454	375953	666360	551	333640	042313	98	957687	7
54	624319	453	375681	666691	551	333309	042372	98	957628	6
55	624591	453	375409	667021	551	332979	042430	98	957570	5
56	624863	453	375137	667352	551	332648	042489	98	957511	4
57	625135	452	374865	667682	550	332318	042548	98	957452	3
58	625406	452	374594	668013	550	331987	042607	98	957393	2
59	625677	452	374323	668343	550	331657	042665	98	957335	1
60	625948	451	374052	668672	550	331328	042724	98	957276	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.625948	451	10.374052	9.668673	550	10.331327	10.042724	98	9.957276	60
1	626219	451	373781	669002	549	330998	042783	98	957217	59
2	626490	451	373510	669332	549	330668	042842	98	957158	58
3	626760	450	373240	669661	549	330339	042901	98	957099	57
4	627030	450	372970	669991	548	330009	042960	98	957040	56
5	627300	450	372700	670320	548	329680	043019	98	956981	55
6	627570	449	372430	670649	548	329351	043079	99	956921	54
7	627840	449	372160	670977	548	329023	043138	99	956862	53
8	628109	449	371891	671306	547	328694	043197	99	956803	52
9	628378	448	371622	671634	547	328366	043256	99	956744	51
10	628647	448	371353	671963	547	328037	043316	99	956684	50
11	9.628916	447	10.371084	9.672291	547	10.327709	10.043375	99	9.956625	49
12	629185	447	370815	672619	546	327381	043434	99	956566	48
13	629453	447	370547	672946	546	327053	043494	99	956506	47
14	629721	446	370279	673274	546	326726	043553	99	956447	46
15	629989	446	370011	673602	546	326398	043613	99	956387	45
16	630257	446	369743	673929	545	326071	043673	99	956327	44
17	630524	446	369476	674257	545	325743	043732	99	956268	43
18	630792	445	369208	674584	545	325416	043792	100	956208	42
19	631059	445	368941	674910	544	325090	043852	100	956148	41
20	631826	445	368674	675237	544	324763	043911	100	956089	40
21	9.631593	444	10.368407	9.675564	544	10.324436	10.043971	100	9.956029	39
22	631859	444	368141	675890	544	324110	044031	100	955969	38
23	632125	444	367875	676216	543	323784	044091	100	955909	37
24	632392	443	367608	676543	543	323457	044151	100	955849	36
25	632658	443	367342	676869	543	323131	044211	100	955789	35
26	632923	443	367077	677194	543	322806	044271	100	955729	34
27	633189	442	366811	677520	542	322480	044331	100	955669	33
28	633454	442	366546	677846	542	322154	044391	100	955609	32
29	633719	442	366281	678171	542	321829	044452	100	955548	31
30	633984	441	366016	678496	542	321504	044512	100	955488	30
31	9.634249	441	10.365751	9.678821	541	10.321179	10.044572	101	9.955428	29
32	634514	440	365486	679146	541	320854	044632	101	955368	28
33	634778	440	365222	679471	541	320529	044693	101	955307	27
34	635042	440	364958	679795	541	320205	044753	101	955247	26
35	635306	439	364694	680120	540	319880	044814	101	955186	25
36	635570	439	364430	680444	540	319556	044874	101	955126	24
37	635834	439	364166	680768	540	319232	044935	101	955065	23
38	636097	438	363903	681092	540	318908	044995	101	955005	22
39	636360	438	363640	681416	539	318584	045056	101	954944	21
40	636623	438	363377	681740	539	318260	045117	101	954883	20
41	9.636886	437	10.363114	9.682063	539	10.317937	10.045177	101	9.954823	19
42	637148	437	362852	682387	539	317613	045238	101	954762	18
43	637411	437	362589	682710	538	317290	045299	101	954701	17
44	637673	437	362327	683033	538	316967	045360	101	954640	16
45	637935	436	362065	683356	538	316644	045421	101	954579	15
46	638197	436	361803	683679	538	316321	045482	102	954518	14
47	638458	436	361542	684001	537	315999	045543	102	954457	13
48	638720	435	361280	684324	537	315676	045604	102	954396	12
49	638981	435	361019	684646	537	315354	045665	102	954335	11
50	639242	435	360758	684968	537	315032	045726	102	954274	10
51	9.639503	434	10.360497	9.685290	536	10.314710	10.045787	102	9.954213	9
52	639764	434	360236	685612	536	314388	045848	102	954152	8
53	640024	434	359976	685934	536	314066	045910	102	954090	7
54	640284	433	359716	686255	536	313745	045971	102	954029	6
55	640544	433	359456	686577	535	313423	046032	102	953968	5
56	640804	433	359196	686898	535	313102	046094	102	953906	4
57	641064	432	358936	687219	535	312781	046155	102	953845	3
58	641324	432	358677	687540	535	312460	046217	102	953783	2
59	641584	432	358416	687861	534	312139	046278	103	953722	1
60	641842	431	358158	688182	534	311818	046340	103	953660	0
	Cosine.		Secant	Cotang.		Tang.	Cosec.		Sine	M.

(26 Degrees.) *Table of Logarithmic Sines,*

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.641842	431	10.358158	9.688182	534	10 311818	10.046340	103	9.953660	60
1	642101	431	357899	688502	534	311498	046401	103	953599	59
2	642360	431	357640	688823	534	311177	046463	103	953537	58
3	642618	430	357382	689143	533	310857	046525	103	953475	57
4	642877	430	357123	689463	533	310537	046587	103	953413	56
5	643135	430	356865	689783	533	310217	046648	103	953352	55
6	643393	430	356607	690103	533	309897	046710	103	953290	54
7	643650	429	356350	690423	533	309577	046772	103	953228	53
8	643908	429	356092	690742	532	309258	046834	103	953166	52
9	644165	429	355835	691062	532	308938	046896	103	953104	51
10	644423	428	355577	691381	532	308619	046958	103	953042	50
11	9.644680	428	10.355320	9.691700	531	10.308300	10.047020	104	9.952980	49
12	644936	428	355064	692019	531	307981	047082	104	952918	48
13	645193	427	354807	692338	531	307662	047145	104	652855	47
14	645450	427	354550	692656	531	307344	047207	104	952793	46
15	645706	427	354294	692975	531	307025	047269	104	952731	45
16	645962	426	354038	693293	530	306707	047331	104	952669	44
17	646218	426	353782	693612	530	306388	047394	104	952606	43
18	646474	426	353526	693930	530	306070	047456	104	952544	42
19	646729	425	353271	694248	530	305752	047519	104	952481	41
20	646984	425	353016	694566	529	305434	047581	104	952419	40
21	9.647240	425	10.352760	9.694883	529	10.305117	10.047644	104	9.952356	39
22	647494	424	352506	695201	529	304799	047706	104	952294	38
23	647749	424	352251	695518	529	304482	047769	104	952231	37
24	648004	424	351996	795836	529	304164	047832	105	952168	36
25	658258	424	351742	696153	528	303847	047894	105	952106	35
26	648512	423	351488	696470	528	303530	047957	105	952043	34
27	648766	423	351234	696787	528	303213	048020	105	951980	33
28	649020	423	350980	697103	528	302897	048083	105	951917	32
29	649274	422	350726	697420	527	302580	048146	105	951854	31
30	649527	422	350473	697736	527	302264	048209	105	951791	30
31	9.649781	422	10.350219	9.698053	527	10.301947	10.048272	105	9.951728	29
32	650034	422	349966	698369	527	301631	048335	105	951665	28
33	650287	421	349713	698685	526	301315	048390	105	951602	27
34	650539	421	349461	699001	526	300999	048461	105	951539	26
35	650792	421	349208	699316	526	300684	048524	105	951476	25
36	651044	420	358956	699632	526	300368	048588	105	951412	24
37	651297	420	348703	699947	526	300053	048651	106	951349	23
38	651549	420	348451	700263	525	299737	048714	106	951286	22
39	651800	419	348200	700578	525	299422	048778	106	951222	21
40	952052	419	347948	700893	525	299107	048841	106	951159	20
41	0.652304	419	10.347696	9.701208	524	10.298792	10.048904	106	9.951096	19
42	652555	418	347445	701523	524	298477	048968	106	951032	18
43	652806	418	347194	701837	524	298163	049032	106	950968	17
44	653057	418	346943	702152	524	297848	049095	106	950905	16
45	653308	418	346692	702466	524	297534	049159	106	950841	15
46	653558	417	346442	702780	523	297220	049222	106	950778	14
47	653808	417	346192	703095	523	296905	049286	106	950714	13
48	654059	417	345941	703409	523	296591	049350	106	950650	12
49	654309	416	345691	703723	523	296277	049414	106	950586	11
50	654558	416	345442	704036	522	295964	049478	107	950522	10
51	9.654808	416	10.345192	9.704350	522	10.295650	10.049542	107	9.950458	9
52	655058	416	344942	704663	522	295337	049606	107	950394	8
53	655307	415	344693	704977	522	295023	049670	107	950330	7
54	655556	415	344444	705290	522	294710	049734	107	950266	6
55	655805	415	344195	705603	521	294397	049798	107	950202	5
56	656054	414	343946	705916	521	294084	049862	107	950138	4
57	956302	414	343698	706228	521	293772	049926	107	950074	3
58	656551	414	343449	706541	521	293459	049990	107	950010	2
59	656799	413	343201	706854	521	293146	050055	107	949945	1
60	657047	413	342953	707166	520	292834	050119	107	949881	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

63 Degrees.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.657047	413	10.342953	9.707166	520	10.292834	10.050119	107	9.949881	60
1	657295	413	342705	707478	520	292522	050184	107	949816	59
2	657542	412	342458	707790	520	292210	050248	107	949752	58
3	657790	412	342210	708102	520	291898	050312	108	949688	57
4	658037	412	341963	708414	519	291586	050377	108	949623	56
5	658284	412	341716	708726	519	291274	050442	108	049558	55
6	658531	411	341469	709037	519	290963	050506	108	949494	54
7	658778	411	341222	709349	519	290651	050571	108	949429	53
8	659025	411	340975	709660	519	290340	050636	108	949364	52
9	659271	410	340729	709971	518	290029	050700	108	949300	51
10	659517	410	340483	710282	518	289718	050765	108	949235	50
11	9.659763	410	10.340237	9.710593	518	10.289407	10.050830	108	9.949170	49
12	660009	409	339991	710904	518	289096	050895	108	949105	48
13	660255	409	339745	711215	518	288785	050960	108	949040	47
14	660501	409	339499	711525	517	288475	051025	108	948975	46
15	660746	409	339254	711836	517	288164	051090	108	948910	45
16	660991	408	339009	712146	517	287854	051155	108	948845	44
17	661236	408	338764	712456	517	287544	051220	109	948780	43
18	661481	408	338519	712766	516	287234	051285	109	948715	42
19	661726	407	338274	713076	516	286924	051350	109	948650	41
20	661970	407	338030	713386	516	286614	051416	109	948584	40
21	9.662214	407	10.337786	9.713696	516	10.286304	10.051481	109	9.948519	39
22	662459	407	337540	714005	516	285995	051546	109	948454	38
23	662703	406	337297	714314	515	285686	051612	109	948388	37
24	662946	406	337054	714624	515	285376	051677	109	948323	36
25	663190	406	336810	714933	515	285067	051743	109	948257	35
26	663433	405	336567	715242	515	284758	051808	109	948192	34
27	663677	405	336323	715551	514	284449	051874	109	948126	33
28	663920	405	336080	715860	514	284140	051940	109	948060	32
29	664163	405	335837	716168	514	283832	052005	100	947995	31
30	664406	404	335594	716477	514	283523	052071	100	947929	30
31	9.664648	404	10.335352	9.716785	514	10.283215	10.052137	110	9.947863	29
32	664891	404	335109	717093	513	282907	052203	110	947797	28
33	665133	403	334867	717401	513	282599	052269	110	947731	27
34	665375	403	334625	717709	513	282291	052335	110	947665	26
35	665617	403	334383	718017	513	281983	052400	110	947600	25
36	665859	402	334141	718325	513	281675	052467	110	947533	24
37	666100	402	333900	718633	512	281367	052533	110	947467	23
38	666342	402	333658	718940	512	281060	052599	110	947401	22
39	666583	402	333417	719248	512	280752	052665	110	947335	21
40	666824	401	333176	719555	512	280445	052731	110	947269	20
41	9.667065	401	10.332935	9.719862	512	10.280138	10.052797	110	9.947203	19
42	667305	401	332695	720169	511	279831	052864	111	947136	18
43	667546	401	332454	720476	511	279524	052930	111	947070	17
44	667786	400	332214	720783	511	279217	052996	111	947004	16
45	668027	400	331973	721089	511	278911	053063	111	946937	15
46	668267	400	331733	721396	511	278604	053129	111	946871	14
47	668506	399	331494	721702	510	278298	053196	111	946804	13
48	668746	399	331254	722009	510	277991	053262	111	946738	12
49	668986	399	331014	722315	510	277685	053329	111	946671	11
50	669225	399	330775	722621	510	277379	053396	111	946604	10
51	9.669464	398	10.330536	9.722927	510	10.277073	10.053462	111	9.946538	9
52	669703	398	330297	723232	509	276768	053529	111	946471	8
53	669942	398	330058	723538	509	276462	053596	111	946404	7
54	670181	397	329819	723844	509	276156	053663	111	946337	6
55	670419	397	329581	724149	509	275851	053730	112	946270	5
56	670658	397	329342	724454	509	275546	053797	112	946203	4
57	670896	397	329104	724759	508	275241	053864	112	946136	3
58	671134	396	328866	725065	508	274935	053931	112	946069	2
59	671372	396	328628	725369	508	274631	053998	112	946002	1
60	671609	396	328391	725674	508	274326	054065	112	945935	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.609313	473	10.390687	9.648583	566	10.351417	10.039270	94	9.960730	60
1	609597	472	390403	648923	566	351077	039326	94	960674	59
2	609880	472	390120	649263	566	350737	039382	94	960618	58
3	610164	472	389836	649602	566	350398	039439	94	960561	57
4	610447	471	389553	649942	565	350058	039495	94	960505	56
5	610729	471	389271	650281	565	349719	039552	94	960448	55
6	611012	470	388988	650620	565	349380	039608	94	960392	54
7	611294	470	388706	650959	564	349041	039665	94	960335	53
8	611576	470	388424	651297	564	348703	039721	94	960279	52
9	611858	469	388142	651636	564	348364	039778	94	960222	51
10	612140	469	387860	651974	563	348026	039835	94	960165	50
11	9.612421	469	10.387579	9.652312	563	10.347688	10.039891	95	9.960109	49
12	612702	468	387298	652650	563	347350	039948	95	960052	48
13	612983	468	387017	652988	563	347012	040005	95	959995	47
14	613264	467	386736	653326	562	346674	040062	95	959938	46
15	613545	467	386455	653663	562	346337	040118	95	959882	45
16	613825	467	386175	654000	562	346000	040175	95	959825	44
17	614105	466	385895	654337	561	345663	040232	95	959768	43
18	614385	466	385615	654674	561	345326	040289	95	959711	42
19	614665	466	385335	655011	561	344989	040346	95	959654	41
20	614944	465	385056	655348	561	344652	040404	95	959596	40
21	9.615223	465	10.384777	9.655684	560	10.344316	10.040461	95	9.959539	39
22	615502	465	384498	656020	560	343980	040518	95	959482	38
23	615781	464	384219	656356	560	343644	040575	95	959425	37
24	616060	464	383940	656692	559	343308	040632	95	959368	36
25	616338	464	383662	657028	559	342972	040690	96	959310	35
26	616616	463	383384	657364	559	342636	040747	96	959253	34
27	616894	463	383106	657699	559	342301	040805	96	959195	33
28	617172	462	382828	658034	558	341966	040862	96	959138	32
29	617450	462	382550	658369	558	341631	040919	96	959081	31
30	617727	462	382273	658704	558	341296	040977	96	959023	30
31	9.618004	461	10.381996	9.659039	558	10 340961	10.041035	96	9.958965	29
32	618281	461	381719	659373	557	340627	041092	96	958908	28
33	618558	461	381442	659708	557	340292	041150	96	958850	27
34	618834	460	381166	660042	557	339958	041208	96	958792	26
35	619110	460	380890	660376	557	339624	041266	96	958734	25
36	619386	460	380614	660710	556	339290	041323	96	958677	24
37	619662	459	380338	661043	556	338957	041381	96	958619	23
38	619938	459	380062	661377	556	338623	041439	96	958561	22
39	620213	459	379787	661710	555	338290	041497	97	958503	21
40	620488	458	379512	662043	555	337957	041555	97	958445	20
41	9.620763	458	10.379237	9.662376	555	10.337624	10.041613	97	9.958387	19
42	621038	457	378962	662709	554	337291	041671	97	958329	18
43	621313	457	378687	663042	554	336958	041729	97	958271	17
44	621587	457	378413	663375	554	336625	041787	97	958213	16
45	621861	456	378139	663707	554	336293	041846	97	958154	15
46	622135	456	377865	664039	553	335961	041904	97	958096	14
47	622409	456	377591	664371	553	335629	041962	97	958038	13
48	622682	455	377318	664703	553	335297	042021	97	957979	12
49	622956	455	377044	665035	553	334965	042079	97	957921	11
50	623229	455	376771	665366	552	334634	042137	97	957863	10
51	9.623502	454	10.376498	9.665697	552	10.334303	10.042196	97	9.957804	9
52	623774	454	376226	666029	552	333971	042254	98	957746	8
53	624047	454	375953	666360	551	333640	042313	98	957687	7
54	624319	453	375681	666691	551	333309	042372	98	957628	6
55	624591	453	375409	667021	551	332979	042430	98	957570	5
56	624863	453	375137	667352	551	332648	042489	98	957511	4
57	625135	452	374865	667682	550	332318	042548	98	957452	3
58	625406	452	374594	668013	550	331987	042607	98	957393	2
59	625677	452	374323	668343	550	331657	042665	98	957335	1
60	625948	451	374052	668672	550	331328	042724	98	957276	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.625948	451	10.374052	9.668673	550	10.331327	10.042724	98	9.957276	60
1	626219	451	373781	669002	549	330998	042783	98	957217	59
2	626490	451	373510	669332	549	330668	042842	98	957158	58
3	626760	450	373240	669661	549	330339	042901	98	957099	57
4	627030	450	372970	669991	548	330009	042960	98	957040	56
5	627300	450	372700	670320	548	329680	043019	98	956981	55
6	627570	449	372430	670649	548	329351	043079	99	956921	54
7	627840	449	372160	670977	548	329023	043138	99	956862	53
8	628109	449	371891	671306	547	328694	043197	99	956803	52
9	628378	448	371622	671634	547	328366	043256	99	956744	51
10	628647	448	371353	671963	547	328037	043316	99	956684	50
11	9.628916	447	10.371084	9.672291	547	10.327709	10.043375	99	9.956625	49
12	629185	447	370815	672619	546	327381	043434	99	956566	48
13	629453	447	370547	672946	546	327053	043494	99	956506	47
14	629721	446	370279	673274	546	326726	043553	99	956447	46
15	629989	446	370011	673602	546	326398	043613	99	956387	45
16	630257	446	369743	673929	545	326071	043673	99	956327	44
17	630524	446	369476	674257	545	325743	043732	99	956268	43
18	630792	445	369208	674584	545	325416	043792	100	956208	42
19	631059	445	368941	674910	544	325090	043852	100	956148	41
20	631326	445	368674	675237	544	324763	043911	100	956089	40
21	9.631593	444	10.368407	9.675564	544	10.324436	10.043971	100	9.956029	39
22	631859	444	368141	675890	544	324110	044031	100	955969	38
23	632125	444	367875	676216	543	323784	044091	100	955909	37
24	632392	443	367608	676543	543	323457	044151	100	955849	36
25	632658	443	367342	676869	543	323131	044211	100	955789	35
26	632923	443	367077	677194	543	322806	044271	100	955729	34
27	633189	442	366811	677520	542	322480	044331	100	955669	33
28	633454	442	366546	677846	542	322154	044391	100	955609	32
29	633719	442	366281	678171	542	321829	044452	100	955548	31
30	633984	441	366016	678496	542	321504	044512	100	955488	30
31	9.634249	441	10.365751	9.678821	541	10.321179	10.044572	101	9.955428	29
32	634514	440	365486	679146	541	320854	044632	101	955368	28
33	634778	440	365222	679471	541	320529	044693	101	955307	27
34	635042	440	364958	679795	541	320205	044753	101	955247	26
35	635306	439	364694	680120	540	319880	044814	101	955186	25
36	635570	439	364430	680444	540	319556	044874	101	955126	24
37	635834	439	364166	680768	540	319232	044935	101	955065	23
38	636097	438	363903	681092	540	318908	044995	101	955005	22
39	636360	438	363640	681416	539	318584	045056	101	954944	21
40	636623	438	363377	681740	539	318260	045117	101	954883	20
41	9.636886	437	10.363114	9.682063	539	10.317937	10.045177	101	9.954823	19
42	637148	437	362852	682387	539	317613	045238	101	954762	18
43	637411	437	362589	682710	538	317290	045299	101	954701	17
44	637673	437	362327	683033	538	316967	045360	101	954640	16
45	637935	436	362065	683356	538	316644	045421	101	954579	15
46	638197	436	361803	683679	538	316321	045482	102	954518	14
47	638458	436	361542	684001	537	315999	045543	102	954457	13
48	638720	435	361280	684324	537	315676	045604	102	954396	12
49	638981	435	361019	684646	537	315354	045665	102	954335	11
50	639242	435	360758	684968	537	315032	045726	102	954274	10
51	9.639503	434	10.360497	9.685290	536	10.314710	10.045787	102	9.954213	9
52	639764	434	360236	685612	536	314388	045848	102	954152	8
53	640024	434	359976	685934	536	314066	045910	102	954090	7
54	640284	433	359716	686255	536	313745	045971	102	954029	6
55	640544	433	359456	686577	535	313423	046032	102	953968	5
56	640804	433	359196	686898	535	313102	046094	102	953906	4
57	641064	432	358936	687219	535	312781	046155	102	953845	3
58	641324	432	358677	687540	535	312460	046217	102	953783	2
59	641584	432	358416	687861	534	312139	046278	103	953722	1
60	641842	431	358158	688182	534	311818	046340	103	953660	0
	Cosine.		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.641842	431	10.358158	9.688182	534	10 311818	10.046340	103	9.953660	60
1	642101	431	357899	688502	534	311498	046401	103	953599	59
2	642360	431	357640	688823	534	311177	046463	103	953537	58
3	642618	430	357382	689143	533	310857	046525	103	953475	57
4	642877	430	357123	689463	533	310537	046587	103	953413	56
5	643135	430	356865	689783	533	310217	046648	103	953352	55
6	643393	430	356607	690103	533	309897	046710	103	953290	54
7	643650	429	356350	690423	533	309577	046772	103	953228	53
8	643908	429	356092	690742	532	309258	046834	103	953166	52
9	644165	429	355835	691062	532	308938	046896	103	953104	51
10	644423	428	355577	691381	532	308619	046958	103	953042	50
11	9.644680	428	10.355320	9.691700	531	10.308300	10.047020	104	9.952980	49
12	644936	428	355064	692019	531	307981	047082	104	952918	48
13	645193	427	354807	692338	531	307662	047145	104	652855	47
14	645450	427	354550	692656	531	307344	047207	104	952793	46
15	645706	427	354294	692975	531	307025	047269	104	952731	45
16	645962	426	354038	693293	530	306707	047331	104	952669	44
17	646218	426	353782	693612	530	306388	047394	104	952606	43
18	646474	426	353526	693930	530	306070	047456	104	952544	42
19	646729	425	353271	694248	530	305752	047519	104	952481	41
20	646984	425	353016	694566	529	305434	047581	104	952419	40
21	9.647240	425	10.352760	9.694883	529	10.305117	10.047644	104	9.952356	39
22	647494	424	352506	695201	529	304799	047706	104	952294	38
23	647749	424	352251	695518	529	304482	047769	104	952231	37
24	648004	424	351996	795836	529	304164	047832	105	952168	36
25	658258	424	351742	696153	528	303847	047894	105	952106	35
26	648512	423	351488	696470	528	303530	047957	105	952043	34
27	648766	423	351234	696787	528	303213	048020	105	951980	33
28	649020	423	350980	697103	528	302897	048083	105	951917	32
29	649274	422	350726	697420	527	302580	048146	105	951854	31
30	649527	422	350473	697736	527	302264	048209	105	951791	30
31	9.649781	422	10.350219	9.698053	527	10.301947	10.048272	105	9.951728	29
32	650034	422	349966	698369	527	301631	048335	105	951665	28
33	650287	421	349713	698685	526	301315	048390	105	951602	27
34	650539	421	349461	699001	526	300999	048461	105	951539	26
35	650792	421	349208	699316	526	300684	048524	105	951476	25
36	651044	420	358956	699632	526	300368	048588	105	951412	24
37	651297	420	348703	699947	526	300053	048651	106	951349	23
38	651549	420	348451	700263	525	299737	048714	106	951286	22
39	651800	419	348200	700578	525	299422	048778	106	951222	21
40	952052	419	347948	700893	525	299107	048841	106	951159	20
41	0.652304	419	10.347696	9.701208	524	10.298792	10.048904	106	9.951096	19
42	652555	418	347445	701523	524	298477	048968	106	951032	18
43	652806	418	347194	701837	524	298163	049032	106	950968	17
44	653057	418	346943	702152	524	297848	049095	106	950905	16
45	653308	418	346692	702466	524	297534	049159	106	950841	15
46	653558	417	346442	702780	523	297220	049222	106	950778	14
47	653808	417	346192	703095	523	296905	049286	106	950714	13
48	654059	417	345941	763409	523	296591	049350	106	950650	12
49	654309	416	345691	703723	523	296277	049414	106	950586	11
50	654558	416	345442	704036	522	295964	049478	107	950522	10
51	9.654808	416	10.345192	9.704350	522	10.295650	10.049542	107	9.950458	9
52	655058	416	344942	704663	522	295337	049606	107	950394	8
53	655307	415	344693	704977	522	295023	049670	107	950330	7
54	655556	415	344444	705290	522	294710	049734	107	950266	6
55	655805	415	344195	705603	521	294397	049798	107	950202	5
56	656054	414	343946	705916	521	294084	049862	107	950138	4
57	956302	414	343698	706228	521	293772	049926	107	950074	3
58	656551	414	343449	706541	521	293459	049990	107	950010	2
59	656799	413	343201	706854	521	293146	050055	107	949945	1
60	657047	413	342953	707166	520	292834	050119	107	949881	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.657047	413	10.342953	9.707166	520	10.292834	10.050119	107	9.949881	60
1	657295	413	342705	707478	520	292522	050184	107	949816	59
2	657542	412	342458	707790	520	292210	050248	107	949752	58
3	657790	412	342210	708102	520	291898	050312	108	949688	57
4	658037	412	341963	708414	519	291586	050377	108	949623	56
5	658284	412	341716	708726	519	291274	050442	108	049558	55
6	658531	411	341469	709037	519	290963	050506	108	949494	54
7	658778	411	341222	709349	519	290651	050571	108	949429	53
8	659025	411	340975	709660	519	290340	050636	108	949364	52
9	659271	410	340729	709971	518	290029	050700	108	949300	51
10	659517	410	340483	710282	518	289718	050765	108	949235	50
11	9.659763	410	10.340237	9.710593	518	10.289407	10.050830	108	9.949170	49
12	660009	409	339991	710904	518	289096	050895	108	949105	48
13	660255	409	339745	711215	518	288785	050960	108	949040	47
14	660501	409	339499	711525	517	288475	051025	108	948975	46
15	660746	409	339254	711836	517	288164	051090	108	948910	45
16	660991	408	339009	712146	517	287854	051155	108	948845	44
17	661236	408	338764	712456	517	287544	051220	109	948780	43
18	661481	408	338519	712766	516	287234	051285	109	948715	42
19	661726	407	338274	713076	516	286924	051350	109	948650	41
20	661970	407	338030	713386	516	286614	051416	109	948584	40
21	9.662214	407	10.337786	9.713696	516	10.286304	10.051481	109	9.948519	39
22	662459	407	337540	714005	516	285995	051546	109	948454	38
23	662703	406	337297	714314	515	285686	051612	109	948388	37
24	662946	406	337054	714624	515	285376	051677	109	948323	36
25	663190	406	336810	714933	515	285067	051743	109	948257	35
26	663433	405	336567	715242	515	284758	051808	109	948192	34
27	663677	405	336323	715551	514	284449	051874	109	948126	33
28	663920	405	336080	715860	514	284140	051940	109	948060	32
29	664163	405	335837	716168	514	283832	052005	100	947995	31
30	664406	404	335594	716477	514	283523	052071	100	947929	30
31	9.664648	404	10.335352	9.716785	514	10.283215	10.052137	110	9.947863	29
32	664891	404	335109	717093	513	282907	052203	110	947797	28
33	665133	403	334867	717401	513	282599	052269	110	947731	27
34	665375	403	334625	717709	513	282291	052335	110	947665	26
35	665617	403	334383	718017	513	281983	052400	110	947600	25
36	665859	402	334141	718325	513	281675	052467	110	947533	24
37	666100	402	333900	718633	512	281367	052533	110	947467	23
38	666342	402	333658	718940	512	281060	052599	110	947401	22
39	666583	402	333417	719248	512	280752	052665	110	947335	21
40	666824	401	333176	719555	512	280445	052731	110	947269	20
41	9.667065	401	10.332935	9.719862	512	10.280138	10.052797	110	9.947203	19
42	667305	401	332695	720169	511	279831	052864	111	947136	18
43	667546	401	332454	720476	511	279524	052930	111	947070	17
44	667786	400	332214	720783	511	279217	052996	111	947004	16
45	668027	400	331973	721089	511	278911	053063	111	946937	15
46	668267	400	331733	721396	511	278604	053129	111	946871	14
47	668506	399	331494	721702	510	278298	053196	111	946804	13
48	668746	399	331254	722009	510	277991	053262	111	946738	12
49	668986	399	331014	722315	510	277685	053329	111	946671	11
50	669225	399	330775	722621	510	277379	053396	111	946604	10
51	9.669464	398	10.330536	9.722927	510	10.277073	10.053462	111	9.946538	9
52	669703	398	330297	723232	509	276768	053529	111	946471	8
53	669942	398	330058	723538	509	276462	053596	111	946404	7
54	670181	397	329819	723844	509	276156	053663	111	946337	6
55	670419	397	329581	724149	509	275851	053730	112	946270	5
56	670658	397	329342	724454	509	275546	053797	112	946203	4
57	670896	397	329104	724759	508	275241	053864	112	946136	3
58	671134	396	328866	725065	508	274935	053931	112	946069	2
59	671372	396	328628	725369	508	274631	053998	112	946002	1
60	671609	396	328391	725674	508	274326	054065	112	945935	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.671609	396	10.328391	9.725674	508	10.274326	10.054065	112	9.945935	60
1	671847	395	328153	725979	508	274021	054132	112	945868	59
2	672084	395	327916	726284	507	273716	054200	112	945800	58
3	672321	395	327679	726588	507	273412	054267	112	945733	57
4	672558	395	327442	726892	507	273108	054334	112	945666	56
5	672795	394	327205	727197	507	272803	054402	112	945598	55
6	673032	294	326968	727501	507	272499	054469	112	945531	54
7	673268	394	326732	727805	506	272195	054536	113	945464	53
8	673505	394	326495	728109	506	271891	054604	113	945396	52
9	673741	393	326259	728412	506	271588	054672	113	945328	51
10	673977	393	326023	728716	506	271284	054739	113	845261	50
11	9.674213	393	10.325787	9.729020	506	10.270980	10.054807	113	9.945193	49
12	674448	392	325552	729323	505	270677	054875	113	945125	48
13	674684	392	325316	729626	505	270374	054942	118	945058	47
14	674919	392	325081	729929	505	270071	055010	113	944990	46
15	675155	392	324845	730233	505	269767	055078	113	944922	45
16	675390	391	324610	730535	505	269465	055146	113	944854	44
17	675624	391	324376	730838	504	269162	055214	113	944786	43
18	675859	391	324141	731141	504	268859	055282	113	944718	42
19	676094	391	323906	731444	504	268556	055350	113	944650	41
20	676328	390	323672	731746	504	268254	055418	114	944582	40
21	9.676562	390	10.323438	9.732048	504	10.267952	10.055486	114	9.944514	39
22	676796	390	323204	732351	503	267649	055554	114	944446	38
23	677030	390	322970	732653	503	267347	055623	114	944377	37
24	677264	389	322736	732955	503	267045	055691	114	944309	36
25	677498	389	322502	733257	503	266743	055759	114	944241	35
26	677731	389	322269	733558	503	266442	055828	114	944172	34
27	677964	388	322036	733860	502	266140	055896	114	944104	33
28	678197	388	321803	734162	502	265838	055964	114	944036	32
29	678430	388	321570	734463	502	265537	056033	114	943967	31
30	678663	388	321337	734764	502	265236	056101	114	943899	30
31	9.678895	387	10.321105	9.735066	502	10.264934	10.056170	114	4.943830	29
32	679128	387	320872	735367	502	264633	056239	114	943761	28
33	679360	387	320640	735668	501	264332	056307	115	943693	27
34	679592	387	320408	735969	501	264031	056376	115	943624	26
35	679824	386	320176	736269	501	263731	056445	115	943555	25
36	680056	386	319944	736570	501	263430	056514	115	943486	24
37	680288	386	319712	736871	501	263129	056583	115	943417	23
38	680519	385	319481	737171	500	262829	056652	115	943348	22
39	680750	385	319250	737471	500	262529	056721	115	943279	21
40	680982	385	319018	737771	500	282229	056790	115	943210	20
41	9.681213	385	10.318787	9.738071	500	10.261929	10.056859	115	9.943141	19
42	681443	384	318557	738371	500	261629	056928	115	943072	18
43	681674	384	318326	738671	499	261329	056997	115	943003	17
44	681905	384	318095	738971	499	261029	057066	115	942934	16
45	682135	384	317865	739271	499	260729	057136	115	942864	15
46	682365	383	317635	739570	499	260430	057205	116	942795	14
47	682595	383	317405	739870	499	230130	057274	116	942726	13
48	682825	383	317175	740169	499	259831	057344	116	942656	12
49	683055	383	316945	740468	498	259532	057413	116	942587	11
50	683284	382	316716	740767	498	259233	057483	116	942517	10
51	9.683514	382	10.316486	9.741066	498	10.258934	10.057552	116	9.942448	9
52	683743	382	316257	741365	498	258635	057623	116	942378	8
53	643972	382	316028	741664	498	258336	057692	116	942308	7
54	684201	381	315799	741962	497	258038	057761	116	942239	6
55	684430	381	315570	742261	497	257739	057831	116	942169	5
56	684658	381	315342	742559	497	257441	057901	116	942099	4
57	684887	380	315113	742858	497	257142	057971	116	942029	3
58	685115	380	314885	743156	497	256844	058041	116	941959	2
59	685343	380	314657	743454	497	256546	058111	117	941889	1
60	685571	380	314429	743752	496	256248	058181	117	941819	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.685572	380	10.314429	9.743752	496	10.256248	10.058181	117	9.941819	60
1	685799	379	314201	744050	496	255950	058251	117	941749	59
2	686027	379	313973	744348	496	255652	058321	117	941679	58
3	686254	379	313746	744645	496	255355	058391	117	941609	57
4	686482	379	313518	744943	496	255057	058461	117	941539	56
5	686709	378	313291	745240	496	254760	058531	117	941469	55
6	686936	378	313064	745538	495	254462	058602	117	941398	54
7	687163	378	312837	745835	495	254165	058672	117	941328	53
8	687389	378	312611	746132	495	253868	058742	117	941258	52
9	687616	377	312384	746429	495	253571	058813	117	941187	51
10	687843	377	312157	746726	495	253274	058883	117	941117	50
11	9.688069	377	10.311931	9.747023	494	10.252977	10.058954	118	9.941046	49
12	688295	377	311705	747319	494	252681	059025	118	940975	48
13	688521	376	311479	747616	494	252384	059095	118	940905	47
14	688747	376	311253	747913	494	252087	059166	118	940834	46
15	688972	376	311028	748209	494	251791	059237	118	940763	45
16	689198	376	310802	748505	493	251495	059307	118	940693	44
17	689423	375	310577	748801	493	251199	059378	118	940622	43
18	689648	375	310352	749097	493	250903	059449	118	940551	42
19	689873	375	310127	749393	493	250607	059520	118	940480	41
20	690098	375	309902	749689	493	250311	059591	118	940409	40
21	9.690323	374	10.309677	9.749985	493	10.250015	10.059662	118	9.940338	39
22	690548	374	309452	750281	492	249719	059733	118	940267	38
23	690772	374	309228	750576	492	249424	059804	118	940196	37
24	690996	374	399004	750872	492	249128	069875	119	940125	36
25	691220	373	308780	751167	492	248833	059946	119	940054	35
26	691444	373	808556	751462	492	248538	060018	119	939982	34
27	691668	373	308332	751757	492	248243	060089	119	939911	33
28	691892	373	308108	752052	491	247948	060160	119	939840	32
29	692115	372	307885	752347	491	247653	060232	119	939768	31
30	692339	372	307661	752642	491	247358	060303	119	939697	30
31	9.692562	372	10.307438	9.752937	491	10.247063	10.060375	119	9.939625	29
32	692785	371	307215	753231	491	246769	060446	119	939554	28
33	693008	371	306992	753526	491	246474	060518	119	939482	27
34	693231	371	306769	753820	490	246180	060590	119	939410	26
35	693453	371	306547	754115	490	245885	060661	119	939339	25
36	693676	370	306324	754409	490	245591	060733	120	939267	24
37	693898	370	306102	754703	490	245297	060805	120	939195	23
38	694120	370	305880	754997	490	245003	060877	120	939123	22
39	694342	370	305658	755291	490	244709	060948	120	939052	21
40	694564	369	305436	755585	489	244415	061020	120	938980	20
41	9.694786	369	10.305214	9.755878	489	10.244122	10.061092	120	9.938908	19
42	695007	369	304993	756172	489	243828	061164	120	938836	18
43	695229	369	304771	756465	489	243535	061237	120	938763	17
44	695450	368	304550	756759	489	243241	061309	120	938691	16
45	695671	368	304329	757052	489	242948	061381	120	938619	15
46	695892	368	304108	757345	488	242655	061453	120	938547	14
47	696113	368	303887	757638	488	242362	061525	120	938475	13
48	696334	367	303666	757931	488	242069	061598	121	938402	12
49	696554	367	303446	758224	488	241776	061670	121	938330	11
50	696775	367	303225	758517	488	241483	061742	121	938258	10
51	9.696995	367	10.303005	9.758810	488	10.241190	10.061815	121	9.938185	9
52	697215	366	302785	759102	487	240898	061887	121	938113	8
53	697435	366	302565	759395	487	240605	061960	121	938040	7
54	697654	366	302346	759687	487	240313	062033	121	937967	6
55	697874	366	302126	759979	487	240021	062105	121	937895	5
56	698094	365	301906	760272	487	239728	062178	121	637822	4
57	698313	365	301687	760564	487	239436	062251	121	937749	3
58	698532	365	301468	760856	486	239144	062324	121	937676	2
59	698751	365	301249	761148	486	238852	062396	121	937604	1
60	698970	364	301030	761439	486	238561	062469	121	937531	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.698970	364	10.301030	9.761439	486	10.238561	10.062469	121	9.937531	60
1	699189	364	300811	761731	486	238269	062542	122	937458	59
2	699407	364	300593	762023	486	237977	062615	122	937385	58
3	699626	364	300374	762314	486	237686	062688	122	937312	57
4	699844	363	300156	762606	485	237394	062762	122	937238	56
5	700062	363	299938	762897	485	237103	062835	122	937165	55
6	700280	363	299720	763188	485	236812	062908	122	937092	54
7	700498	363	299502	763479	485	236521	062981	122	937019	53
8	700716	363	299285	763770	485	236230	063054	122	936946	52
9	700933	362	299067	764061	485	235939	063128	122	936872	51
10	701151	362	298849	764352	484	235648	063201	122	936799	50
11	9.701368	362	10.298632	9.764643	484	10.235357	10.063275	122	9.936725	49
12	701585	362	298415	764933	484	235067	063348	123	936652	48
13	701802	361	298198	765224	484	234776	063422	123	936578	47
14	702019	361	297981	765514	484	234486	063495	123	936505	46
15	702236	361	297764	765805	484	234195	063569	123	936431	45
16	702452	361	297548	766095	484	233905	063643	123	936357	44
17	702669	360	297331	766385	483	233615	063716	123	936284	43
18	702885	360	297115	766675	483	233325	063790	123	936210	42
19	703101	360	296899	766965	483	233035	063864	123	936136	41
20	703317	360	296683	767255	483	232745	063938	123	936062	40
21	9.703533	359	10.296467	9.767545	483	10.232455	10.064012	123	9.935988	39
22	703749	359	266251	767834	483	232166	064086	123	935914	38
23	703964	359	296036	768124	482	231876	064160	123	935840	37
24	704179	359	295821	768413	482	231587	064234	124	935766	36
25	704395	359	295605	768703	482	231297	064308	124	935692	35
26	704610	358	295390	768992	482	231008	064382	124	935618	34
27	704825	358	295175	769281	482	230719	004457	124	935543	33
28	705040	358	294960	769570	482	230430	064531	124	935469	32
29	705254	358	294746	769860	481	230140	064605	124	935395	31
30	705469	357	294531	770148	481	229852	064680	124	935320	30
31	9.705683	357	10.294317	9.770437	481	10.229563	10.064754	124	9.935246	29
32	705898	357	294102	770726	481	229274	064829	124	935171	28
33	706112	357	293888	771015	481	228985	064903	124	935097	27
34	706326	356	293674	771303	481	228697	064978	124	935022	26
35	706539	356	293461	771592	481	228408	065052	124	934948	25
36	706753	356	293247	771880	480	228120	065127	124	934873	24
37	706967	356	293033	772168	480	227832	065202	125	934798	23
38	707180	355	292820	772457	480	227543	065277	125	934723	22
39	707393	355	292607	772745	480	227255	065351	125	934649	21
40	707606	355	292394	773033	480	226967	065426	125	934574	20
41	9.707819	355	10.292181	9.773321	480	10.226679	10.065501	125	9.934499	19
42	708032	354	291968	773608	479	226392	065576	125	934424	18
43	708245	354	291755	773896	479	226104	065651	125	934349	17
44	708458	354	291542	774184	479	225816	065726	125	934274	16
45	708670	354	291330	774471	479	225529	065801	125	934199	15
46	768882	353	291118	774759	479	225241	065877	125	934123	14
47	709094	353	290906	775046	479	224954	065952	125	934048	13
48	709306	353	290694	775333	479	224667	066027	125	933973	12
49	709518	353	290482	775621	478	224379	066102	126	933898	11
50	709730	353	290270	775908	478	224092	066178	126	933822	10
51	9.709941	352	10.290059	9.776195	478	10.223805	10.066253	126	9.933747	9
52	710153	352	289847	776482	478	223518	066329	126	933671	8
53	710364	352	289636	776769	478	223231	066404	126	933596	7
54	710575	352	289425	777055	478	222945	066480	126	933520	6
55	710786	351	289214	777342	478	222658	066555	126	933445	5
56	710997	351	289003	777628	477	222372	066631	126	933369	4
57	711208	351	288792	777915	477	222085	066707	126	933293	3
58	711419	351	288581	778201	477	221799	066783	126	933217	2
59	711629	350	288371	778487	477	221512	066859	126	933141	1
60	711839	350	288161	778774	477	221226	066934	126	933066	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.711839	350	10.288161	9.778774	477	10.221226	10.066934	126	9.933066	60
1	712050	350	287950	779060	477	220940	067010	127	932990	59
2	712260	350	287740	779346	476	220654	067086	127	932914	58
3	712469	349	287531	779632	476	220368	067162	127	932838	57
4	712679	349	287321	779918	476	220082	067238	127	932762	56
5	712889	349	287111	780203	476	219797	067315	127	932685	55
6	713098	349	286902	780489	476	219511	067391	127	932609	54
7	713303	349	286692	780775	476	219225	067467	127	932533	53
8	713517	348	286483	781060	476	218940	067543	127	932457	52
9	713726	348	286274	781346	475	218654	067620	127	932380	51
10	713935	348	286065	781631	475	218369	067696	127	932304	50
11	9.714144	348	10.285856	9.781916	475	10.218084	10.067772	127	9.932228	49
12	714352	347	285648	782201	475	217799	067849	127	932151	48
13	714561	347	285439	782486	475	217514	067925	128	932075	47
14	714769	347	285231	782771	475	217229	068002	128	931998	46
15	714978	347	285022	783056	475	216944	068079	128	931921	45
16	715186	347	284814	783341	475	216659	068155	128	931845	44
17	715394	346	284606	783626	474	216374	068232	128	931768	43
18	715602	346	284398	783910	474	216090	068309	128	931691	42
19	715809	346	284191	784195	474	215805	068386	128	931614	41
20	716017	346	283983	784479	474	215521	068463	128	931537	40
21	9.716224	345	10.283776	9.784764	474	10.215236	10.068540	128	9.931460	39
22	716432	345	283568	785048	474	214952	068617	128	931383	38
23	716639	345	283361	785332	473	214668	068694	128	931306	37
24	716846	345	283154	785616	473	214384	068771	129	931229	36
25	717053	345	282947	785900	473	214100	068848	129	931152	35
26	717259	344	282741	786184	473	213816	068925	129	931075	34
27	717466	344	282534	786468	473	213532	069002	129	930998	33
28	717673	344	282327	786752	473	213248	069079	129	930921	32
29	717879	344	282121	787036	473	212964	069157	129	930843	31
30	718085	343	281915	787319	472	212681	069234	129	930766	30
31	9.718291	343	10.281709	9.787603	472	10.212397	10.069312	129	9.930688	29
32	718497	343	281503	787886	472	212114	069389	129	930611	28
33	718703	343	281297	788170	472	211830	069467	129	930533	27
34	718909	343	281091	788453	472	211547	069544	129	930456	26
35	719114	342	280886	788736	472	211264	069622	129	930378	25
36	719320	342	280680	789019	472	210981	069700	130	930300	24
37	719525	342	280475	789302	471	210698	069777	130	930223	23
38	719730	342	280270	789585	471	210415	069855	130	930145	22
39	719935	341	280065	789868	471	210132	069933	130	930067	21
40	720140	341	279860	790151	471	209849	070011	130	929989	20
41	9.720345	341	10.279655	9.790433	471	10.209567	10.070889	130	9.929911	19
42	720549	341	279451	790716	471	209284	070167	130	929833	18
43	720754	340	279246	790999	471	209001	070245	130	929755	17
44	720958	340	279042	791281	471	208719	070323	130	929677	16
45	721162	340	278838	791563	470	208437	070401	130	929599	15
46	721366	340	278634	791846	470	208154	070479	130	929521	14
47	721570	340	278430	792128	470	207872	070558	130	929442	13
48	721774	339	278226	792410	470	207590	070636	131	929364	12
49	721978	339	278022	792692	470	207308	070714	131	929286	11
50	722181	339	277819	792974	470	207026	070793	131	929207	10
51	9.722385	339	10.277615	9.793256	470	10.206744	10.070871	131	9.929129	9
52	722588	339	277412	793538	469	206462	071950	131	929050	8
53	722791	338	277209	793819	469	206181	071028	131	928972	7
54	722994	338	277006	794101	469	205899	071107	131	928893	6
55	723197	338	276803	794383	469	205617	071185	131	928815	5
56	723400	338	276600	794664	469	205336	071264	131	928736	4
57	723603	337	276397	794945	469	205055	071343	131	928657	3
58	723805	337	276195	795227	469	204773	071422	131	928578	2
59	724007	337	275993	795508	468	204492	071501	131	928499	1
60	724210	337	275790	795789	468	204211	. 071580	131	928420	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine.	
0	9.724210	337	10 275790	9.795789	468	10.204211	10.071580	132	9.928420	60
1	724412	337	275588	796070	468	203930	071658	132	928342	59
2	724614	336	275386	796351	468	203649	071737	132	928263	58
3	724816	336	275184	796632	468	203368	071817	132	928183	57
4	725017	336	274983	796913	468	203087	071896	132	928104	56
5	725219	336	274781	797194	468	202806	071975	132	928025	55
6	725420	335	274580	797475	468	202525	072054	132	927946	54
7	725622	335	274378	797755	468	202245	072133	132	927867	53
8	725823	335	274177	798036	467	201964	072213	132	927787	52
9	726024	335	273976	798316	467	201684	072292	132	927708	51
10	726225	335	273775	798596	467	201404	072371	132	927629	50
11	9.726426	334	10.273574	9.798877	467	10.201123	10.072451	132	9.927549	49
12	726626	334	273374	799157	467	200843	072530	133	927470	48
13	726827	334	273173	799437	467	200563	072610	133	927390	47
14	727027	334	272973	799717	467	200283	072690	133	927310	46
15	727228	334	272772	799997	466	200003	072769	133	927231	45
16	727428	333	272572	800277	466	199723	072849	133	927151	44
17	727629	333	272372	800557	466	199443	072929	133	927071	43
18	727828	333	272172	800836	466	109164	073009	133	926991	42
19	728027	333	271973	801116	466	198884	073089	133	926917	41
20	728227	333	271773	801396	466	198604	073169	133	926831	40
21	9.728427	332	10.271573	9 801675	466	10.198325	10.073249	133	9.926751	39
22	728626	332	271374	801955	466	198045	073329	133	926671	38
23	728825	332	271175	802234	465	197766	073409	133	926591	37
24	729024	332	270976	802513	465	197487	073489	134	926511	36
25	729223	331	270777	802792	465	197208	073569	134	926431	35
26	729432	331	270578	803072	465	196928	073649	134	926351	34
27	729621	331	270379	803351	465	196649	073730	134	926270	33
28	729820	331	270180	803630	465	196370	073810	134	926190	32
29	730018	330	269982	803908	465	196092	073890	134	926110	31
30	730216	330	269784	804187	465	195813	073971	134	926029	30
31	9.730415	330	10.269585	9.804466	464	10.195534	10 074051	134	9.925949	29
32	730613	330	269387	804745	464	195255	074132	134	925868	28
33	730811	330	269189	805023	464	194977	074212	134	925788	27
34	731009	329	268991	805302	464	194698	074293	134	925707	26
35	731206	329	268794	805580	464	194420	074374	134	925626	25
36	731404	329	268596	805859	464	194141	074455	135	925545	24
37	731602	329	268398	806137	464	193863	074535	135	925465	23
38	731799	329	268201	806415	463	193585	074616	135	925384	22
39	731996	328	268004	806693	463	193307	074697	135	925303	21
40	732193	328	267807	806971	463	193029	074778	135	925222	20
41	9.732390	328	10.267610	9.807249	463	10.192751	10.074859	135	9.925141	19
42	732587	328	267413	807527	463	192473	074940	135	925060	18
43	732784	328	267216	907805	463	192195	075021	135	924979	17
44	732980	327	267020	808083	463	191917	075103	135	924897	16
45	733177	327	266823	808361	463	191639	075184	135	924816	15
46	733373	327	266627	808638	462	191362	075265	136	924735	14
47	733569	327	266431	808916	462	191084	075346	136	924654	13
48	733765	327	266235	809193	462	190807	075428	136	924572	12
49	733961	326	266039	809471	462	190529	075509	136	924491	11
50	734157	326	265843	809748	462	190252	075591	136	924409	10
51	9.734353	326	10 265647	9.810025	462	10.189975	10 075672	136	9.924328	9
52	734549	326	264451	810302	462	189698	075754	136	924246	8
53	734744	325	265256	810580	462	189420	075836	136	924164	7
54	734939	325	265061	810857	462	189143	075917	136	924083	6
55	735135	325	264865	811134	461	188866	075999	136	924001	5
56	735330	325	364670	811410	461	188590	076081	136	923919	4
57	735525	325	264475	811687	461	188313	076163	136	923837	3
58	735719	324	264281	811964	461	188036	076245	137	923755	2
59	735914	324	264086	812241	461	187759	076327	137	923673	1
60	736109	324	263891	812517	461	187483	076409	137	923591	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.736109	324	10.263891	9.812517	461	10.187482	10.076409	137	9.923591	60
1	736303	324	263697	812794	461	187206	076491	137	923509	59
2	736498	324	263502	813070	461	186930	076573	137	923427	58
3	736692	323	263308	813347	460	186653	076655	137	923345	57
4	736886	323	263114	813623	460	186377	076737	137	923263	56
5	737080	323	262920	813899	460	186101	076819	137	923181	55
6	737274	323	262726	814175	460	185825	076902	137	923098	54
7	737467	323	262533	814452	460	185548	076984	137	923016	53
8	737661	322	262339	814728	460	185272	077067	137	922933	52
9	737855	322	262145	815004	460	184996	077149	137	922851	51
10	738048	322	261952	815279	460	184721	077232	138	922768	50
11	9.738241	322	10.261759	9.815555	459	10.184445	10.077314	138	9.922686	49
12	738434	322	261566	815831	459	184169	077397	138	922603	48
13	738627	321	261373	816107	459	183893	077480	138	922520	47
14	738820	321	261180	816382	459	183618	077562	138	922438	46
15	739013	321	260987	816658	459	183342	077645	138	922355	45
16	739206	321	260794	816933	459	183067	077728	138	922272	44
17	739398	321	260602	817209	459	182791	077811	138	922189	43
18	739590	320	260410	817484	459	182516	077894	138	922106	42
19	739783	320	260217	817759	459	182241	077977	138	922023	41
20	739975	320	260025	818035	458	181965	078060	138	921940	40
21	9.740167	320	10.259833	9.818310	458	10.181690	10.078143	139	9.921857	39
22	740359	320	259641	818585	458	181415	078226	139	921774	38
23	740550	319	259450	818860	458	181140	078309	139	921691	37
24	740742	319	259258	819135	458	180865	078393	139	921607	36
25	740934	319	259066	819410	458	180590	078476	139	921524	35
26	741125	319	258875	819684	458	180316	078559	139	921441	34
27	741316	319	258684	819959	458	180041	078643	139	921357	33
28	741508	318	258492	820234	458	179766	078726	139	921274	32
29	741699	318	258301	820508	457	179492	078810	139	921190	31
30	741889	318	258111	820783	457	179217	078893	139	921107	30
31	9.742080	318	10.257920	9.821057	457	10.178943	10.078977	139	9.921023	29
32	742271	318	257729	821332	457	178668	079061	140	920939	28
33	742462	317	257538	821606	457	178394	079144	140	920856	27
34	742652	317	257348	821880	457	178120	079228	140	920772	26
35	742842	317	257158	822154	457	177846	079312	140	920688	25
36	743033	517	256967	822429	457	177571	079396	140	920604	24
37	743223	317	256777	822703	457	177297	079480	140	920520	23
38	743413	316	256587	822977	456	177023	079564	140	920436	22
39	743602	316	256398	823250	456	176750	079648	140	920352	21
40	743792	316	256208	823524	456	176476	079732	140	920268	20
41	9.743982	316	10.256018	9.823798	456	10.176202	10.079816	140	9.920184	19
42	744171	316	255829	824072	456	175928	079901	140	920099	18
43	744361	315	255639	824345	456	175655	079985	140	920015	17
44	744550	315	255450	824619	456	175381	080069	141	919931	16
45	744739	315	255261	824893	456	175107	080154	141	919846	15
46	744928	315	255072	825166	456	174834	080238	141	919762	14
47	745117	315	254883	825439	455	174561	080323	141	919677	13
48	745306	314	254694	825713	455	174287	080407	141	919593	12
49	745494	314	254506	825986	455	174014	080492	141	919508	11
50	745683	314	254317	826259	455	173741	080576	141	919424	10
51	9.745871	314	10.254129	9.826532	455	10.173468	10.080661	141	9.919339	9
52	746059	314	253941	826805	455	173195	080746	141	919254	8
53	746248	313	253752	827078	455	172922	080831	141	919169	7
54	746436	313	253564	827351	455	172649	080915	141	919085	6
55	746624	313	253376	827624	455	172376	081000	141	919000	5
56	746812	313	253188	827897	454	172103	081085	142	918915	4
57	746999	313	253001	828170	454	171830	081170	142	918830	3
58	747187	312	252813	828442	454	171558	081255	142	918745	2
59	747374	312	252626	828715	454	171285	081341	142	918659	1
60	747562	312	252438	828987	454	171013	081426	142	918574	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9 747562	312	10.252438	9.828987	454	10.171013	10.081426	142	9.918574	60
1	747749	312	252251	829260	454	170740	081511	142	918489	59
2	747936	312	252064	829532	454	170468	081596	142	918404	58
3	748123	311	251877	829805	454	170195	081682	142	918318	57
4	748310	311	251690	830077	454	169923	081767	142	918233	56
5	748497	311	251503	830349	453	169651	081853	142	918147	55
6	748683	311	251317	830621	453	169379	081938	142	918062	54
7	748870	311	251130	830893	453	169107	082024	143	917976	53
8	749056	310	250944	831165	453	168835	082109	143	917891	52
9	749243	310	250757	831437	453	168563	082195	143	917805	51
10	749429	310	250571	831709	453	168291	082281	143	917719	50
11	9.749615	310	10.250385	9.831981	453	10.168019	10.082366	143	9.917634	49
12	749801	310	250199	832253	453	167747	082452	143	917548	48
13	749987	309	250013	882525	453	167475	082538	143	917462	47
14	750172	309	249828	832796	453	167204	082624	143	917376	46
15	750358	309	249642	833068	452	166932	082710	143	917290	45
16	750543	309	249457	833339	452	166661	082796	143	917204	44
17	750729	309	249271	833611	452	166389	082882	144	917118	43
18	751914	308	249086	833882	452	166118	082968	144	917032	42
19	751099	308	248901	834154	452	165846	083054	144	916946	41
20	751284	308	248716	834425	452	165575	083141	144	916859	40
21	9.751469	308	10.248531	9.834696	452	10.165304	10.083227	144	9.916773	39
22	751654	308	248346	834967	452	165033	083313	144	916687	38
23	751839	308	248161	835238	452	164762	083400	144	916600	37
24	752023	307	247977	835509	452	164491	083486	144	916514	36
25	752208	307	247792	835780	451	164220	083573	144	916427	35
26	752392	307	247608	836051	451	163949	083659	144	916341	34
27	752576	307	247424	836322	451	163678	083746	144	916254	33
28	752760	307	247240	836593	451	163407	083833	145	916167	32
29	752944	306	247056	836864	451	163136	083919	145	916081	31
30	753128	306	246872	837134	451	162866	084006	145	915994	30
31	9.753312	306	10.246688	9.837405	451	10 162595	10.084093	145	9.915907	29
32	753495	306	246505	837675	451	162325	084180	145	915820	28
33	753679	306	246321	837946	451	162054	084267	145	915733	27
34	753862	305	246138	838216	451	161784	084354	145	915646	26
35	754046	305	245954	838487	450	161513	084441	145	915559	25
36	754229	205	245771	838757	450	161243	084528	145	915472	24
37	754412	305	245588	839027	450	160973	084615	145	915385	23
38	754595	305	245405	839297	450	160703	084703	145	915297	22
39	754778	304	245222	839568	450	160432	084790	145	915210	21
40	754960	304	245040	839838	450	160162	084877	146	915132	20
41	9.755143	304	10.244857	9.840168	450	10.159892	10.084965	146	9 915035	19
42	755326	304	244674	840378	450	159622	085052	146	914948	18
43	755508	304	244492	840647	450	159353	085140	146	914860	17
44	755690	304	244310	840917	449	159083	085227	146	914773	16
45	755872	303	244128	841187	449	158813	085315	146	914685	15
46	756054	303	243946	841457	449	158543	085402	146	914598	14
47	756236	303	243764	841726	449	158274	085490	146	914510	13
48	756418	303	243582	841996	449	158004	085578	146	914422	12
49	756600	303	243400	842266	449	157734	085666	146	914334	11
50	756782	302	243218	842535	449	157465	085754	147	914246	10
51	9.756963	302	10.243037	9.842805	449	10.157195	10.085842	147	9.914158	9
52	757144	302	242856	843074	449	156926	085930	147	914070	8
53	757326	302	242674	843343	449	156657	086018	147	913982	7
54	757507	302	242493	843612	449	156388	086106	147	913894	6
55	757688	301	242312	843882	448	156118	086194	147	913806	5
56	757869	301	242131	844151	448	155849	086282	147	913718	4
57	758050	301	241950	844420	448	155580	086370	147	913630	3
58	758230	301	241770	844689	448	155311	086459	147	913541	2
59	758411	301	241589	844958	448	155042	086547	147	913453	1
60	758591	301	241409	845227	448	154773	086635	147	913365	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.758591	301	10.241409	9.845227	448	10.154773	10.086635	147	9.913365	60
1	758772	300	241228	845496	448	154504	086724	147	913276	59
2	758952	300	241048	845764	448	154236	086813	148	913187	58
3	759132	300	240868	846033	448	153967	086901	148	913099	57
4	759312	300	240688	846302	448	153698	086990	148	913010	56
5	759492	300	240508	846570	447	153430	087078	148	912922	55
6	759672	299	240328	846839	447	153161	087167	148	912833	54
7	759852	299	240148	847107	447	152893	087256	148	912744	53
8	760031	299	239969	847376	447	152624	087345	148	912655	52
9	760211	299	239789	847644	447	152356	087434	148	912566	51
10	760390	299	239610	847913	447	152087	087523	148	912477	50
11	9.760569	298	10.239431	9.848181	447	10.151819	10.087612	148	9.912388	49
12	760748	298	239252	848449	447	151551	087701	149	912299	48
13	760927	298	239073	848717	447	151283	087790	149	912210	47
14	761106	298	238894	848986	447	151014	087879	149	912121	46
15	761285	298	238715	849254	447	150746	087969	149	912031	45
16	761464	298	238536	849522	447	150478	088058	149	911942	44
17	761642	297	238358	849790	446	150210	088147	149	911853	43
18	761821	297	238179	850058	446	149942	088237	149	911763	42
19	761999	297	238001	850325	446	149675	088326	149	911674	41
20	762177	297	237823	850593	446	149407	088416	149	911584	40
21	9.762356	297	10.237644	9.850861	446	10.149139	10.088505	149	9.911495	39
22	762534	296	237466	851129	446	148871	088595	149	911405	38
23	762712	296	237288	851396	446	148604	088685	150	911315	37
24	762889	296	237111	851664	446	148336	088774	150	911226	36
25	763067	296	236933	851931	446	148069	088864	150	911136	35
26	763245	296	236755	852199	446	147801	088954	150	911046	34
27	763422	296	236578	852466	446	147534	089044	150	910956	33
28	763600	295	236400	852733	445	147267	089134	150	910866	32
29	763777	295	236223	853001	445	146999	089224	150	910776	31
30	763954	295	236046	853268	445	146732	089314	150	910686	30
31	9.764131	295	10.235869	9.853535	445	10.146465	10.089404	150	9.910596	29
32	764308	295	235692	853802	445	146198	089494	150	910506	28
33	764485	294	235515	854069	445	145931	089585	150	910415	27
34	764662	294	235338	854336	445	145664	089675	151	910325	26
35	764838	294	235162	854603	445	145397	089765	151	910235	25
36	765015	294	234985	854870	445	145130	089856	151	910144	24
37	765191	294	234809	855137	445	144863	089946	151	910054	23
38	765367	294	234633	855404	445	144596	090037	151	909963	22
39	765544	293	234456	855671	444	144329	090127	151	909873	21
40	765720	293	234280	855938	444	144062	090218	151	909782	20
41	9.765896	293	10.234104	9.856204	444	10.143796	10.090309	151	9.909691	19
42	766072	293	233928	856471	444	143529	090399	151	909601	18
43	766247	293	233753	856737	444	143263	090490	151	909510	17
44	766423	293	233577	857004	444	142996	090581	151	909419	16
45	766598	292	233402	857270	444	142730	090672	152	909328	15
46	766774	292	233226	857537	444	142463	090763	152	909237	14
47	766949	292	233051	857803	444	142197	090854	152	909146	13
48	767124	292	232876	858069	444	141931	090945	152	909055	12
49	767300	292	232700	858336	444	141664	091036	152	908964	11
50	767475	291	232525	858602	443	141398	091127	152	908873	10
51	9.767649	291	10.232351	9.858868	443	10.141132	10.091219	152	9.908781	9
52	767824	291	232176	859134	443	140866	091310	152	908690	8
53	767999	291	232001	859400	443	140600	091401	152	908599	7
54	768173	291	231827	859666	443	140334	091493	152	908507	6
55	768348	290	231652	859932	443	140068	091584	153	908416	5
56	768522	290	231478	860198	443	139802	091676	153	908324	4
57	768697	290	231303	860464	443	139536	091767	153	908233	3
58	768871	290	231129	860730	443	139270	091859	153	908141	2
59	769045	290	230955	860995	443	139005	091951	153	908049	1
60	769219	290	230781	861261	443	138739	092042	153	907958	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.769219	290	10.230781	9.861261	443	10.138739	10.092042	153	9.907958	60
1	769393	289	230607	861527	443	138473	092134	153	907866	59
2	769566	289	230434	861792	442	138208	092226	153	907774	58
3	769740	289	230260	862058	442	137942	092318	153	907682	57
4	769913	289	230087	862323	442	137677	092410	153	907590	56
5	770087	289	229913	862589	442	137411	092502	153	907498	55
6	770260	288	229740	862854	442	137146	092594	153	907406	54
7	770433	288	229567	863119	442	136881	092686	154	907314	53
8	770606	288	229394	863385	442	136615	092778	154	907222	52
9	770779	288	229221	863650	442	136350	092871	154	907129	51
10	770952	288	229048	863915	442	136085	092963	154	907037	50
11	9.771125	288	10.228875	9.864180	442	10.135820	10.093055	154	9.906945	49
12	771298	287	228702	864445	442	135555	093148	154	906852	48
13	771470	287	228530	864710	442	135290	093240	154	906760	47
14	771643	287	228357	864975	441	135025	093333	154	906667	46
15	771815	287	228185	865240	441	134760	093425	154	906575	45
16	771987	287	228013	865505	441	134495	093518	154	906482	44
17	772159	287	227841	865770	441	134230	093611	155	906389	43
18	772331	286	227669	866035	441	133965	093704	155	906295	42
19	772503	286	227497	866300	441	133700	093796	155	906204	41
20	772675	286	227325	866564	441	133436	093889	155	906111	40
21	9.772847	286	10.227153	9.866829	441	10.133171	10.093982	155	9.906018	39
22	773018	286	226982	867094	441	132906	004075	155	905925	38
23	773190	286	226810	867358	441	132642	094168	155	905832	37
24	773361	285	226639	867623	441	132377	094261	155	905739	36
25	773533	285	226467	867887	441	132113	094355	155	905645	35
26	773704	285	226296	868152	440	131848	094448	155	905552	34
27	773875	285	226125	868416	440	131584	094541	155	905459	33
28	774046	285	225954	868680	440	131320	094634	156	905366	32
29	774217	285	225783	868945	440	131055	094728	156	905272	31
30	774388	284	225612	869209	440	130791	094821	156	905179	30
31	9.774558	284	10.225442	9.869473	440	10.130527	10.094915	156	9.905085	29
32	774729	284	225271	869737	440	130263	095008	156	904992	28
33	774899	284	225101	870001	440	129999	095102	156	904898	27
34	775070	284	224930	870265	440	129735	095196	156	904804	26
35	775240	284	224760	870529	440	129471	095289	156	904711	25
36	775410	283	224590	870793	440	129207	095383	156	904617	24
37	775580	283	224420	871057	440	128943	095477	156	904523	23
38	775750	283	224250	871321	440	128679	095571	157	904429	22
39	775920	283	224080	871585	440	128415	095665	157	904335	21
40	776090	283	223910	871849	439	128151	095759	157	904241	20
41	9.776259	283	10.223741	9.872112	439	10.127888	10.095853	157	9.904147	19
42	776429	282	223571	872376	439	127624	095947	157	904053	18
43	776598	282	223402	872640	439	127360	096041	157	903959	17
44	776768	282	223232	872903	439	127097	096136	157	903864	16
45	776937	282	223063	873167	439	126833	096230	157	903770	15
46	777106	282	222894	873430	439	126570	096324	157	903676	14
47	777275	281	222725	873694	439	126306	096419	157	903581	13
48	777444	281	222556	873957	439	126043	096513	157	903487	12
49	777613	281	222387	874220	439	125780	096608	158	903392	11
50	777781	281	222219	874484	439	125516	096702	158	903298	10
51	9.777950	281	10.222050	9.874747	439	10.125253	10.096797	158	9.093203	9
52	778119	281	221881	875010	439	124990	096892	158	903108	8
53	778287	280	221713	875273	438	124727	096986	158	903014	7
54	778455	280	221545	875536	438	124464	097081	158	902919	6
55	778624	280	221376	875800	438	124200	097176	158	902824	5
56	778792	280	221208	876063	438	123937	097271	158	902729	4
57	778960	280	221040	876326	438	123674	097365	158	902634	3
58	779128	280	220872	876589	438	123411	097461	159	902539	2
59	779295	279	220705	876851	438	123149	097556	159	902444	1
60	779463	279	220537	877114	438	122886	097651	159	902349	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9 779463	279	10.220537	9.877114	438	10.122886	10.097651	159	9 902349	60
1	779631	279	220369	877377	438	122623	097747	159	902253	59
2	779798	279	220202	877640	438	122360	097842	159	902158	58
3	779966	279	220034	877903	438	122097	097937	159	902063	57
4	780133	279	219867	878165	438	121835	098033	159	901967	56
5	780300	278	219700	878428	438	121572	098128	159	901872	55
6	780467	278	219533	878691	438	121309	098224	159	901776	54
7	780634	278	219366	878953	437	121047	098319	159	901681	53
8	780801	278	219199	879216	437	120784	098415	159	901585	52
9	780968	278	219032	879478	437	120522	098510	159	901490	51
10	781134	278	218866	879741	437	120259	098606	160	901394	50
11	9 781301	277	10.218699	9.880003	437	10.119997	10.098702	160	9.901298	49
12	781468	277	218532	880265	437	119735	098798	160	901202	48
13	781634	277	218366	880528	437	119472	098894	160	901106	47
14	781800	277	218200	880790	437	119210	098990	160	901010	46
15	781966	277	218034	881052	437	118948	099086	160	900914	45
16	782132	277	217868	881314	437	118686	099182	160	900818	44
17	782298	276	217702	881576	437	118424	999278	160	900722	43
18	782464	276	217536	881839	437	118161	099374	160	900626	42
19	782630	276	217370	882101	437	117899	099471	160	900529	41
20	782796	276	217204	882363	436	117637	099567	161	900433	40
21	9.782961	276	10.217039	9 882625	436	10.117375	10 099663	161	9 900337	39
22	783127	276	216873	882887	436	117113	099760	161	900240	38
23	783292	275	216708	883148	436	116852	099856	161	900144	37
24	783458	275	216542	883410	436	116590	099953	161	900047	36
25	783623	275	216377	883672	436	116328	100049	161	899951	35
26	783788	275	216212	883934	436	116066	100146	161	899854	34
27	783953	275	216047	884196	436	115804	100243	161	899757	33
28	783118	275	.215882	884457	436	115543	100340	161	899660	32
29	783282	274	215718	884719	436	115281	100436	161	899564	31
30	784447	274	215553	884980	436	115020	100533	162	899467	30
31	9.784612	274	10.215388	9 885242	436	10.114758	10.100630	162	9.899370	29
32	784776	274	215224	885503	436	114497	100727	162	899273	28
33	784941	274	215059	885765	436	114235	100824	162	899176	27
34	785105	274	214895	886026	436	113974	100922	162	899078	26
35	785269	273	214731	886288	436	113712	101019	162	898981	25
36	785433	273	214567	886549	435	113451	101116	162	898884	24
37	785597	273	214403	886810	435	113190	101213	162	898787	23
38	785761	273	214239	887072	435	112928	101311	162	898689	22
39	785925	273	214075	887333	435	112667	101408	162	898592	21
40	786089	273	213911	887594	435	112406	101506	163	898494	20
41	9.786252	272	10.213748	9 887855	435	10.112145	10.101603	163	9.898397	19
42	786416	272	213584	888116	435	111884	101701	163	898299	18
43	786579	272	213421	888377	435	111623	101798	163	898202	17
44	786742	272	213258	888639	435	111361	101896	163	898104	16
45	786906	272	213094	888900	435	111100	101994	163	898006	15
46	787069	272	212931	889160	435	110840	102092	163	897908	14
47	787232	271	212768	889421	435	110579	102190	163	897810	13
48	787395	271	212605	889682	435	110318	102288	163	897712	12
49	787557	271	212443	889943	435	110057	102386	163	897614	11
50	787720	271	212280	890204	434	109796	102484	163	897516	10
51	9 787883	271	10.212117	9.890465	434	10.109535	10.102532	164	9 897418	9
52	788045	271	211955	890725	434	109275	102680	164	897320	8
53	788208	271	211792	890986	434	109014	102778	164	897222	7
54	788370	270	211630	891247	434	108753	102877	164	897123	6
55	788532	270	211468	891507	434	108493	102975	164	897025	5
56	788694	270	211306	891768	434	108232	103074	164	896926	4
57	788856	270	211144	892028	434	107972	103172	164	896828	3
58	789018	270	210982	892289	434	107711	103271	164	896729	2
59	789180	270	210820	892549	434	107451	103369	164	896631	1
60	789342	269	210658	892810	434	107190	103468	164	896532	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.808067	251	10.191933	9.923813	427	10.076187	10.115746	177	9.884254	60
1	808218	251	191782	924070	427	075930	115852	177	884148	59
2	808368	251	191632	924327	427	075673	115958	177	884042	58
3	808519	250	191481	924583	427	075417	116064	177	883936	57
4	808669	250	191331	924840	427	075160	116171	177	883829	56
5	808819	250	191181	925096	427	074904	116277	177	883723	55
6	808969	250	191031	925352	427	074648	116383	177	883617	54
7	809119	250	190881	925609	427	074391	116490	177	883510	53
8	809269	250	190731	925865	427	074135	116596	177	883404	52
9	809419	249	190581	926122	427	073878	116703	178	883297	51
10	809569	249	190431	926378	427	073622	116809	178	883191	50
11	9.809718	249	10.190282	9.926634	427	10.073366	10.116916	178	9.883084	49
12	809868	249	190132	926890	427	073110	117023	178	882977	48
13	810017	249	189983	927147	427	072853	117129	178	882871	47
14	810167	249	189833	927403	427	072597	117236	178	882764	46
15	810316	248	189684	927659	427	078341	117343	178	882657	45
16	810465	248	189535	927915	427	072085	117450	178	882550	44
17	810614	248	189386	928171	427	071829	117557	178	882443	43
18	810763	248	189237	928427	427	071573	117664	179	882336	42
19	810912	248	189088	928683	427	071317	117771	179	882229	41
20	811061	248	188939	928940	427	071060	117879	179	882121	40
21	9.811210	248	10.188790	9.929196	427	10.070804	10.117986	179	9.882014	39
22	811358	247	188642	929452	427	070548	118093	179	881907	38
23	811507	247	188493	929708	427	070292	118201	179	881799	37
24	811655	247	188345	929964	426	070036	118308	179	881692	36
25	811804	247	188196	930220	426	069780	118416	179	881584	35
26	811952	247	188048	930475	426	069525	118523	179	881477	34
27	812100	247	187900	930731	426	069269	118631	179	881369	33
28	812248	247	187752	930987	426	069013	118739	180	881261	32
29	812396	246	187604	931243	426	068757	118847	180	881153	31
30	812544	246	187456	931499	426	068501	118954	180	881046	30
31	9.812692	246	10.187308	9.931755	426	10.068245	10.119062	180	9.880938	29
32	812840	246	187160	932010	426	067990	119170	180	880830	28
33	812988	246	187012	932266	426	067734	119278	180	880722	27
34	813135	246	186865	932522	426	067478	119387	180	880613	26
35	813283	246	186717	932778	426	067222	119495	180	880505	25
36	813430	245	186570	933033	426	066967	119603	180	880397	24
37	813578	245	186422	933289	426	066711	119711	181	880289	23
38	813725	245	186275	933545	426	066455	119820	181	880180	22
39	813872	245	186128	933800	426	066200	119928	181	880072	21
40	814019	245	185981	934056	426	065944	120037	181	879963	20
41	9.814166	245	10.185834	9.934311	426	10.065689	10.120145	181	9.879855	19
42	814313	245	185687	934567	426	065433	120254	181	879746	18
43	814460	244	185540	934823	426	065177	120363	181	879637	17
44	814607	244	185393	935078	426	064922	120471	181	879529	16
45	814753	244	185247	935333	426	064667	120580	181	879420	15
46	814900	244	185100	935589	426	064411	120689	181	879311	14
47	815046	244	184954	935844	426	064156	120798	182	879202	13
48	815193	244	184807	936100	426	063900	120907	182	879093	12
49	815339	244	184661	936355	426	063645	121016	182	878984	11
50	815485	243	184515	936610	426	063390	121125	182	878875	10
51	9.815631	243	10.184368	9.936866	425	10.063134	10.121234	182	9.878766	9
52	815778	243	184222	937121	425	062879	121344	182	878656	8
53	815924	243	184076	937376	425	062624	121453	182	878547	7
54	816060	243	183931	937632	425	062368	121562	182	878438	6
55	816215	243	183785	937887	425	062113	121672	182	878328	5
56	816361	243	183639	938142	425	061858	121781	183	878219	4
57	816507	242	183493	938398	425	061602	121891	183	878109	3
58	816652	242	183348	938653	425	061347	122001	183	877999	2
59	816798	242	183202	938908	425	061092	122110	183	877890	1
60	816943	242	183057	939163	425	060837	122220	183	877780	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.816943	242	10.183057	9.939163	425	10.060837	10.122220	183	9.877780	60
1	817088	242	182912	939418	425	060582	122330	183	877670	59
2	817233	242	182767	939673	425	060327	122440	183	877560	58
3	817379	242	182621	939928	425	060072	122550	183	877450	57
4	817524	241	182476	940183	425	059817	122660	183	877340	56
5	817668	241	182332	940438	425	059562	122770	184	877230	55
6	817813	241	182187	940694	425	059306	122880	184	877120	54
7	817958	241	182042	940949	425	059051	122990	184	877010	53
8	818103	241	181897	941204	425	058796	123101	184	876899	52
9	818247	241	181753	941458	425	058542	123211	184	876789	51
10	818392	241	181608	941714	425	058286	123322	184	876678	50
11	9.818536	240	10.181464	9.941968	425	10.058032	10.123432	184	9.876568	49
12	818681	240	181319	942223	425	057777	123543	184	876457	48
13	818825	240	181175	942478	425	057522	123653	184	876347	47
14	818969	240	181031	942733	425	057267	123764	185	876236	46
15	819113	240	180887	942988	425	057012	123875	185	876125	45
16	819257	240	180743	943243	425	056757	123986	185	876014	44
17	819401	240	180599	943498	425	056502	124096	185	875904	43
18	819545	239	180455	943752	425	056248	124207	185	875793	42
19	819689	239	180311	944007	425	055993	124318	185	875682	41
20	819832	239	180168	944262	425	055738	124429	185	875571	40
21	9.819976	239	10.180024	9.944517	425	10.055483	10.124541	185	9.875459	39
22	820120	239	179880	944771	424	055229	124652	185	875348	38
23	820263	239	179737	945026	424	054974	124763	185	875237	37
24	820406	239	179594	945281	424	054719	124874	186	875126	36
25	820550	238	179450	945535	424	054465	124986	186	875014	35
26	820693	238	179307	945790	424	054210	125097	186	874903	34
27	820836	238	179164	946045	424	053955	125209	186	874791	33
28	820979	238	179021	946299	424	053701	125320	186	874680	32
29	821122	238	178878	946554	424	053446	125432	186	874568	31
30	821265	238	178735	946808	424	053192	125544	186	874456	30
31	9.821407	238	10.178593	9.947063	424	10.052937	10.125656	186	9.874344	29
32	821550	238	178450	947318	424	052682	125768	187	874232	28
33	821693	237	178307	947572	424	052428	125879	187	674121	27
34	821835	237	178165	947826	424	052174	125991	187	874009	26
35	821977	237	178023	948081	424	051919	126104	187	873896	25
36	822120	237	177880	948336	424	051664	126216	187	873784	24
37	822262	237	177738	948590	424	051410	126328	187	873672	23
38	822404	237	177596	948844	424	051156	126440	187	873560	22
39	822546	237	177454	949099	424	050901	126552	187	873448	21
40	822688	236	177312	949353	424	050647	126665	187	873335	20
41	9.822830	236	10.177170	9.949607	424	10.050393	10.126777	187	9.873223	19
42	822972	236	177028	949862	424	050138	126890	188	873110	18
43	823114	236	176886	950116	424	049884	127002	188	872998	17
44	823255	236	176745	950370	424	049630	127115	188	872885	16
45	823397	236	176603	950625	424	049375	127228	188	872772	15
46	823539	236	176461	950879	424	049121	127341	188	872659	14
47	823680	235	176320	951133	424	048867	127453	188	872547	13
48	823821	235	176179	951388	424	048612	127566	188	872434	12
49	823963	235	176037	951642	424	048358	127679	188	872321	11
50	824104	235	175896	951896	424	048104	127792	188	872208	10
51	9.824245	235	10.175755	9.952150	424	10.047850	10.127905	189	9.872095	9
52	824386	235	175614	952405	424	047595	128019	189	871981	8
53	824527	235	175473	952659	424	047341	128132	189	871868	7
54	824668	234	175332	952913	424	047087	128245	189	871755	6
55	824808	234	175192	953167	423	046833	128359	189	871641	5
56	824949	234	175051	953421	423	046579	128472	189	871528	4
57	825090	234	174910	953675	423	046325	128586	189	871414	3
58	825230	234	174770	953929	423	046071	128699	189	871301	2
59	825371	234	174629	954183	423	045817	128813	189	871187	1
60	825511	234	174489	954437	423	045563	128927	190	871073	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.808067	251	10.191933	9.923813	427	10.076187	10.115746	177	9.884254	60
1	808218	251	191782	924070	427	075930	115852	177	884148	59
2	808368	251	191632	924327	427	075673	115958	177	884042	58
3	808519	250	191481	924583	427	075417	116064	177	883936	57
4	808669	250	191331	924840	427	075160	116171	177	883829	56
5	808819	250	191181	925096	427	074904	116277	177	883723	55
6	808969	250	191031	925352	427	074648	116383	177	883617	54
7	809119	250	190881	925609	427	074391	116490	177	883510	53
8	809269	250	190731	925865	427	074135	116596	177	883404	52
9	809419	249	190581	926122	427	073878	116703	178	883297	51
10	809569	249	190431	926378	427	073622	116809	178	883191	50
11	9.809718	249	10.190282	9.926634	427	10.073366	10.116916	178	9.883084	49
12	809868	249	190132	926890	427	073110	117023	178	882977	48
13	810017	249	189983	927147	427	072853	117129	178	882871	47
14	810167	249	189833	927403	427	072597	117236	178	882764	46
15	810316	248	189684	927659	427	072341	117343	178	882657	45
16	810465	248	189535	927915	427	072085	117450	178	882550	44
17	810614	248	189386	928171	427	071829	117557	178	882443	43
18	810763	248	189237	928427	427	071573	117664	179	882336	42
19	810912	248	189088	928683	427	071317	117771	179	882229	41
20	811061	248	188939	928940	427	071060	117879	179	882121	40
21	9.811210	248	10.188790	9.929196	427	10.070804	10.117986	179	9.882014	39
22	811358	247	188642	929452	427	070548	118093	179	881907	38
23	811507	247	188493	929708	427	070292	118201	179	881799	37
24	811655	247	188345	929964	426	070036	118308	179	881692	36
25	811804	247	188196	930220	426	069780	118416	179	881584	35
26	811952	247	188048	930475	426	069525	118523	179	881477	34
27	812100	247	187900	930731	426	069269	118631	179	881369	33
28	812248	247	187752	930987	426	069013	118739	180	881261	32
29	812396	246	187604	931243	426	068757	118847	180	881153	31
30	812544	246	187456	931499	426	068501	118954	180	881046	30
31	9.812692	246	10.187308	9.931755	426	10.068245	10.119062	180	9.880938	29
32	812840	246	187160	932010	426	067990	119170	180	880830	28
33	812988	246	187012	932266	426	067734	119278	180	880722	27
34	813135	246	186865	932522	426	067478	119387	180	880613	26
35	813283	246	186717	932778	426	067222	119495	180	880505	25
36	813430	245	186570	933033	426	066967	119603	180	880397	24
37	813578	245	186422	933289	426	066711	119711	181	880289	23
38	813725	245	186275	933545	426	066455	119820	181	880180	22
39	813872	245	186128	933800	426	066200	119928	181	880072	21
40	814019	245	185981	934056	426	065944	120037	181	879963	20
41	9.814166	245	10.185834	9.934311	426	10.065689	10.120145	181	9.879855	19
42	814313	245	185687	934567	426	065433	120254	181	879746	18
43	814460	244	185540	934823	426	065177	120363	181	879637	17
44	814607	244	185393	935078	426	064922	120471	181	879529	16
45	814753	244	185247	935333	426	064667	120580	181	879420	15
46	814900	244	185100	935589	426	064411	120689	181	879311	14
47	815046	244	184954	935844	426	064156	120798	182	879202	13
48	815193	244	184807	936100	426	063900	120907	182	879093	12
49	815339	244	184661	936355	426	063645	121016	182	878984	11
50	815485	243	184515	936610	426	063390	121125	182	878875	10
51	9.815631	243	10.184368	9.936866	425	10.063134	10.121234	182	9.878766	9
52	815778	243	184222	937121	425	062879	121344	182	878656	8
53	815924	243	184076	937376	425	062624	121453	182	878547	7
54	816060	243	183931	937632	425	062368	121562	182	878438	6
55	816215	243	183785	937887	425	062113	121672	182	878328	5
56	816361	243	183639	938142	425	061858	121781	183	878219	4
57	816507	242	183493	938398	425	061602	121891	183	878109	3
58	816652	242	183348	938653	425	061347	122001	183	877999	2
59	816798	242	183202	938908	425	061092	122110	183	877890	1
60	816943	242	183057	939163	425	060837	122220	183	877780	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.816943	242	10.183057	9.939163	425	10.060837	10.122220	183	9.877780	60
1	817088	242	182912	939418	425	060582	122330	183	877670	59
2	817233	242	182767	939673	425	060327	122440	183	877560	58
3	817379	242	182621	939928	425	060072	122550	183	877450	57
4	817524	241	182476	940183	425	059817	122660	183	877340	56
5	817668	241	182332	940438	425	059562	122770	184	877230	55
6	817813	241	182187	940694	425	059306	122880	184	877120	54
7	817958	241	182042	940949	425	059051	122990	184	877010	53
8	818103	241	181897	941204	425	058796	123101	184	876899	52
9	818247	241	181753	941458	425	058542	123211	184	876789	51
10	818392	241	181608	941714	425	058286	123322	184	876678	50
11	9.818536	240	10.181464	9.941968	425	10.058032	10.123432	184	9.876568	49
12	818681	240	181319	942223	425	057777	123543	184	876457	48
13	818825	240	181175	942478	425	057522	123653	184	876347	47
14	818969	240	181031	942733	425	057267	123764	185	876236	46
15	819113	240	180887	942988	425	057012	123875	185	876125	45
16	819257	240	180743	943243	425	056757	123986	185	876014	44
17	819401	240	180599	943498	425	056502	124096	185	875904	43
18	819545	239	180455	943752	425	056248	124207	185	875793	42
19	819689	239	180311	944007	425	055993	124318	185	875682	41
20	819832	239	180168	944262	425	055738	124429	185	875571	40
21	9.819976	239	10.180024	9.944517	425	10.055483	10.124541	185	9.875459	39
22	820120	239	179880	944771	424	055229	124652	185	875348	38
23	820263	239	179737	945026	424	054974	124763	185	875237	37
24	820406	239	179594	945281	424	054719	124874	186	875126	36
25	820550	238	179450	945535	424	054465	124986	186	875014	35
26	820693	238	179307	945790	424	054210	125097	186	874903	34
27	820836	238	179164	946045	424	053955	125209	186	874791	33
28	820979	238	179021	946299	424	053701	125320	186	874680	32
29	821122	238	178878	946554	424	053446	125432	186	874568	31
30	821265	238	178735	946808	424	053192	125544	186	874456	30
31	9.821407	238	10.178593	9.947063	424	10.052937	10.125656	186	9.874344	29
32	821550	238	178450	947318	424	052682	125768	187	874232	28
33	821693	237	178307	947572	424	052428	125879	187	674121	27
34	821835	237	178165	947826	424	052174	125991	187	874009	26
35	821977	237	178023	948081	424	051919	126104	187	873896	25
36	822120	237	177880	948336	424	051664	126216	187	873784	24
37	822262	237	177738	948590	424	051410	126328	187	873672	23
38	822404	237	177596	948844	424	051156	126440	187	873560	22
39	822546	237	177454	949099	424	050901	126552	187	873448	21
40	822688	236	177312	949353	424	050647	126665	187	873335	20
41	9.822830	236	10.177170	9.949607	424	10.050393	10.126777	187	9.873223	19
42	822972	236	177028	949862	424	050138	126890	188	873110	18
43	823114	236	176886	950116	424	049884	127002	188	872998	17
44	823255	236	176745	950370	424	049630	127115	188	872885	16
45	823397	236	176603	950625	424	049375	127228	188	872772	15
46	823539	236	176461	950879	424	049121	127341	188	872659	14
47	823680	235	176320	951133	424	048867	127453	188	872547	13
48	823821	235	176179	951388	424	048612	127566	188	872434	12
49	823963	235	176037	951642	424	048358	127679	188	872321	11
50	824104	235	175896	951896	424	048104	127792	188	872208	10
51	9.824245	235	10.175755	9.952150	424	10.047850	10.127905	189	9.872095	9
52	824386	235	175614	952405	424	047595	128019	189	871981	8
53	824527	235	175473	952659	424	047341	128132	189	871868	7
54	824668	234	175332	952913	424	047087	128245	189	871755	6
55	824808	234	175192	953167	423	046833	128359	189	871641	5
56	824949	234	175051	953421	423	046579	128472	189	871528	4
57	825090	234	174910	953675	423	046325	128586	189	871414	3
58	825230	234	174770	953929	423	046071	128699	189	871301	2
59	825371	234	174629	954183	423	045817	128813	189	871187	1
60	825511	234	174489	954437	423	045563	128927	190	871073	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.808067	251	10.191933	9.923813	427	10.076187	10.115746	177	9.884254	60
1	808218	251	191782	924070	427	075930	115852	177	884148	59
2	808368	251	191632	924327	427	075673	115958	177	884042	58
3	808519	250	191481	924583	427	075417	116064	177	883936	57
4	808669	250	191331	924840	427	075160	116171	177	883829	56
5	808819	250	191181	925096	427	074904	116277	177	883723	55
6	808969	250	191031	925352	427	074648	116383	177	883617	54
7	809119	250	190881	925609	427	074391	116490	177	883510	53
8	809269	250	190731	925865	427	074135	116596	177	883404	52
9	809419	249	190581	926122	427	073878	116703	178	883297	51
10	809569	249	190431	926378	427	073622	116809	178	883191	50
11	9.809718	249	10.190282	9.926634	427	10.073366	10.116916	178	9.883084	49
12	809868	249	190132	926890	427	073110	117023	178	882977	48
13	810017	249	189983	927147	427	072853	117129	178	882871	47
14	810167	249	189833	927403	427	072597	117236	178	882764	46
15	810316	248	189684	927659	427	078341	117343	178	882657	45
16	810465	248	189535	927915	427	072085	117450	178	882550	44
17	810614	248	189386	928171	427	071829	117557	178	882443	43
18	810763	248	189237	928427	427	071573	117664	179	882336	42
19	810912	248	189088	928683	427	071317	117771	179	882229	41
20	811061	248	188939	928940	427	071060	117879	179	882121	40
21	9.811210	248	10.188790	9.929196	427	10.070804	10.117986	179	9.882014	39
22	811358	247	188642	929452	427	070548	118093	179	881907	38
23	811507	247	188493	929708	427	070292	118201	179	881799	37
24	811655	247	188345	929964	426	070036	118308	179	881692	36
25	811804	247	188196	930220	426	069780	118416	179	881584	35
26	811952	247	188048	930475	426	069525	118523	179	881477	34
27	812100	247	187900	930731	426	069269	118631	179	881369	33
28	812248	247	187752	930987	426	069013	118739	180	881261	32
29	812396	246	187604	931243	426	068757	118847	180	881153	31
30	812544	246	187456	931499	426	068501	118954	180	881046	30
31	9.812692	246	10.187308	9.931755	426	10.068245	10.119062	180	9.880938	29
32	812840	246	187160	932010	426	067990	119170	180	880830	28
33	812988	246	187012	932266	426	067734	119278	180	880722	27
34	813135	246	186865	932522	426	067478	119387	180	880613	26
35	813283	246	186717	932778	426	067222	119495	180	880505	25
36	813430	245	186570	933033	426	066967	119603	180	880397	24
37	813578	245	186422	933289	426	066711	119711	181	880289	23
38	813725	245	186275	933545	426	066455	119820	181	880180	22
39	813872	245	186128	933800	426	066200	119928	181	880072	21
40	814019	245	185981	934056	426	065944	120037	181	879963	20
41	9.814166	245	10.185834	9.934311	426	10.065689	10.120145	181	9.879855	19
42	814313	245	185687	934567	426	065433	120254	181	879746	18
43	814460	244	185540	934823	426	065177	120363	181	879637	17
44	814607	244	185393	935078	426	064922	120471	181	879529	16
45	814753	244	185247	935333	426	064667	120580	181	879420	15
46	814900	244	185100	935589	426	064411	120689	181	879311	14
47	815046	244	184954	935844	426	064156	120798	182	879202	13
48	815193	244	184807	936100	426	063900	120907	182	879093	12
49	815339	244	184661	936355	426	063645	121016	182	878984	11
50	815485	243	184515	936610	426	063390	121125	182	878875	10
51	9.815631	243	10.184368	9.936866	425	10.063134	10.121234	182	9.878766	9
52	815778	243	184222	937121	425	062879	121344	182	878656	8
53	815924	243	184076	937376	425	062624	121453	182	878547	7
54	816060	243	183931	937632	425	062368	121562	182	878438	6
55	816215	243	183785	937887	425	062113	121672	182	878328	5
56	816361	243	183639	938142	425	061858	121781	183	878219	4
57	816507	242	183493	938398	425	061602	121891	183	878109	3
58	816652	242	183348	938653	425	061347	122001	183	877999	2
59	816798	242	183202	938908	425	061092	122110	183	877890	1
60	816943	242	183057	939163	425	060837	122220	183	877780	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.816943	242	10.183057	9.939163	425	10.060837	10.122220	183	9.877780	60
1	817088	242	182912	939418	425	060582	122330	183	877670	59
2	817233	242	182767	939673	425	060327	122440	183	877560	58
3	817379	242	182621	939928	425	060072	122550	183	877450	57
4	817524	241	182476	940183	425	059817	122660	183	877340	56
5	817668	241	182332	940438	425	059562	122770	184	877230	55
6	817813	241	182187	940694	425	059306	122880	184	877120	54
7	817958	241	182042	940949	425	059051	122990	184	877010	53
8	818103	241	181897	941204	425	058796	123101	184	876899	52
9	818247	241	181753	941458	425	058542	123211	184	876789	51
10	818392	241	181608	941714	425	058286	123322	184	876678	50
11	9.818536	240	10.181464	9.941968	425	10.058032	10.123432	184	9.876568	49
12	818681	240	181319	942223	425	057777	123543	184	876457	48
13	818825	240	181175	942478	425	057522	123653	184	876347	47
14	818969	240	181031	942733	425	057267	123764	185	876236	46
15	819113	240	180887	942988	425	057012	123875	185	876125	45
16	819257	240	180743	943243	425	056757	123986	185	876014	44
17	819401	240	180599	943498	425	056502	124096	185	875904	43
18	819545	239	180455	943752	425	056248	124207	185	875793	42
19	819689	239	180311	944007	425	055993	124318	185	875682	41
20	819832	239	180168	944262	425	055738	124429	185	875571	40
21	9.819976	239	10.180024	9.944517	425	10.055483	10.124541	185	9.875459	39
22	820120	239	179880	944771	424	055229	124652	185	875348	38
23	820263	239	179737	945026	424	054974	124763	185	875237	37
24	820406	239	179594	945281	424	054719	124874	186	875126	36
25	820550	238	179450	945535	424	054465	124986	186	875014	35
26	820693	238	179307	945790	424	054210	125097	186	874903	34
27	820836	238	179164	946045	424	053955	125209	186	874791	33
28	820979	238	179021	946299	424	053701	125320	186	874680	32
29	821122	238	178878	946554	424	053446	125432	186	874568	31
30	821265	238	178735	946808	424	053192	125544	186	874456	30
31	9.821407	238	10.178593	9.947063	424	10.052937	10.125656	186	9.874344	29
32	821550	238	178450	947318	424	052682	125768	187	874232	28
33	821693	237	178307	947572	424	052428	125879	187	674121	27
34	821835	237	178165	947826	424	052174	125991	187	874009	26
35	821977	237	178023	948081	424	051919	126104	187	873896	25
36	822120	237	177880	948336	424	051664	126216	187	873784	24
37	822262	237	177738	948590	424	051410	126328	187	873672	23
38	822404	237	177596	948844	424	051156	126440	187	873560	22
39	822546	237	177454	949099	424	050901	126552	187	873448	21
40	822688	236	177312	949353	424	050647	126665	187	873335	20
41	9.822830	236	10.177170	9.949607	424	10.050393	10.126777	187	9.873223	19
42	822972	236	177028	949862	424	050138	126890	188	873110	18
43	823114	236	176886	950116	424	049884	127002	188	872998	17
44	823255	236	176745	950370	424	049630	127115	188	872885	16
45	823397	236	176603	950625	424	049375	127228	188	872772	15
46	823539	236	176461	950879	424	049121	127341	188	872659	14
47	823680	235	176320	951133	424	048867	127453	188	872547	13
48	823821	235	176179	951388	424	048612	127566	188	872434	12
49	823963	235	176037	951642	424	048358	127679	188	872321	11
50	824104	235	175896	951896	424	048104	127792	188	872208	10
51	9.824245	235	10.175755	9.952150	424	10.047850	10.127905	189	9.872095	9
52	824386	235	175614	952405	424	047595	128019	189	871981	8
53	824527	235	175473	952659	424	047341	128132	189	871868	7
54	824668	234	175332	952913	424	047087	128245	189	871755	6
55	824808	234	175192	953167	423	046833	128359	189	871641	5
56	824949	234	175051	953421	423	046579	128472	189	871528	4
57	825090	234	174910	953675	423	046325	128586	189	871414	3
58	825230	234	174770	953929	423	046071	128699	189	871301	2
59	825371	234	174629	954183	423	045817	128813	189	871187	1
60	825511	234	174489	954437	423	045563	128927	190	871073	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.825511	234	10.174489	9.954437	423	10.045563	10.128927	190	9.871073	60
1	825651	233	174349	954691	423	045309	129040	190	870960	59
2	825791	233	174209	954945	423	045055	129154	190	870846	58
3	825931	233	174069	955200	423	044800	129268	190	870732	57
4	826071	233	173929	955454	423	044546	129382	190	870618	56
5	826211	233	173789	955707	423	044293	129496	190	870504	55
6	826351	233	173649	955961	423	044039	129610	190	870390	54
7	826491	233	173509	956215	423	043785	129724	190	870276	53
8	826631	233	173369	956469	423	043531	129839	190	870161	52
9	826770	232	173230	956723	423	043277	129953	191	870047	51
10	826910	232	173090	956977	423	043023	130067	191	869933	50
11	9.827049	232	10.172951	9.957231	423	10.042769	10.130182	191	9.869818	49
12	827189	232	172811	957485	423	042515	130296	191	869704	48
13	827328	232	172672	957739	423	042261	130411	191	869589	47
14	827467	232	172533	957993	423	042007	130526	191	869474	46
15	827606	232	172394	958246	423	041754	130640	191	869360	45
16	827745	232	172255	958500	423	041500	130755	191	869245	44
17	827884	231	172116	958754	423	041246	130870	191	869130	43
18	828023	231	171977	959008	423	040992	130985	192	869015	42
19	828162	231	171838	959262	423	040738	131100	192	868900	41
20	828301	231	171699	959516	423	040484	131215	192	868785	40
21	9.828439	231	10.171561	9.959769	423	10.040231	10.131330	192	9.868670	39
22	828578	231	171422	960023	423	039977	131445	192	868555	38
23	828716	231	171284	960277	423	039723	131560	192	868440	37
24	828855	230	171145	960531	423	039469	131676	192	868324	36
25	828993	230	171007	960784	423	039216	131791	192	868209	35
26	829131	230	170869	961038	423	038962	131907	192	868093	34
27	829269	230	170731	961291	423	038709	132022	193	867978	33
28	829407	230	170593	961545	423	038455	132138	193	867862	32
29	829545	230	170455	961799	423	038201	132253	193	867747	31
30	829683	230	170317	962052	423	037948	132369	193	867631	30
31	9.829821	229	10.170179	9.962306	423	10.037694	10.132485	193	9 867515	29
32	829959	229	170041	962560	423	037440	132601	193	867399	28
33	830097	229	169903	962813	423	037187	132717	193	867283	27
34	830234	229	169766	963067	423	036933	132833	193	867167	26
35	830372	229	169628	963320	423	036680	132949	193	867051	25
36	830509	229	169491	963574	423	036426	133065	194	866935	24
37	830646	229	169354	963827	423	036173	133181	194	866819	23
38	830784	229	169216	964081	423	035919	133297	194	866703	22
39	830921	228	169079	964335	423	035665	133414	194	866586	21
40	831058	228	168942	964588	422	035412	133530	194	866470	20
41	9.831195	228	10.168805	9 964842	422	10.035158	10.133647	194	9.866353	19
42	831332	228	168668	965095	422	034905	133763	194	866237	18
43	831469	228	168531	965349	422	034651	133880	194	866120	17
44	831606	228	168394	965602	422	034398	133996	195	866004	16
45	831742	228	168258	965855	422	034145	134113	195	865887	15
46	831879	228	168121	966109	422	033891	134230	195	865770	14
47	832015	227	167985	966362	422	033638	134347	195	865653	13
48	832152	227	167848	966616	422	033384	134464	195	865536	12
49	832288	227	167712	966869	422	033131	134581	195	865419	11
50	832425	227	167575	967123	422	032877	134698	195	865302	10
51	9.832561	227	10.167439	9.967376	422	10.032624	10.134815	195	9.865185	9
52	832697	227	167303	967629	422	032371	134932	195	865068	8
53	832833	227	167167	967883	422	032117	135050	195	864950	7
54	832969	226	167031	968136	422	031864	135167	196	864833	6
55	833105	226	166895	968389	422	031611	135284	196	864716	5
56	833241	226	166759	968643	422	031357	135402	196	864598	4
57	833377	226	166623	968896	422	031104	135519	196	864481	3
58	833512	226	166488	969149	422	030851	135637	196	864363	2
59	833648	226	166352	969403	422	030597	135755	196	864245	1
60	833783	226	166217	969656	422	030344	135873	196	864127	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang.	D.	Cotang.	Secant	D.	Cosine	
0	9.833783	226	10.166217	9.969656	422	10.030344	10.135873	196	9.864127	60
1	833919	225	166081	969909	422	030091	135990	196	864010	59
2	834054	225	165946	970162	422	029838	136108	197	863892	58
3	834189	225	165811	970416	422	029584	136226	197	863774	57
4	834325	225	165675	970669	422	029331	136344	197	863656	56
5	834460	225	165540	970922	422	029078	136462	197	863538	55
6	834595	225	165405	971175	422	028825	136581	197	863419	54
7	834730	225	165270	971429	422	028571	136699	197	863301	53
8	834865	225	165135	971682	422	028318	136817	197	863183	52
9	834999	224	165001	971935	422	028065	136936	197	863064	51
10	835134	224	164866	972188	422	027812	137054	198	862946	50
11	9.835269	224	10.164731	9.972441	422	10.027559	10.137173	198	9.862827	49
12	835403	224	164597	972694	422	027306	137291	198	862709	48
13	835538	224	164462	972948	422	027052	137410	198	862590	47
14	835672	224	164328	973201	422	026799	137529	198	862471	46
15	835807	224	164193	973454	422	026546	137647	198	862353	45
16	835941	224	164059	973707	422	026293	137766	198	862234	44
17	836075	223	163925	973960	422	026040	137885	198	862115	43
18	836209	223	163791	974213	422	025787	138004	198	861996	42
19	836343	223	163657	974466	422	025534	138123	198	861877	41
20	836477	223	163523	974719	422	025281	138242	198	861758	40
21	9.836611	223	10.163389	9.974973	422	10.025027	10.138362	199	9.861638	39
22	836745	223	163255	975226	422	024774	138481	199	861519	38
23	836878	223	163122	975479	422	024521	138600	199	861400	37
24	837012	222	162988	975732	422	024268	138720	199	861280	36
25	837146	222	162854	975985	422	024015	138839	199	861161	35
26	837279	222	162721	976238	422	023762	138959	199	861041	34
27	837412	222	162588	976491	422	023509	139078	199	860922	33
28	837546	222	162454	976744	422	023256	139197	199	860802	32
29	837679	222	162321	976997	422	023003	139318	200	860682	31
30	837812	222	162188	977250	422	022750	139338	200	860562	30
31	9.837945	222	10.162055	9.977503	422	10.022497	10.139558	200	9.860442	29
32	838078	221	161922	977756	422	022244	139678	200	860322	28
33	838211	221	161789	978009	422	021991	139798	200	860202	27
34	838344	221	161656	978262	422	021738	139918	200	860082	26
35	838477	221	161523	978515	422	021485	140038	200	859962	25
36	838610	221	161390	978768	422	021232	140158	200	859842	24
37	838742	221	161258	979021	422	020979	140279	201	859721	23
38	838875	221	161125	979274	422	020726	140399	201	859601	22
39	839007	221	160993	979527	422	020476	140520	201	859480	21
40	839140	220	160860	979780	422	020220	140640	201	859360	20
41	9.839272	220	10.160728	9.980033	422	10 019967	10.140761	201	9.859239	19
42	839404	220	160596	980286	422	019714	140881	201	859119	18
43	839536	220	160464	980538	422	019462	141003	201	858998	17
44	839668	220	160332	980791	421	019209	141123	201	858877	16
45	839800	220	160200	981044	421	018956	141244	202	858756	15
46	839932	220	160068	981297	421	018703	141365	202	858635	14
47	840064	219	159936	981550	421	018450	141486	202	858514	13
48	840196	219	159804	981803	421	018197	141607	202	858393	12
49	840328	219	159672	982056	421	017944	141728	202	858272	11
50	840459	219	159541	982309	421	017691	141849	202	858151	10
51	9.840591	219	10.159409	9.982562	421	10.017438	10.141971	202	9.858029	9
52	840722	219	159278	982814	421	017186	142092	202	857908	8
53	840854	219	159146	983067	421	016933	142214	202	857786	7
54	840985	219	159015	983320	421	016680	142335	203	857665	6
55	841116	218	158884	983573	421	016427	142457	203	857543	5
56	841247	218	158753	983826	421	016174	142578	203	857422	4
57	841378	218	158622	984079	421	015921	142700	203	857300	3
58	841509	218	158491	984331	421	015669	142822	203	857178	2
59	841640	218	158360	984584	421	015416	142944	203	857056	1
60	841771	218	158229	984837	421	015163	143066	203	856934	0
	Cosine		Secant	Cotang.		Tang.	Cosec.		Sine	M.

M.	Sine	D.	Cosec.	Tang	D.	Cotang.	Secant	D.	Cosine	
0	9.841771	218	10.158229	9.984837	421	10.015163	10.143066	203	9.856934	60
1	841902	218	158098	985090	421	014910	143188	203	856812	59
2	842033	218	157967	985343	421	014657	143310	204	856690	58
3	842163	217	157837	985596	421	014404	143432	204	856568	57
4	842294	217	157706	985848	421	014152	143554	204	856446	56
5	842424	217	157576	986101	421	013899	143677	204	856323	55
6	842555	217	157445	986354	421	013646	143799	204	856201	54
7	842685	217	157315	986607	421	013393	143922	204	856078	53
8	842815	217	157185	986860	421	013140	144044	204	855956	52
9	842946	217	157054	987112	421	012888	144167	204	855833	51
10	843076	217	156924	987365	421	012635	144289	205	855711	50
11	9.843206	216	10.156794	9.987618	421	10.012382	10.144412	205	9.855588	49
12	843336	216	156664	987871	421	012129	144535	205	855465	48
13	843466	216	156534	988123	421	011877	144658	205	855342	47
14	843595	216	156405	988376	421	011624	144781	205	855219	46
15	843725	216	156275	988629	421	011371	144904	205	855096	45
16	843855	216	156145	988882	421	011118	145027	205	854973	44
17	843984	216	156016	989134	421	010866	145150	205	854850	43
18	844114	215	155886	989387	421	010613	145273	206	854727	42
19	844243	215	155757	989640	421	010360	145397	206	854603	41
20	844372	215	155628	989893	421	010107	145520	206	854480	40
21	9.844502	215	10.155498	9.990145	421	10.009855	10.145644	206	9.854356	39
22	844631	215	155369	990398	421	009602	145767	206	854233	38
23	844760	215	155240	990651	421	009349	145891	206	854109	37
24	844889	215	155111	990903	421	009097	146014	206	853986	36
25	845018	215	154982	991156	421	008844	146138	206	853862	35
26	845147	215	154853	991409	421	008591	146262	206	853738	34
27	845276	214	154724	991662	421	008338	146386	207	853614	33
28	845405	214	154595	991914	421	008086	146510	207	853490	32
29	845533	214	154467	992167	421	007833	146634	207	853366	31
30	845662	214	154338	992420	421	007580	146758	207	853242	30
31	9.845790	214	10.154210	9.992672	421	10.007328	10.146882	207	9.853118	29
32	845919	214	154081	992925	421	007075	147006	207	852994	28
33	846047	214	153953	993178	421	006822	147131	207	852869	27
34	846175	214	153825	993430	421	006570	147255	207	852745	26
35	846304	214	153696	993683	421	006317	147380	207	852620	25
36	846432	213	153568	993936	421	006064	147504	208	852496	24
37	846560	213	153440	994189	421	005811	147629	208	852371	23
38	836688	213	153312	994441	421	005559	147753	208	852247	22
39	846816	213	153184	994694	421	005306	147878	208	852122	21
40	846944	213	153056	994947	421	005053	148003	208	851997	20
41	9.847071	213	10.152929	9.995199	421	10.004801	10.148128	208	9.851872	19
42	847199	213	152801	995452	421	004548	148253	208	851747	18
43	847327	213	152673	995705	421	004295	148378	208	851622	17
44	847454	212	152546	995957	421	004043	148503	209	851497	16
45	847582	212	152418	996210	421	003790	148628	209	851372	15
46	837709	212	152291	996463	421	003537	148754	209	851246	14
47	847836	212	152164	996715	421	003285	148879	209	851121	13
48	847964	212	152036	996968	421	003032	149004	209	850996	12
49	848091	212	151909	997221	421	002779	149130	209	850870	11
50	848218	212	151782	997473	421	002527	149255	209	850745	10
51	9.848345	212	10 151655	9.997726	421	10.002274	10.149381	209	9.850619	9
52	848472	211	151528	997979	421	002021	149507	210	850493	8
53	848599	211	151401	998231	421	001769	149632	210	850368	7
54	848726	211	151274	998484	421	001516	149758	210	850242	6
55	848852	211	151148	998737	421	001263	149884	210	850116	5
56	848979	211	151021	998989	421	001011	150010	210	849990	4
57	849106	211	150894	999242	421	000758	150136	210	849864	3
58	849232	211	150768	999495	421	000505	150263	210	849738	2
59	849359	211	150641	999747	421	000253	150389	210	849611	1
60	849485	211	150515	10.00000	421	000000	150515	210	849485	0
	Cosine.		Secant	Cotang.		Tang.	Cosec.		Sine	M.

45 Degrees.

A TABLE

OF

NATURAL SINES.

A Table of Natural Sines.

M.	0 Deg. N.sin.	0 Deg. N.cos.	1 Deg. N.sin.	1 Deg. N.cos.	2 Deg. N.sin.	2 Deg. N.cos.	3 Deg. N.sin.	3 Deg. N.cos.	4 Deg. N.sin.	4 Deg. N.cos.	
0	0	100000	1745	99985	3490	99939	5234	99863	6976	99756	60
1	29	100000	1774	99984	3519	99938	5263	99861	7005	99754	59
2	58	100000	1803	99984	3548	99937	5292	99860	7034	99752	58
3	87	100000	1832	99983	3577	99936	5321	99858	7063	99750	57
4	116	100000	1862	99983	3606	99935	5350	99857	7092	99748	56
5	145	100000	1891	99982	3635	99934	5379	99855	7121	99746	55
6	175	100000	1920	99982	3664	99933	5408	99854	7150	99744	54
7	204	100000	1949	99981	3693	99932	5437	99852	7179	99742	53
8	233	100000	1978	99980	3723	99931	5466	99851	7208	99740	52
9	262	100000	2007	99980	3752	99930	5495	99849	7237	99738	51
10	291	100000	2036	99979	3781	99929	5524	99847	7266	99736	50
11	320	99999	2065	99979	3810	99927	5553	99846	7295	99734	49
12	349	99999	2094	99978	3839	99926	5582	99844	7324	99731	48
13	378	99999	2123	99977	3868	99925	5611	99842	7353	99729	47
14	407	99999	2152	99977	3897	99924	5640	99841	7382	99727	46
15	436	99999	2181	99976	3926	99923	5669	99839	7411	99725	45
16	465	99999	2211	99976	3955	99922	5698	99838	7440	99723	44
17	495	99999	2240	99975	3984	99921	5727	99836	7469	99721	43
18	524	99999	2269	99974	4013	99919	5756	99834	7498	99719	42
19	553	99998	2298	99974	4042	99918	5785	99833	7527	99716	41
20	582	99998	2327	99973	4071	99917	5814	99831	7556	99714	40
21	611	99998	2356	99972	4100	99916	5844	99829	7585	99712	39
22	640	99998	2385	99972	4129	99915	5873	99827	7614	99710	38
23	669	99998	2414	99971	4159	99913	5902	99826	7643	99708	37
24	698	99998	2443	99970	4188	99912	5931	99824	7672	99705	36
25	727	99997	2472	99969	4217	99911	5960	99822	7701	99703	35
26	756	99997	2501	99969	4246	99910	5989	99821	7730	99701	34
27	785	99997	2530	99968	4275	99909	6018	99819	7759	99699	33
28	814	99997	2560	99967	4304	99907	6047	99817	7788	99696	32
29	844	99996	2589	99966	4333	99906	6076	99815	7817	99694	31
30	873	99996	2618	99966	4362	99905	6105	99813	7846	99692	30
31	902	99996	2647	99965	4391	99904	6134	99812	7875	99689	29
32	931	99996	2676	99964	4420	99902	6163	99810	7904	99687	28
33	960	99995	2705	99963	4449	99901	6192	99808	7933	99685	27
34	989	99995	2734	99963	4478	99900	6221	99806	7962	99683	26
35	1018	99995	2763	99962	4507	99898	6250	99804	7991	99680	25
36	1047	99994	2792	99961	4536	99897	6279	99803	8020	99678	24
37	1076	99994	2821	99960	4565	99896	6308	99801	8049	99676	23
38	1105	99994	2850	99959	4594	99894	6337	99799	8078	99673	22
39	1134	99994	2879	99959	4623	99893	6366	99797	8107	99671	21
40	1164	99993	2908	99958	4653	99892	6395	99795	8136	99668	20
41	1193	99993	2938	99957	4682	99890	6424	99793	8165	99666	19
42	1222	99993	2967	99956	4711	99889	6453	99792	8194	99664	18
43	1251	99992	2996	99955	4740	99888	6482	99790	8223	99661	17
44	1280	99992	3025	99954	4769	99886	6511	99788	8252	99659	16
45	1309	99991	3054	99953	4798	99885	6540	99786	8281	99657	15
46	1338	99991	3083	99952	4827	99883	6569	99784	8310	99654	14
47	1367	99991	3112	99952	4856	99882	6598	99782	8339	99652	13
48	1396	99990	3141	99951	4885	99881	6627	99780	8368	99649	12
49	1425	99990	3170	99950	4914	99879	6656	99778	8397	99647	11
50	1454	99939	3199	99949	4943	99878	6685	99776	8426	99644	10
51	1483	99989	3228	99948	4972	99876	6714	99774	8455	99642	9
52	1513	99989	3257	99947	5001	99875	6743	99772	8484	99639	8
53	1542	99988	3286	99946	5030	99873	6773	99770	8513	99637	7
54	1571	99988	3316	99945	5059	99872	6802	99768	8542	99635	6
55	1600	99987	3345	99944	5088	99870	6831	99766	8571	99632	5
56	1629	99987	3374	99943	5117	99869	6860	99764	8600	99630	4
57	1658	99986	3403	99942	5146	99867	6889	99762	8629	99627	3
58	1687	99986	3432	99941	5175	99866	6918	99760	8658	99625	2
59	1716	99985	3461	99940	5205	99864	6947	99758	8687	99622	1
60	1745	99985	3490	99939	5234	99863	6976	99756	8716	99619	0
	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin	N.cos.	N.sin.	M.
	89 Deg.		88 Deg.		87 Deg.		86 Deg.		85 Deg.		

A Table of Natural Sines.

	5 Deg.		6 Deg.		7 Deg.		8 Deg.		9 Deg.		
M.	N.sin.	N.cos.	N.sin	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	
0	8716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	8745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	8774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	8803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	8831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	8860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	8889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	8918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	8947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	8976	99596	10713	99424	12447	99222	14177	98990	15902	98728	51
10	9005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	9034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	9063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	9092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	9121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	9150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	9179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	9208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	9237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	9266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	9295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	9324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	9353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	9382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	9411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	9440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	9469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	9498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	9527	99545	11263	99364	12995	99152	14723	98910	16447	98638	32
29	9556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	9585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	9614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	9642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	9671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	9700	99528	11436	99344	13168	99129	14896	98884	16620	98609	26
35	9729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	9758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	9787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	9816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	9845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	9874	99511	11609	99324	13341	99106	15069	98858	16792	98580	20
41	9903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	9932	99506	11667	99317	13399	99098	15126	98849	16849	98570	18
43	9961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	9990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15299	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	98531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99655	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
60	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	0
	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	M.
	84 Deg.		83 Deg.		82 Deg.		81 Deg.		80 Deg.		

A Table of Natural Sines.

	10 Deg.		11 Deg.		12 Deg.		13 Deg.		14 Deg.		
M.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	47424	24249	97015	58
3	17451	98466	19167	08146	20877	87797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	77754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96637	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24954	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
60	19081	98163	20791	97815	22495	97437	24192	97030	25882	96593	0
	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N cos.	N.sin.	N.cos.	N.sin.	M.
	79 Deg.		78 Deg.		77 Deg.		76 Deg.		75 Deg.		

	15 Deg.		16 Deg.		17 Deg.		18 Deg.		19 Deg.		
M.	N.sin.	N.cos.	N.sin	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96435	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	30265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94645	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	93009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
60	27564	96126	29237	95630	30902	95106	32557	94552	34202	93969	0
	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	M.
	74 Deg.		73 Deg.		72 Deg.		71 Deg.		70 Deg.		

A Table of Natural Sines.

M.	20 Deg.		21 Deg.		22 Deg.		23 Deg.		24 Deg.		
	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N. sin.	N.cos.	N. sin.	N.cos.	
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	01140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	.41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35184	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
60	35837	93358	37461	92718	39073	92050	40674	91355	42262	90631	0
	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N. sin.	N.cos.	N. sin.	M.
	69 Deg.		68 Deg.		67 Deg.		66 Deg.		65 Deg.		

M.	25 Deg. N. sin.	25 Deg. N. cos.	26 Deg. N. sin.	26 Deg. N. cos.	27 Deg. N. sin.	27 Deg. N. cos.	28 Deg. N. sin.	28 Deg. N. cos.	29 Deg. N. sin.	29 Deg. N. cos.	
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	33418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48254	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	64 Deg.		63 Deg.		62 Deg.		61 Deg.		60 Deg.		

X

	30 Deg.		31 Deg.		32 Deg.		33 Deg.		34 Deg.		
M.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	63228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	88131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0
	N.cos.	N.sin.	N.cos.	N.sin	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	M.
	59 Deg.		58 Deg.		57 Deg.		56 Deg.		55 Deg.		

	35 Deg.		36 Deg.		37 Deg.		38 Deg.		39 Deg.		
M.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57281	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78693	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	80713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	61645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	50669	80247	61069	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59762	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79016	62660	77934	64011	76828	12
49	58519	81089	59926	80059	61314	78998	62683	77916	64033	76810	11
50	56543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0
	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	M.
	54 Deg.		53 Deg.		52 Deg.		51 Deg.		50 Deg.		

A Table of Natural Sines.

	40 Deg.		41 Deg.		42 Deg		43 Deg.		44 Deg.		
M.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	79339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	N.cos.	N.sin.	N.cos.	N.sin	N.cos.	N.sin.	N.cos.	N.sin.	N.cos.	N.sin.	M.
	49 Deg.		48 Deg.		47 Deg.		46 Deg.		45 Deg.		

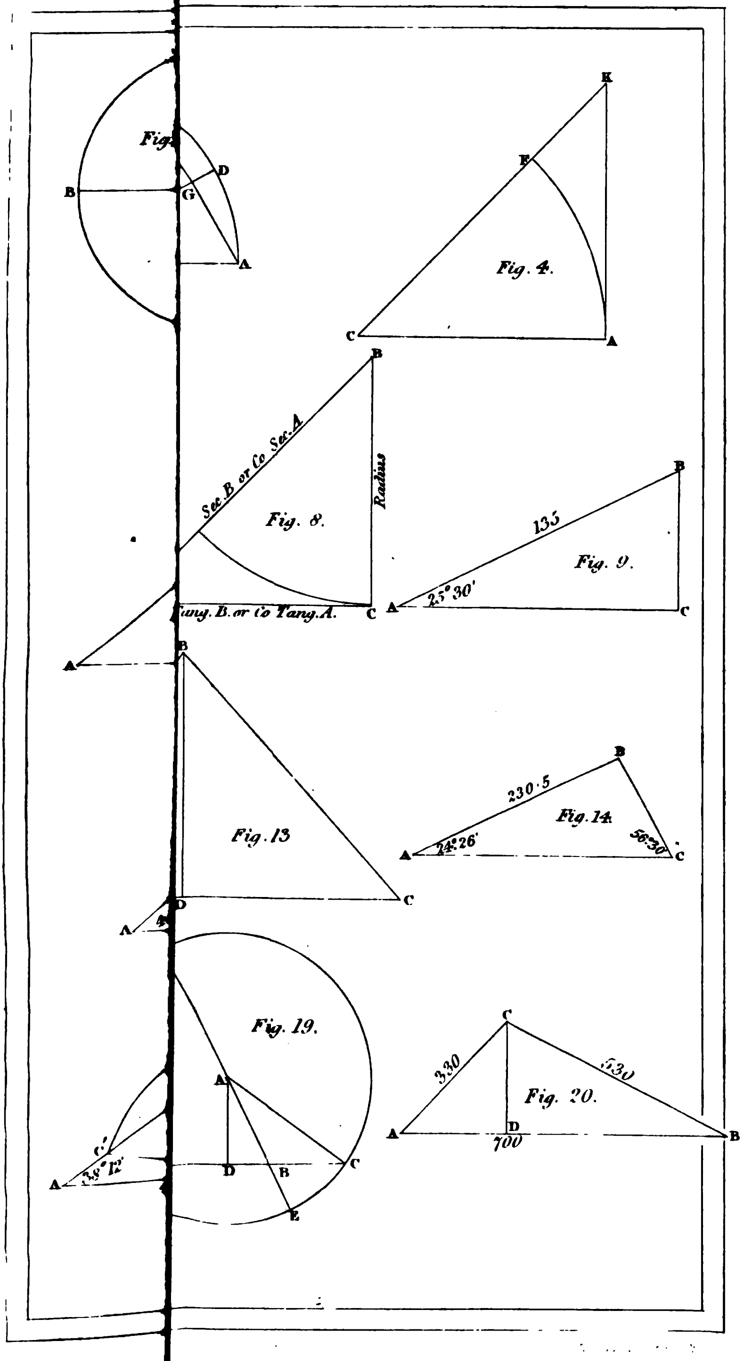

Fig.
B
G
D
A
K
F
Fig. 4.
C
A
B
Sec. B or Co Sec. A
Radius
Fig. 8.
Tang. B. or Co Tang. A.
C
A
B
135
Fig. 9.
25° 30'
A
C
B
Fig. 13
D
C
A
B
230·5
Fig. 14
24°.26'
56°30'
A
C
Fig. 19.
A
C
38° 12'
A
D
B
C
E
C
330
530
Fig. 20.
A
D
700
B

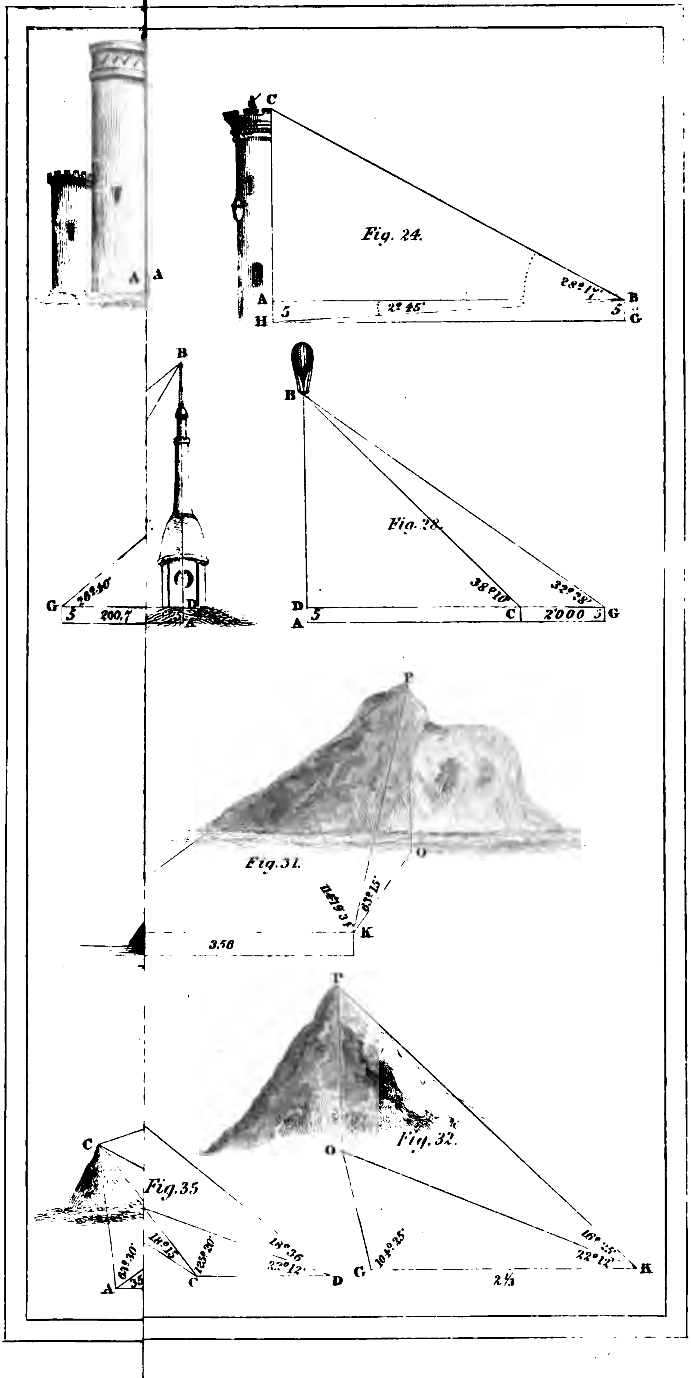
Fig. 24.
C
A
H
B
G
28°17'
2°45'
Fig. 28.
B
D
A
C
G
38°10'
32°28'
2000
200.7
Fig. 31.
P
O
K
63°15'
3.56
P
Fig. 32.
O
G
K
104°25'
22°12'
2⅓
C
Fig. 35
A
D
63°30'
125°20'
18°36'

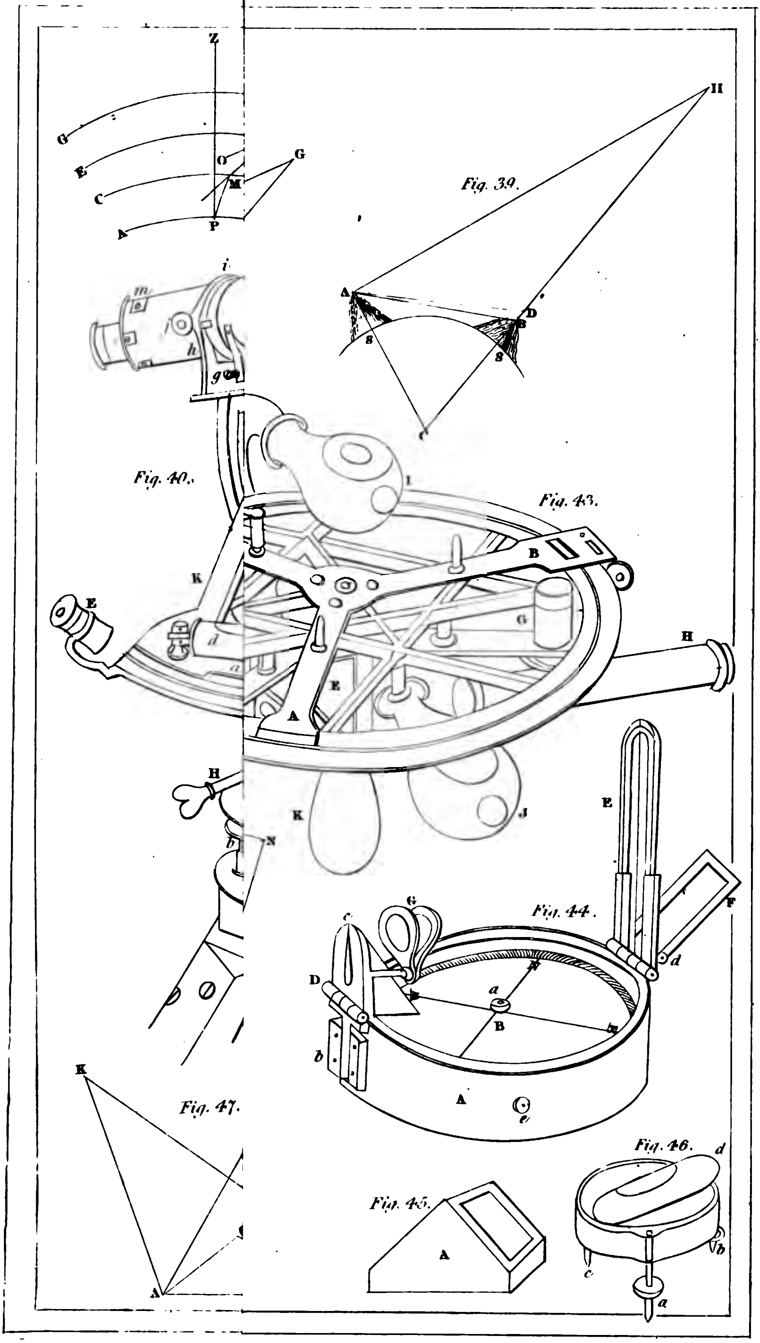
Fig. 39.
Fig. 40.
Fig. 43.
Fig. 44.
Fig. 45.
Fig. 46.
Fig. 47.

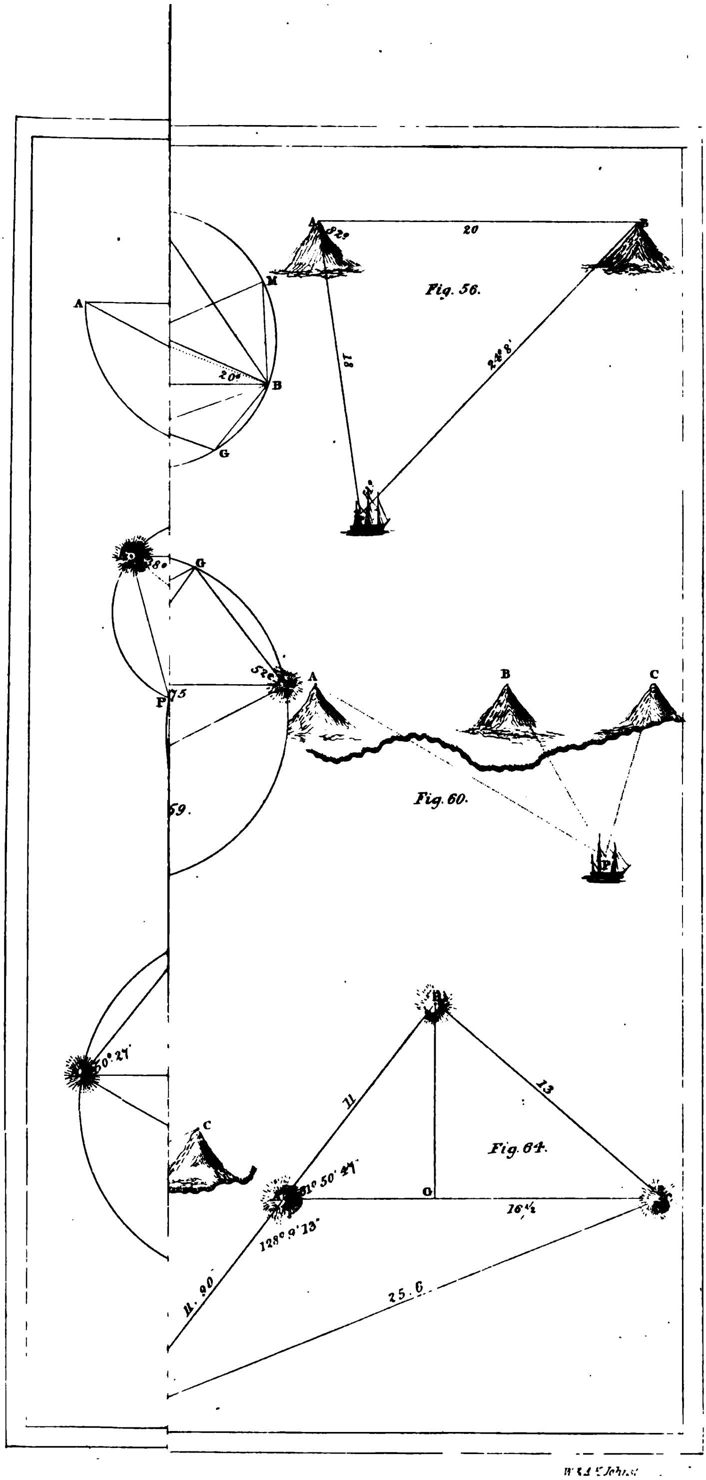
Fig. 56.
A
B
M
G
20
18
24° 8'
20°
Fig. 60.
A
B
C
P
75
69.
Fig. 64.
11
13
16½
25.6
11.90
51° 50' 47"
128° 9' 13"
50° 27'
C
G

W & A K Johnst

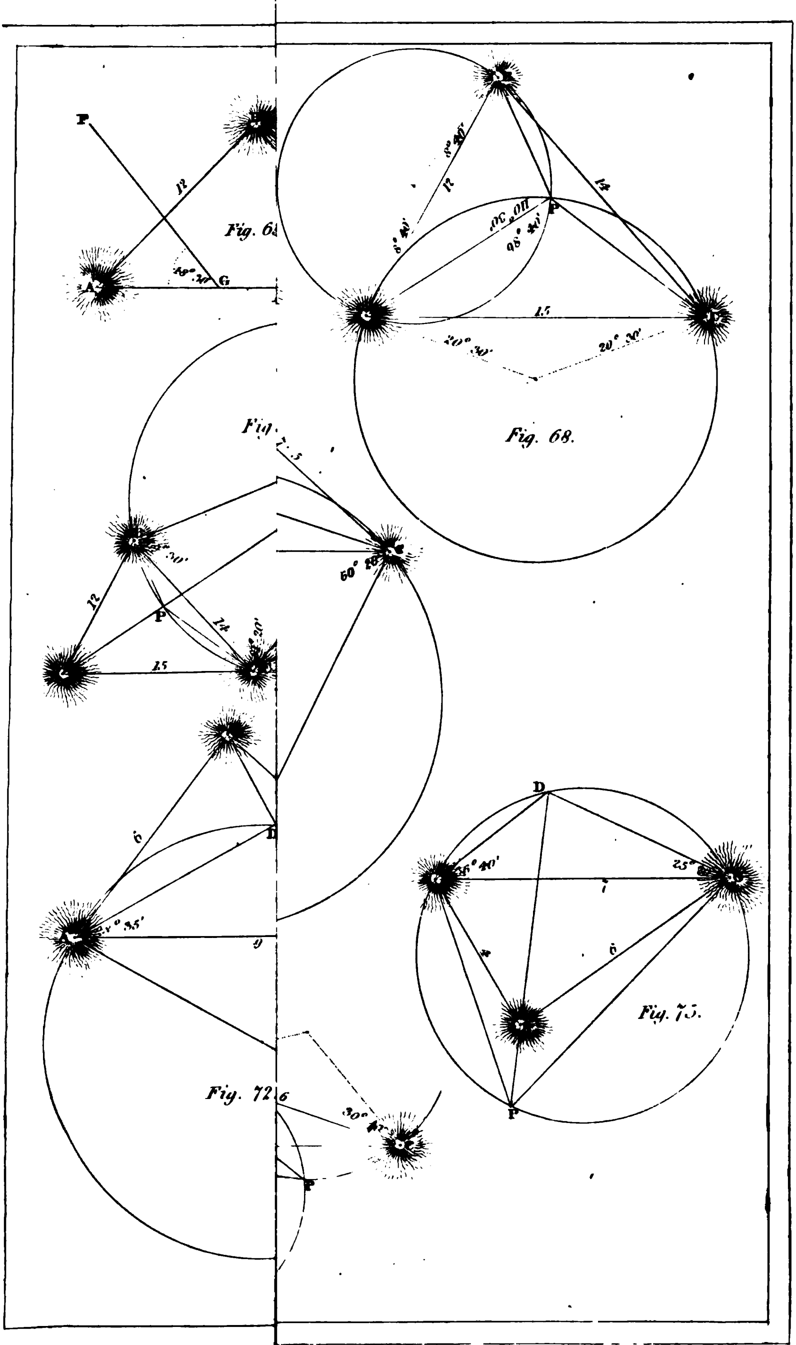

Fig. 68.

Fig. 72.

Fig. 75.

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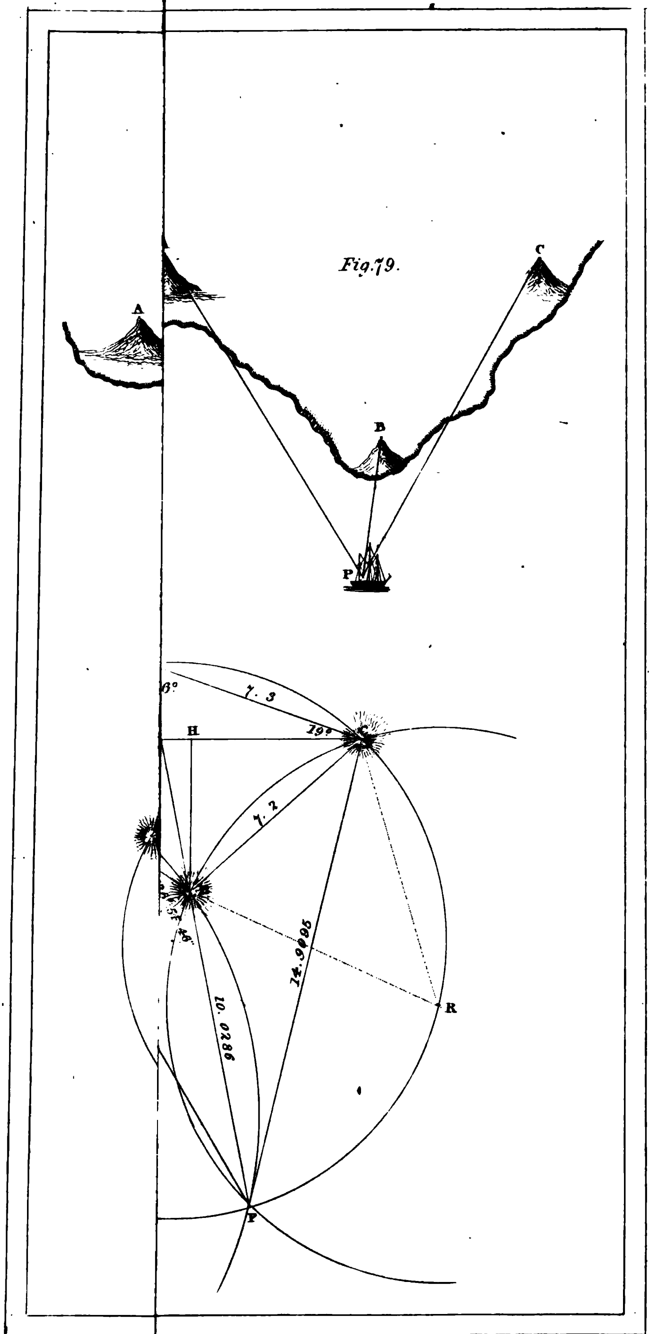

Fig. 79.

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PLATE 5.

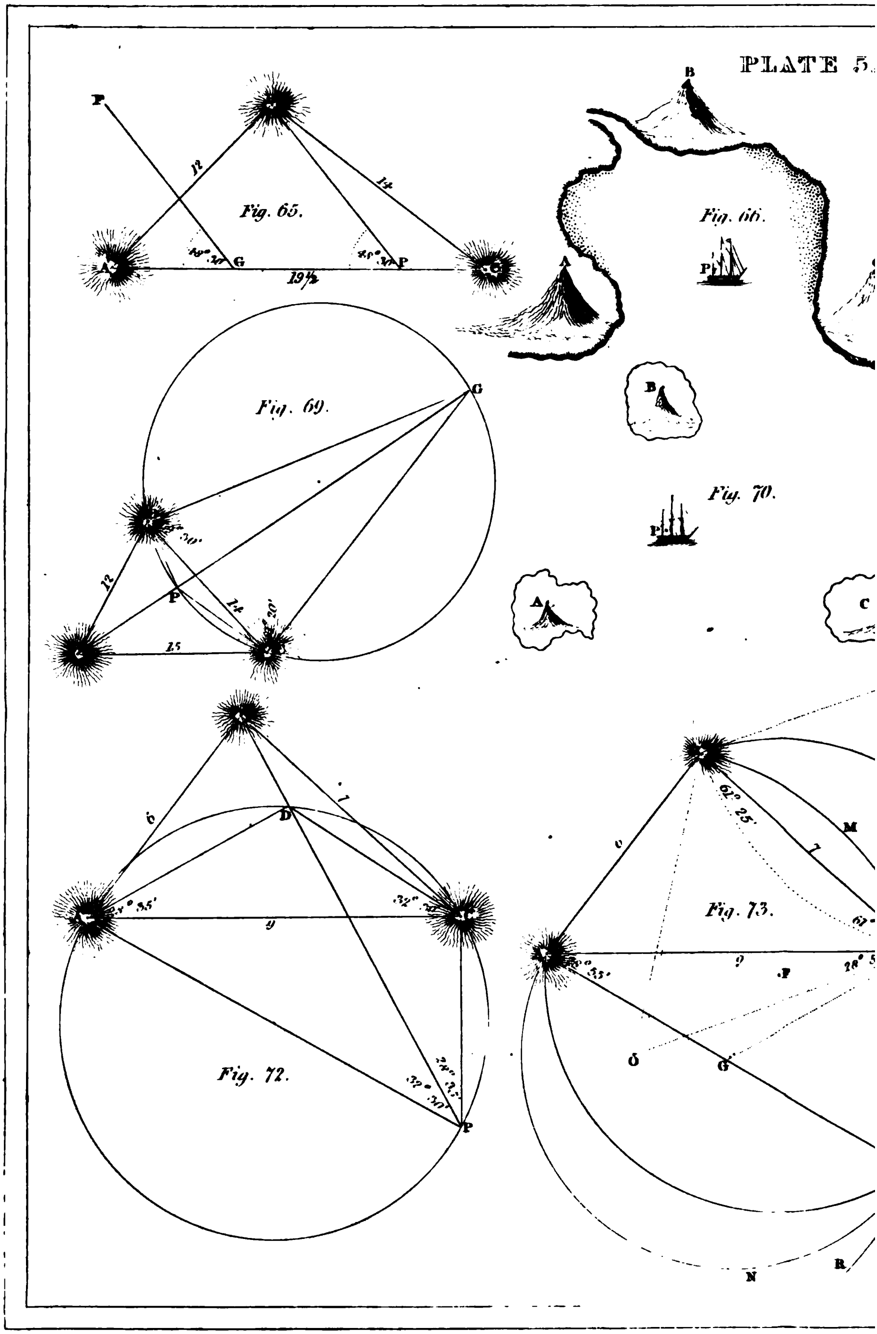


Fig. 65.
P
12
14
A
G
19½
P
C
B
Fig. 66.
P
A
Fig. 69.
G
30'
12
P
14
20'
15
B
Fig. 70.
P
A
C
6
7
D
9
P
Fig. 72.
32° 30'
62° 25'
M
Fig. 73.
9
P
O
G
N
R

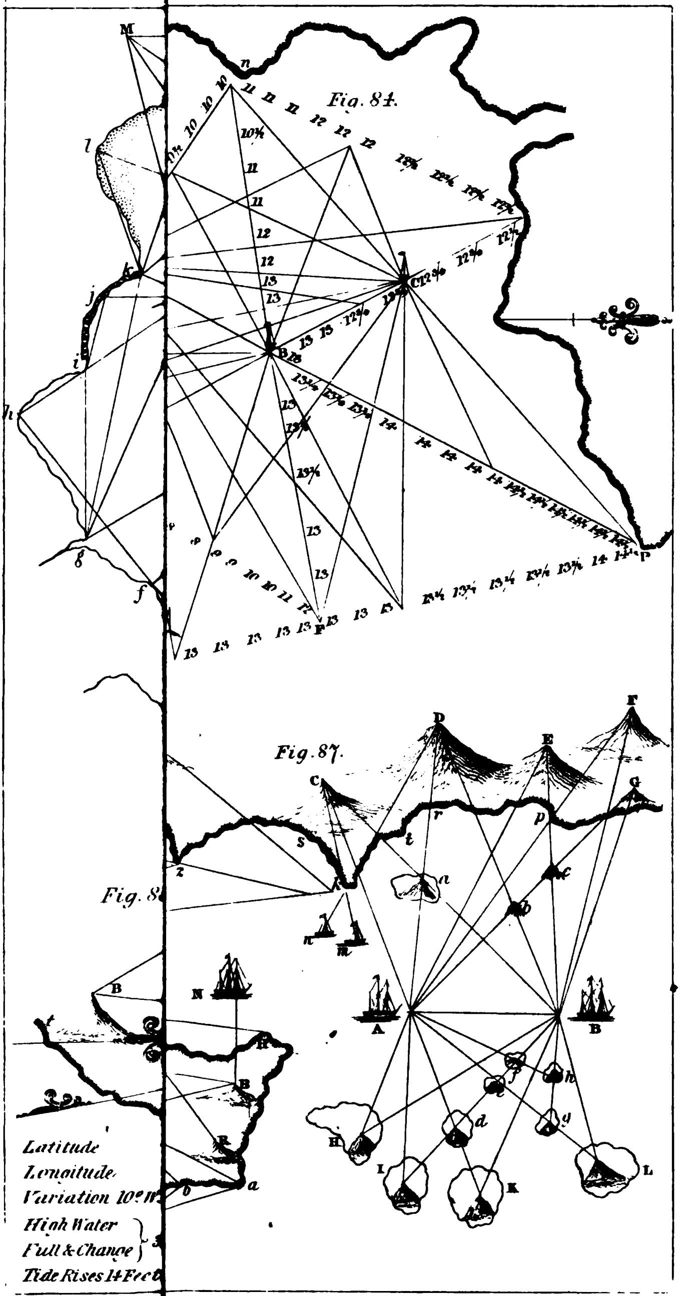
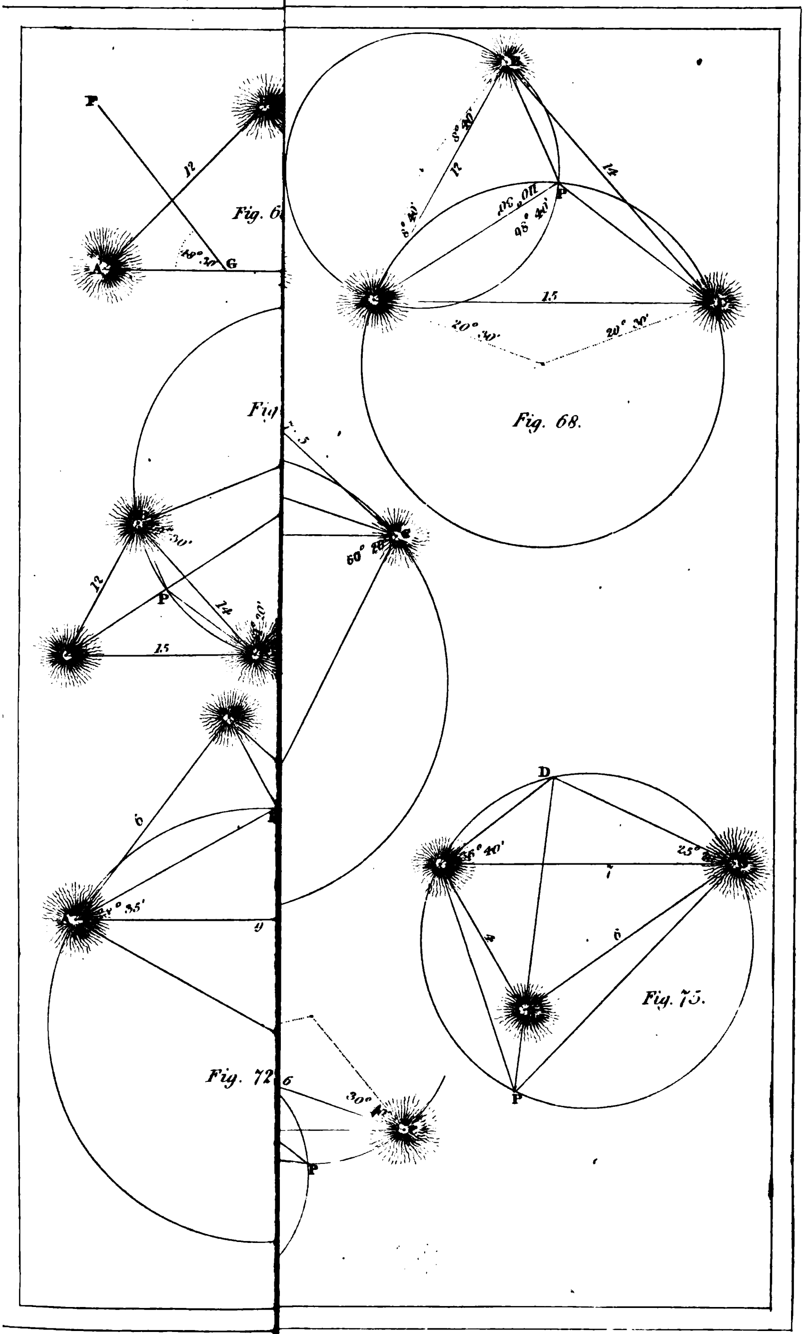


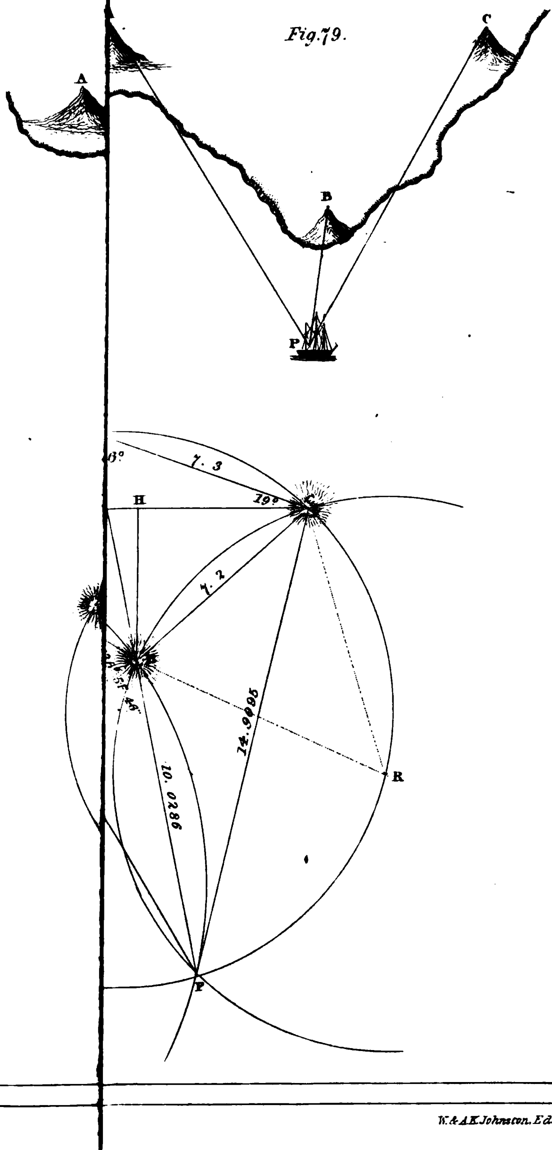
Fig. 84.
Fig. 87.
Fig. 8
Latitude
Longitude
Variation 10° W.
High Water
Full & Change
Tide Rises 14 Feet

W. & A.K.



W. & A. K. Johnston Edin

Fig. 79.



W. & A.K. Johnston, Edinr

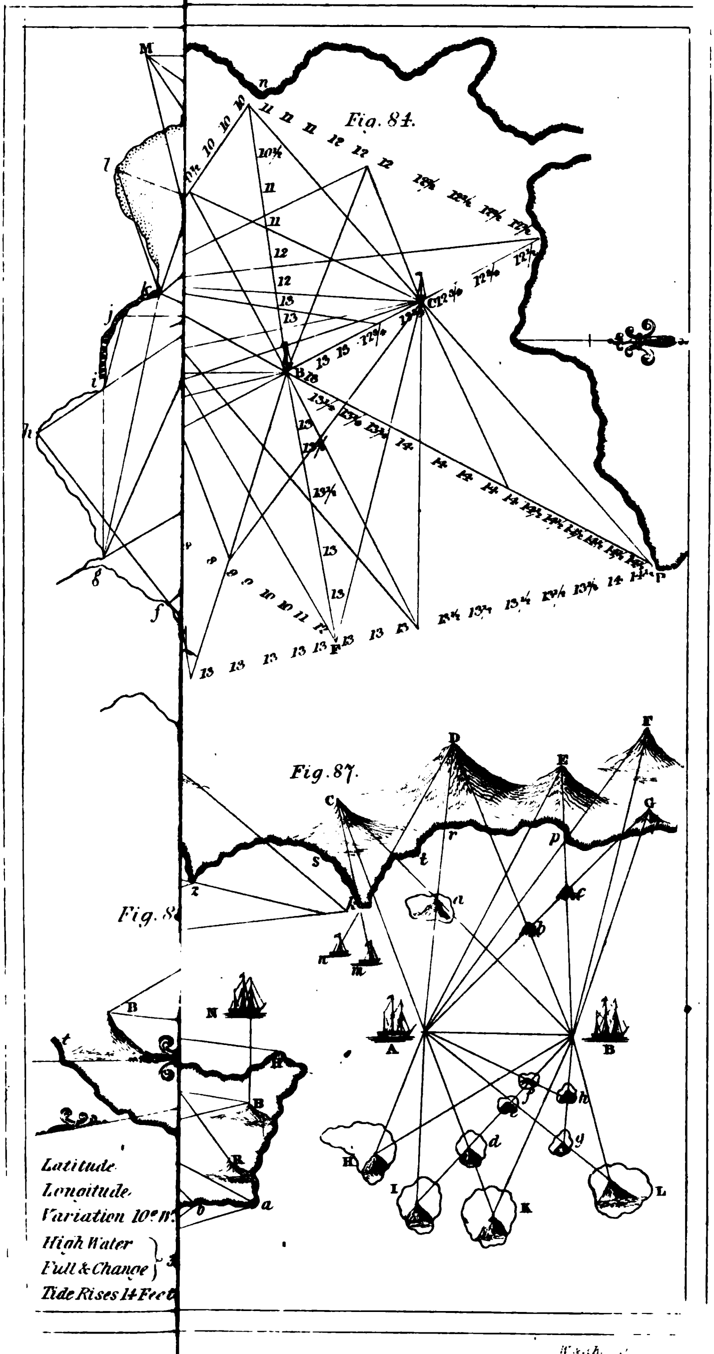


Fig. 84.
Fig. 87.
Fig. 8
Latitude
Longitude
Variation 10° W
High Water
Full & Change
Tide Rises 14 Feet